

Tipping in Two-Sided Software Markets:
An Investigation of Asymmetric Cost Differences

by

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Abstract

This paper models a two-sided market with two horizontally-differentiated platforms that conduct a game in prices. Platforms set only membership fees to both buyers and sellers. The game is single-stage as platforms choose prices and agents simultaneously choose which platform to join. Given the equilibrium prices to each side, the paper investigates the effects of inter-platform cost asymmetries. It finds that the magnitude of the effects of inter-platform cost differences on equilibrium membership depends on whether the asymmetric costs are borne by the platform or by developers on the platform. Under regimes with high network effects, differences in the platforms' costs of acquiring and serving customers have only limited impact, while the impact of cost differences for component production are less clear.

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Part I

Introduction

Networks of many types are essential for the successful functioning of firms and the well-being of consumers in the modern economy. Telephone and computer networks allow for the near-instantaneous communication that is the hallmark of the 21st century, social networks like Facebook, LinkedIn, and professional organizations like the American Economic Association help consumers and firms filter the incredible load of data at their fingertips, and virtual networks of users of the same types of technology make it possible to send digital files back and forth and purchase add-ons of all sorts. Frictionless transfers of data are a must in the information economy and networks of different sorts are the links that permit these interactions.

In many cases, markets involving networks exhibit *network effects*—the sometimes uninternalized impact that increasing the number of parties in one part of the market has on the expected utilities or expected profits of other agents. In general, network effects are beneficial¹ and markets with network effects often exhibit mutual reinforcement in decision-making and economies of scale in consumption. When markets include multiple incompatible networks, switching costs and consumer lock-in are probable and the self-reinforcing nature of network membership can easily lead to bandwagon effects, cascades, and tipping.

One important market for a network good is that for PC operating systems. The choice of an operating system relies in large part on software availability

¹Counter-examples, like telemarketers joining a telephone network, do exist (assuming that telephone users prefer not to get solicitation calls). However, network effects will be assumed positive for the duration of this paper.

and what other people are using, indicating the importance of network effects in the market. Furthermore, because an operating system acts as a portal to software and content, choosing an operating system applies an implicit constraint to users' behavior. Low levels of operating system inter-compatibility, difficulties in switching from one operating system to another, and the twenty-year dominion of Microsoft Windows in the home and business PC operating system markets make it obvious that switching costs, lock-in, and tipping are distinct possibilities in the market.

Computer operating system markets are part of a particular subset of markets with network effects known as *two-sided markets*. In a two-sided market, platforms must attract parties from two different groups (referred to as sides) in order to remain in the market. Without software on an operating system, it is almost totally useless; without consumers to buy software, there is no reason for developers to write software for a particular platform. In the case of a two-sided market, the network effects are mostly inter-side—consumers care about the presence of software developers and developers care about the number of end-users, but neither cares particularly strongly about the number of agents on their own side of the market.

In this paper, I model a two-sided market for PC operating systems² based on Armstrong's 2006 model where both sides join only one platform. Unlike Armstrong's work, which assumes away tipping, I carefully examine inter-platform differences in costs and evaluate the likelihood that the market will experience cascading and tipping to one side or the other. After a more careful introduction to network effects, two sided markets, and tipping, Part II provides a review of

²Although the model is based directly on PC operating systems, it clearly has broader applicability to any two-sided market with only membership fees where one side develops components that the other side consumes.

the literature and definition of relevant terms, Part III provides new analysis, Part IV concludes, and Part V includes appendices with mathematical proofs of the claims made throughout the paper.

1 Network Effects

Network effects (or network externalities) come in two main flavors, direct and indirect. Direct network effects involve the impact of two members being able to interact with each other. The canonical example of direct network effects (that although outdated, makes the idea crystal clear) is a telephone grid. As more users have telephone access, the potential benefit each user gains from being a network member increases as they can contact more people.³

Indirect network effects describe the value of one network member to another as a result of the expanded provision of complementary goods that accompanies an increase in network size. In a market with indirect network effects, firms that provide complementary goods have more potential consumers and higher potential profits as network membership increases. For example, there is a greater incentive for someone to provide higher-quality voicemail services as a telephone network grows since the voicemail firm can expect to reach a larger customer base. Even though the addition of one more party to the network does not directly provide better voicemail services, it increases incentives to provide those services to all users, making the telephone product itself more useful.

Two-sided computer operating system markets clearly feature both direct and indirect network effects. The market's weak intra-side network effects are direct—when the number of software developers on the platform goes up, the number

³Although recent research suggests that local network topology may be more important than total network size (See Swann 2002, Weitzel, Wendt, Westarp, and König 2003).

of programmers familiar with the platform increases, lowering production costs, and when the number of end-users goes up, more people are using compatible file formats, increasing their usefulness.

The market's stronger inter-side effects are indirect. Since developers of software products and end-users interact with each other to exchange money and software, the presence of more agents on one side indirectly increases utility or profits on that side. More users provides incentive for firms to develop increased software variety, assumed boost the valuation of the platform, and the presence of more software developers encourages end-users to join the network, increasing the potential profits of all of the software developers.

2 Two-Sided Markets

In general, a market is “two-sided” (or multi-sided) if potential utility or profit for one side of the market is affected by the other side's choices in membership or usage⁴. Canonical examples of products sold in two-sided markets include computer software, payment cards, and the services of heterosexual dating agencies and nightclubs⁵. Computer software developers can expect higher profits if there are more end-users on the platform and end-users prefer a higher level of software variety on their chosen hardware platform. Cardholders care about the number of merchants that honor cards they hold and merchants will only choose to accept a card if there are enough cardholders to justify the costs of adoption. The more potential partners using a matchmaking service or attending a nightclub, the greater the likelihood of a match and the more a potential user will value the agency or club. An essential element of these examples and of all two-sided markets is the

⁴For a more precise definition, see Section 5.1

⁵Obviously the example also works for gay nightclubs and dating agencies, but the fact that the market is trying to attract *two* distinct populations is not so immediately obvious.

platform or intermediary–computer operating systems, payment card companies, and nightclubs or dating services are obviously critical in their respective markets.

Since interaction between the two sides of the market is governed by the intermediary, its nature, role, and structure is of central importance in two-sided markets. The intermediary holds such a crucial position in two-sided markets that the literature generally focuses on the actions of the intermediary, particularly on how prices are set to each side (Rysman 2009). The pricing choices of the firms that produce computer hardware and operating systems, that provide payment cards, and that run nightclubs and dating agencies are obviously of crucial importance to the proper functioning of those markets.

3 Tipping

In markets with multiple incompatible networks, switching costs are often implied. Since utility and profit in network markets are increasing functions of platform membership, membership growth in networks tends to be self-reinforcing and highly asymmetric or even monopolistic structures can easily be the equilibrium market outcomes of competition. On a more basic level, when network effects exist in a market, “the resulting positive-feedback effects have proven troublesome to economic theory...in terms of market performance (the fundamental theorems of welfare economics may not apply).” (Katz and Shapiro 1994, 94). Since the Coase theorem does not apply, determination of the proper regulatory stance in these important markets is relatively difficult.

In the literature, when a market reaches an equilibrium where everyone joins only one of the networks or platforms, the market is called “tipped.” However, tipping is not just a theoretical outcome in economics papers; real world ex-

amples are plentiful. Recently, the market for high-definition blue laser video discs standardized on a format in a manner that can only be properly described as tipping. Neither Toshiba’s HD DVD format nor Sony’s Blu-Ray format garnered significant market traction until January 4, 2008 when Warner Bros. Entertainment announced they would be producing exclusively in the Blu-Ray format (Carnoy 2008). The commitment by Warner Bros. was enough to start a cascade of other retailers announcing that they were committing to Blu-Ray⁶, and it is now clear that Blu-Ray will be the only format for high-definition blue laser video discs going forward.⁷ It is hard to imagine that without the influence of network effects, one producer announcing which platform they will join would induce standardization in the market. However, the precise mechanism of how and when tipping occurs is ill-explored in the literature, an issue this paper works to rectify.

Part II

Literature Review

4 Network Effects

Katz and Shapiro (1985) develop a “simple, static model of oligopoly to analyze markets in which consumption externalities are present,” (Katz and Shapiro 1985, 425) in what is considered by many to be the first paper in the current body of literature on network effects. Their delineation of direct and indirect network externalities and their emphasis on compatibility and interoperability properly defining the borders of a network are central to the literature.

⁶Even with Toshiba’s prior commitment from Paramount to only release in HD DVD.

⁷It is interesting to note that this was Sony’s second go-around in a format war with a two-sided multimedia product, but only their first win. They lost the well-known VHS versus Betamax format competition to JVC’s VHS format almost 20 years earlier.

Although Katz and Shapiro used the term network externalities, the term network effect is preferred because some models have demonstrated that it is possible to have the same consumer behavior without network externalities. For example, Chou and Shy (1990) models a computer software market in which there are no network externalities but complementary products (software) exhibit increasing returns to scale in production because of high fixed costs and low marginal costs. They show that consumer behavior is qualitatively the same as in markets with network externalities.

Data on network effects is hard to obtain, as network markets often have only a few players, and these firms have proprietary data they wish not to share. However, some studies have tackled precisely that issue and have found significant evidence regarding the existence and importance of network effects in industries including PC software, fax machines, music and video media, mobile telecommunications, home video games, and electronic payment systems (Birke 2009). Of particular relevance to PC operating system markets, Gandal (1995) studies the value of file compatibility standards in the PC software market and finds that LOTUS 1-2-3 (an early spreadsheet application) provided significant help to IBM in the 1980s in selling PCs to businesses, indicating the presence of network effects in those markets. Furthermore, Goolsbee and Klenow (2002) examine home computer diffusion data and find support for the fact that local knowledge spillovers and network effects played a role in the diffusion of home computers. Beyond establishing the existence and influence of network effects, the bulk of the literature on network effects remains theoretical because of data availability issues.

Markets with network effects fall under the umbrella of activities where agents' strategies are mutually reinforcing. One agent's decision to behave in a particular way increasing the likelihood that other players will utilize the same strategy is

clearly definitional both of activities with mutual reinforcement in strategies and a market with network effects. An examination of games where players' strategies exhibit mutual reinforcement, mathematically modeled as increasing differences (Heal and Kunreuther 2010), finds that cascading, entrapment, and tipping are all natural outcomes in these settings. This conclusion carries directly into thinking on network effects. High degrees of asymmetry in network markets are "often a natural feature of equilibrium, rather than an historical aberration or an event that should be explained either by out-of-the equilibrium considerations or by non-economic considerations," (Economides and Flyer 1997, 3). Given the importance of network goods, high probabilities of cascading and tipping in the markets for those goods means that network markets are worth further examination.

The observation of high levels of asymmetry was realized starting in the very earliest research on network effects. In fact, Katz and Shapiro (1985) demonstrate that consumption externalities lead to demand-side economies of scale and that there may be multiple equilibria in network markets. In an examination of network effects within the paradigm of hardware-software markets, Katz and Shapiro (1994) examine issues of asymmetry. They point out that economies of scale, product differentiation, and the frequent existence of network externalities leads to (sometimes transitory) monopolies.

Additionally, limited empirical work confirms increased concentration and tipping in markets for home video game systems resultant of network effects. Video game systems in particular are used for empirical studies because the relatively simultaneous release of several competing consoles in "generations" allows for proper data collection.⁸ Dube, Hitsch, and Chintagunta (2008) find significant empirical evidence for an increase in market concentration caused by network ef-

⁸Corts and Lederman (2009) names six generations of video game consoles.

fects. Using counter-factual simulations of videogame markets where there are no indirect network effects, they find that in real markets (with network effects) the individual firm concentration ratio is up 23% relative to a market with no indirect network effects. They argue that, “Two standards...could be identical *ex-ante* but...due to the emergence of positive feedback and the role of expectations, markets with indirect network effects may become concentrated, i.e. tip towards one of the competing standards,” (3). Additionally, Corts and Lederman (2009) study different generations of consoles in the U.S. home video game market and provide evidence for early dominance of the market by one firm (Nintendo), with higher degrees of market sharing becoming evident in later generations. They argue that the increasing prevalence of non-exclusive software constitutes a form of compatibility and allows for indirect network effects between owners of competing and incompatible hardware, decreasing market concentration relative to under competing standards with exclusive network effects.

Many theoretical papers expand on tipping and provide further reasons to be interested in asymmetric equilibria in network markets. Economides and Flyer (1997) examine markets with significant network externalities where firms have the option of adhering to common technical standards to enjoy the benefits of a larger network or to horizontally differentiate by creating incompatible proprietary networks. They find that firms balance these two incentives and that even when two firms produce functionally identical products and have the same cost structure, network effects in the market can cause large differences between the two products in sales, prices, and profits that increase as network effects do. Furthermore, they find that entrance has little effect on equilibrium and actually slightly lowers total surplus.⁹

⁹Since the segment of consumers who have strong intrinsic preferences for the entrant’s product do not internalize the lost consumer surplus resulting from their exit from the larger network, entrance can lower total surplus.

Switching costs, the fact that changing from one incompatible product to another is costly, frequently occur simultaneously with network effects in a market. Even without network effects, in markets with switching costs, the non-cooperative oligopoly equilibria can be the same as collusive oligopoly outcomes without switching costs because of the *ex-post* differentiation of *ex-ante* homogeneous products by switching costs (Klemperer 1987). Furthermore, with the addition of network effects, such equilibria can be made permanent. Unlike in non-network markets where switching costs tend to cause “fat-cat” effects where that actually prevent tipping because large firms price less aggressively, the combination of switching costs and network effects can lead to the ossification of market structures¹⁰ (Chen 2009). However, the combination of switching costs and network effects tend to intensify competition when the networks are of the same size before the market tips and firms are competing *for* the market (For a thorough treatment of the differences between competition for and in a market, see Chapter 8: Competition and Compatibility in Shapiro and Varian 1999).

Given that the bounds of networks are delineated by inter-compatibility, issues in choosing a compatibility level are common. When the degree of inter-compatibility of products is at issue (Katz and Shapiro 1985), it can be difficult for firms to agree upon how much compatibility is the right level. Smaller competitors or entrants generally prefer to integrate their network with larger competitors, often oversupplying compatibility, while larger firms prefer to keep their networks proprietary, often choosing less than socially optimal levels of interconnection. Even more strikingly, Economides and Flyer (1997) find that for goods with relatively small network externalities, full industry compatibility can be a non-cooperative equilibrium but for goods with large network effects, there is no non-cooperative equilibrium, or (if approval is required to join a tech-

¹⁰However, switching costs and network effects do not have a monotonic relationship.

nical standard) total incompatibility is the unique equilibrium. Moreover, when software developers make provision decisions based on the membership choices of users who have strong preferences for variety, a socially suboptimal level of hardware standardization is supplied by the market (Church and Gandal 1992). Furthermore, Cremer, Rey, and Tirole (2000) show in a backbone model of the Internet that compatibility can be used strategically by firms. In their model, the larger backbone prefers a lower quality interconnection over a smaller one and may even utilize a strategy of targeted degradation towards its smaller rivals.

Moreover, vendors and consumers have opposing interests in when to set an industry standard. Lee and Mendelson (2007) find that although the aggregate value of fully compatible systems may be higher than the aggregate value of incompatible competing networks, vendors are better off agreeing on a standard ahead of time.¹¹ On the other hand, consumers (particularly early adopters who are given price incentive) are better off when no *de jure* standard is chosen ahead of time since the strong head-to-head competition yields lower prices and strong incentives to innovate. Under this regime, either a *de facto* standard or a split market outcome can obtain depending on the value of variation.

Taking tipping and *de facto* standardization (which can be Pareto optimal) as a given, it still is possible to standardize on the *wrong* technology. Katz and Shapiro (1986) find that in competition to take a network, when two non-proprietary technologies compete the one that is superior today has an advantage and will likely dominate. However, if one technology is sponsored and the other is not (i.e. one has a proprietary owner and the other does not), the sponsored technology is likely to be adopted even when it is inferior. In Church and Gandal (1993) address

¹¹The exception is a firm that can enter early with an incompatible technology can gain significant first-mover advantage by expanding their network early even in the absence of switching costs

the case of a network characterized by complementary products developed by different firms. They find that the hardware that has lower software development costs is adopted even when it may be socially optimal to adopt the other hardware due to discrepancies between private and social benefits of having a large network.

5 Two-Sided Markets

5.1 Definition of Two-Sidedness

I begin by providing a more precise definition of two-sided markets and then proceed to provide some general findings of the literature. Rysman (2009) argues that a market is two-sided if there is an intermediary in the market and if there is some kind of interdependence or externality between the two groups the market serves. He acknowledges that this definition is extremely broad and could potentially define too many markets as two-sided but reasons that, “The interesting question is often not whether a market can be defined as two-sided—virtually all markets might be two-sided to some extent—but how important two-sided issues are in determining outcomes of interest,” (Rysman 2009, 127). For that reason, Rysman encourages thinking in terms of two-sided strategies, leading to the conclusion that a market where two-sided strategies are a must for rational profit-maximization might be correctly called two-sided.

Rochet and Tirole (2006) provide an overview of research on two-sided markets and argue that a market is two-sided if, “the volume of transactions between end-users depends on the structure and not only the overall level of the fees charged by the platform,” (Rochet and Tirole 2006, 646). Mathematically, their definition states that a market with total price level $p = p_1 + p_2$, where p_1 is the price to side 1 and p_2 is the price to side 2, is two-sided if changes in p_1 and p_2 induce

changes in equilibrium membership or usage even when p is held constant. This definition seems very attractive and lines up well with expectations about the nature of two-sided markets.¹² For example, the very popular social networking site Facebook provides users access for free and charges advertisers to generate revenue.¹³ It is clear that were the reverse to be true, even if the overall price level remained the same, there would be no users, indicating that Facebook is an example of a two-sided product by the Rochet and Tirole definition.

This paper examines PC operating system markets where software providers and end-users clearly have significant interdependence in their choice of computer operating system. While there is no counter-factual evidence to directly support the fact that the market meets the Rochet and Tirole definition, the industry’s “conventional wisdom” is that Microsoft’s strategy to charge end-users for their product while subsidizing development kits for software providers served them well in their competition with Apple and IBM. Neither Apple nor IBM structured prices similarly and have both fared relatively poorly in the operating system market (Rochet and Tirole 2003).

5.2 General Findings About Two-Sided Markets

Two-sided markets are special cases of markets with network effects, and it therefore comes as no surprise that several of the main themes of the network effects literature, including asymmetry, carry over to the two-sided realm. Some important issues that are particular to two-sided markets include the role of inter-side effects, which side pays, whether they are levied on a fixed or per-transaction

¹²They further argue that the non-neutrality of the price structure requires that the two sides cannot simply negotiate away any differential between the one that exhibits and the optimum, implying a failure of the Coase theorem (although they demonstrate such a failure is not sufficient for two-sidedness).

¹³This is an interesting case where advertisers’ network effect on users is negative since users would likely prefer an advertisement-free Facebook.

basis, and the choice whether to single-home or multihome—i.e. join only one platform or several (Armstrong 2007). This paper focuses mainly on the role of inter-side effects and the allocation of one-time membership fees between platforms. Therefore, I will mainly present findings about these from the literature (although other elements will be mentioned when relevant).

Probably the most central finding specific to the literature on two-sided markets is that prices need not reflect the underlying cost structure to be efficient. Wright (2003) presents eight common mistakes in evaluating two-sided markets using one-sided reasoning in a regulatory framework. Most concern how mismatches between costs and prices can still be efficient.¹⁴ Bakos and Katsamakas (2008) show that asymmetries in intermediary pricing as a result of each side's valuation of the other can be the efficient cost structure in two-sided markets even without the existence of asymmetric costs.

Other models further examine pricing (Armstrong 2006) and find that it is determined by magnitudes of the cross-group effects, whether payment is structured as membership or usage fees, and whether agents join several platforms or only one. In general, lower prices are provided to the side of the market that is more competitive or provides more benefit to the other side. Parker and Alstyne (2005) go a step further and investigate why a firm might choose to provide a product to one side of the market without charging them for it even without competition and find that the decision of which side to give the product to relies on the magnitude of the inter-side network effects.

Armstrong and Wright (2007) expand upon the model in Armstrong (2006) by allowing for variation in product differentiation across the two sides of the market

¹⁴Others concern how competition may not help bring about a higher level of efficiency.

(i.e. sellers are *ex-ante* indifferent between the two platforms, while buyers may have a preference for one side over the other) and endogenizing the choice to multihome. They find that platforms do not compete directly for sellers, but instead compete indirectly by attracting more buyers, a finding that is borne out in this paper as well. Additionally, they find that sellers endogenously choose to multihome if product differentiation between the two platforms is sufficiently small.

Hagiu (2009) further extends Armstrong’s (2006) model of competitive bottlenecks by adding a dimension of intra-platform competition. He is able to explain the strange discrepancy in pricing between video game and computer software markets. In video game markets, consumers purchase consoles relatively cheaply, sometimes below marginal cost of production, while developers pay significant fees to obtain development kits. On the other hand, in personal computer markets, it is often the case that developers join for free¹⁵ while consumers pay significant costs for operating systems. He argues that the prices charged are a function of developer and consumer demand elasticities. The opposing pricing schemes are explained by different consumer elasticities in the markets as a result of the importance of variety in video game markets relative to productivity-oriented software since, “games get ‘played out’ whereas software with practical applications is theoretically infinitely durable (technological obsolescence notwithstanding of course),” (Hagiu 2009, 1024).

Concerns about asymmetry and tipping carry into the two-sided literature and some amount of work has been done on tipping in a two-sided context. Ambrus and Argenziano (2009) model pricing and network choices in a two-sided market

¹⁵Or are even subsidized with software development kits (SDKs) or by platforms’ expensive commitments to application programming interfaces (APIs) several years before operating system releases.

that allows for heterogeneity of consumer valuation of the network externality. They demonstrate that under monopoly or competition, multiple asymmetric networks can exist in equilibrium if there is sufficient consumer heterogeneity. They show that for all asymmetric equilibria, one platform is cheaper and larger on one side and the other is the opposite.

Most of the literature on two-sided markets focus on the way pricing works between the two sides and either assumes a monopoly platform or *ex-ante* rules out tipping by assuming some requisite level of symmetry (Hagiu 2009, Armstrong and Wright 2007, Armstrong 2006, Rochet and Tirole 2003). This paper focuses on the sources and mechanism of tipping, further exploring an important aspect of two-sided markets where little work has been done.

Part III

Model

This paper models a market with two proprietary platforms in Bertrand competition. Platforms set membership fees¹⁶ to both sides of the market while agents simultaneously decide which platform to join. I assume market coverage and single-homing such that each agent joins exactly one platform. Agents are grouped into the sides that the platform serves based on whether they are end-users or software developers. It is assumed that on-market transactions are competitive and that both buyers and developers are price-takers.

The model is based on Armstrong's (2006) model of a two-sided market where both sides exogenously decide to single-home (Armstrong 2006, Section 4). Although it seems likely that a competitive bottleneck model, where one side chooses to single-home and the other to multihome, would more accurately mirror the real world, a more complicated model that allows for multihoming was not used for analytical tractability. By having both sides single-home and assuming market coverage, I am able to state one platform's equilibrium membership as the total number of agents minus the number of agents who join the other platform. This formulation facilitates the solution of more complete results than would be possible otherwise.

Furthermore, the qualitative outcomes of Armstrong's competitive bottleneck model seem to be precisely the opposite of those observed in PC operating system markets. He finds that under a competitive bottleneck, the side which single-homes is treated well and the side that multihomes finds their concerns ignored

¹⁶There are no usage fees, reflecting the way pricing is done in PC software markets.

by the platform. This does not seem to be the case in the PC operating system market, where the side that more strictly single-homes (end-users) bears almost all of the costs and the side that is more likely to multihome (software developers) are treated very well. For that reason, I use a model where both sides exogenously decide to single-home and will point out differences in modeling from Armstrong when they are relevant.¹⁷

The platforms are modeled as completely incompatible. In this context, that means that components designed for one platform do not function on the other. The platform is considered to encompass both the operating system and the physical hardware (See Church and Gandal 1992, p.86). Although the provision of physical hardware is clearly relevant when buying a computer, most users buy their operating system and hardware as a single package, making the theoretically separate choices of hardware and operating system mutually implicative.¹⁸ I derive the Nash equilibrium of the Bertrand game in prices and examine how much perturbation the equilibrium can tolerate via inter-platform cost differences before the market tips entirely to one platform.

6 Basic Setup

6.1 Definition of Variables

In the model there are two sides, sellers (developers) denoted by S and buyers (end-users) denoted by B , such that for platforms $i = 1, 2$ and sides $k = B, S$, $n_k^{(i)}$ ¹⁹ is the number of agents on side k of platform i , normalized such that $n_k^{(1)} +$

¹⁷See section 9.1 for comparisons between this model and Armstrong’s (2006) model in qualitative results.

¹⁸This clearly ignores some portion of the population that engages in various nonstandard “hacking” behavior—i.e. installing more than one operating system on a single physical machine.

¹⁹Convention throughout the paper will be that superscript letters within parentheses indicate to which platform the term belongs, subscript letters will denote to which side of the platform

$n_k^{(2)} = n_k = 1$. $p_k^{(i)}$ is platform i 's price to side k such that $p_k^{(i)} \in [\underline{p}, \bar{p}]$, a compact and convex subset of \mathbb{R} . α_k is a parameter describing the strength of the network effect for side k , indicating how important the presence of agents on side $l \neq k$ is to those on side k . α_k is assumed to always be greater than zero. The basic model of utility and profits is very straightforward such that for both sides, net utility or profit is simply the level of network effects times the number of agents on the other side less the costs of interacting with the other side.

For end-users, the cost of interacting with developers is simply the price to join the platform, $p_B^{(i)}$. For producers, the cost of interacting with end users is their total cost function, $C_S^{(i)}$ for a firm on the developers' side of platform i . To give some more detail to the structure of the market, $C_S^{(i)}$ is considered $C_S^{(i)}(p_S^{(i)}, \gamma^{(i)}, \rho^{(i)})$, where $\gamma^{(i)}$ and $\rho^{(i)}$ are exogenous parameters describing software quality and costliness of production on the platform.

Making software quality ($\gamma^{(i)}$) an exogenous variable is contrived, as it is competitively determined, but starting the analysis at that level is outside the scope of this paper.²⁰ $\rho^{(i)}$ is a parameter describing the intrinsic costliness of doing business on the platform which reflects the way the platform operates. Elements like choice of programming language, hardware requirements, existence of knowledge spillovers, and the quality of application programming interfaces (APIs) are controllable by the platform and influence the cost of production on each platform.

Since the model is based on a PC operating system market, I consider platforms that charge membership fees but no usage fees (For further details about the difference, see Rochet and Tirole 2006). Therefore, $p_S^{(i)}$ is considered to

the term belongs, and unadorned superscripts indicate exponentiation.

²⁰However, it is clear that increasing quality is costly to software producers such that $\frac{\partial C_S^{(i)}}{\partial \gamma^{(i)}} > 0$.

be an additively separable fixed cost in the developer's cost function such that $C_S^{(i)}(p_S^{(i)}, \gamma^{(i)}, \rho^{(i)}) = c_S^{(i)}(\gamma^{(i)}, \rho^{(i)}) + p_S^{(i)}$. I place no restrictions on $p_k^{(i)}$ as there is anecdotal evidence indicating that in many two-sided markets one side is charged a zero price or is even subsidized (in the form of development kits and other non-monetary incentives in software markets) and the platform's revenue is generated entirely on the other side (For more on asymmetric inter-side pricing, see Armstrong 2006).

6.2 Inter-platform Differences

Any story about tipping is largely one about asymmetric competitors, so I introduce the following notational conveniences to make the focus on inter-platform differences clear. Note that for all the terms below, the term describes platform i 's advantage.

1. $\Delta_k = p_k^{(j)} - p_k^{(i)}$, platform i 's advantage in the price the platform charges
2. $\delta_C = c_S^{(j)} - c_S^{(i)}$, platform i 's advantage in component production costs (other than the platform's price to developers)
3. $\Delta_C = C_S^{(j)} - C_S^{(i)} = \Delta_S + \delta_C$, platform i 's advantage in total developer costs
4. $\delta_k = f_k^{(j)} - f_k^{(i)}$ where $f_k^{(i)} \geq 0$ is the marginal cost to the platform of serving side k , so that δ_k is side i 's advantage in marginal cost of serving side k

Note that uppercase deltas indicate differences in platform choice variables (prices) whereas lowercase deltas indicate differences in exogenously determined parameters.

I provide interpretations of the three δ 's to provide some insight into what a particular level of any δ might mean. Recall that $c_S^{(i)}$ is a function of $\gamma^{(i)}$ and

$\rho^{(i)}$, where $\gamma^{(i)}$ is the quality of software on the platform, and $\rho^{(i)}$ is a parameter representing the intrinsic costliness of doing business on the platform. In general, platforms do not exercise very much control over software quality, although it is possible that a platform could require some minimum quality level. However, every platform has significant influence in $\rho^{(i)}$. By manipulating the level of service to developers, the platform can change $\rho^{(i)}$. A platform could choose to produce a more thorough or easy-to-use API for developers' benefit, could choose a more open programming language or one that speeds development time for their platform, or could ensure that it is easy for developers to reach potential consumers. Through any of these channels, the platform might be able to lower the costs of production on their platform, and change δ_C in their favor.

δ_S and δ_B give the differences in the marginal costs of serving users on the two platforms. Interpreting $f_k^{(i)}$ as the cost to the platform of acquiring and keeping the marginal consumer, the platform can exercise some level of control over $f_k^{(i)}$. A platform with higher $f_k^{(i)}$ must have some level of inefficiency in their modes of consumer acquisition and service. For example, one platform may have access to patented technology, a better-designed model to deal with consumer complaints, or more experience in the market, all of which can lower their marginal costs of serving consumers relative to another platform. Clearly, by adopting better practices, the platform can try to lower marginal costs of serving agents on one side and change δ_k in their favor.

6.3 Consumer Utility and Developer Profit

In the model, consumer utility and developer profit on platforms $i = 1, 2$ is specified by

$$U^{(i)} = \alpha_B n_S^{(i)} - p_B^{(i)} \quad \text{and} \quad \pi^{(i)} = \alpha_S n_B^{(i)} - C_S^{(i)}, \quad (1)$$

where S is for developer and B is for buyer (end-user). Following the example of Church and Gandal (1992, p. 88), which models the provision decisions of software firms in hardware-software markets, hardware is modeled with no intrinsic value—an operating system with no software to run is considered to have no value to consumers.

Buyers and developers with a total mass $n_B = n_S = 1$ are assumed to be distributed along independent unit intervals with the duopolistic platforms at the endpoints. Thus $t_B, t_S > 0$ are considered to be product differentiation (transportation cost) parameters for the two sides. I solve for the indifferent consumer à la Hotelling, and find that

$$n_B^{(i)} = \frac{1}{2} + \frac{U^{(i)} - U^{(j)}}{2t_B} \quad \text{and} \quad n_S^{(i)} = \frac{1}{2} + \frac{\pi^{(i)} - \pi^{(j)}}{2t_S}. \quad (2)$$

Since software markets are competitive on the platform, producers and consumers are price takers, and I can solve for the number of agents on each side by plugging (1) into (2). Furthermore, I build in the market coverage assumption that $n_k^{(j)} = 1 - n_k^{(i)}$. The number of users on each side is given by

$$n_S^i = \frac{1}{2} + \frac{t_B \Delta_C + \alpha_S \Delta_B}{2(t_B t_S - \alpha_B \alpha_S)} \quad \text{and} \quad n_B^i = \frac{1}{2} + \frac{\alpha_B \Delta_C + t_S \Delta_B}{2(t_B t_S - \alpha_B \alpha_S)}. \quad (3)$$

7 Platform Profit

Given that the platform i incurs fixed cost F and marginal costs $f_k^{(i)}$ to serve each side of the market, the platform's decision is described by their profit function,

$$\pi^{(i)} = (p_S^{(i)} - f_S^{(i)})n_S^{(i)} + (p_B^{(i)} - f_B^{(i)})n_B^{(i)} - F$$

Plugging in the expressions from (3) yields

$$\begin{aligned} \pi^{(i)} &= (p_S^{(i)} - f_S^{(i)}) \left[\frac{1}{2} + \frac{t_B \Delta_C + \alpha_S \Delta_B}{2(t_B t_S - \alpha_B \alpha_S)} \right] \\ &+ (p_B^{(i)} - f_B^{(i)}) \left[\frac{1}{2} + \frac{\alpha_B \Delta_C + t_S \Delta_B}{2(t_B t_S - \alpha_B \alpha_S)} \right] - F. \end{aligned}$$

7.1 First Order Conditions

Taking derivatives with respect to both of the prices the platform chooses, the first order conditions each platform must solve for their individual optimization problem are given by

$$\begin{aligned} \frac{\partial \pi^{(i)}}{\partial p_S^{(i)}} &= n_S^{(i)} - (p_S^{(i)} - f_S^{(i)}) \left[\frac{t_B}{2(t_B t_S - \alpha_B \alpha_S)} \right] \\ &- (p_B^{(i)} - f_B^{(i)}) \left[\frac{\alpha_B}{2(t_B t_S - \alpha_B \alpha_S)} \right] = 0 \end{aligned} \quad (4)$$

and

$$\begin{aligned} \frac{\partial \pi^{(i)}}{\partial p_B^{(i)}} &= n_B^{(i)} - (p_S^{(i)} - f_S^{(i)}) \left[\frac{\alpha_S}{2(t_B t_S - \alpha_B \alpha_S)} \right] \\ &- (p_B^{(i)} - f_B^{(i)}) \left[\frac{t_S}{2(t_B t_S - \alpha_B \alpha_S)} \right] = 0. \end{aligned} \quad (5)$$

For a full derivation of the first order conditions, see appendix A.

7.2 Second Order Conditions

The second order conditions to ensure that the first order conditions find a *maximum* on the profit function are that

$$t_B t_S > \alpha_B \alpha_S \tag{6}$$

and that

$$4t_B t_S \geq (\alpha_B + \alpha_S)^2. \tag{7}$$

If both conditions hold strictly, then there is a unique maximum on the firm's profit function. For a full derivation of the second order conditions, see appendix B. We provide a lemma regarding the second order conditions. To aid intuition about what the various claims in the paper mean, lemmas and propositions throughout the paper will be stated twice when appropriate—once in plain English and once in mathematical notation.

Lemma 1

Only the second of these conditions is necessary since if (7) holds strictly, then so does (6). See proof in Appendix C.

Mathematically: $(\alpha_B + \alpha_S)^2 < 4t_B t_S$ implies $\alpha_B \alpha_S < t_B t_S$.

The fact that (7) holds strictly such that the firm's profit function has a unique maximum will be assumed through the duration of this analysis.

8 Derivation of Nash Equilibrium

8.1 Best Reaction Functions

Given that the second order condition holds, the first order conditions solve for a unique price pair and can be solved to find explicit reaction functions for equilibrium prices as a function of the other platform's prices. Solving (4) and (5) for $p_S^{(i)}$ and $p_B^{(i)}$, respectively, yields best-reaction functions

$$p_S^{(i)} = \frac{t_S + \delta_C + p_S^{(j)} + f_S^{(i)}}{2} + \frac{\alpha_S \Delta_B - \alpha_B \alpha_S - \alpha_B (p_B^{(i)} - f_B^{(i)})}{2t_B} \quad (8)$$

and

$$p_B^{(i)} = \frac{t_B + p_B^{(j)} + f_B^{(i)}}{2} + \frac{\alpha_B \Delta_C - \alpha_B \alpha_S - \alpha_S (p_S^{(i)} - f_S^{(i)})}{2t_S}. \quad (9)$$

Both prices are linear functions of product differentiation, the other platform's price to that side, and the marginal costs of serving that side. Furthermore, price to developers is a linear function of the platform's advantage in component production costs. Price is also influenced by a more complex relationship involving ratios of strength of network effects and product differentiation as well as the cost advantage on the other side of the market and net revenue from serving the other side. Since the best response functions will eventually be solved as functions of parameters alone, I do not dwell on the interpretation of these reaction functions.

8.1.1 Existence and Uniqueness of Nash Equilibrium

By the Brouwer fixed point theorem, there exists a fixed point (Nash equilibrium) for the $p_k^{(i)}$ best response functions if they are continuous and the sets from which all variables are drawn are compact and convex. The $p_k^{(i)}$ functions are clearly continuous (since $t_B, t_S > 0$) and it was already assumed that $p_k^{(i)}$ is from a

compact and convex subset of \mathbb{R} . Therefore, for a given combination of parameter values that obeys the second order condition, there exists a fixed point on the reaction functions and therefore a Nash equilibrium in platforms' prices.

The reaction functions, (8) and (9) are contraction mappings and cross exactly once such that the Nash equilibrium in prices is unique if

$$\alpha_B < t_S \text{ and } \alpha_S < t_B. \tag{10}$$

See derivation in appendix D. Intuitively, if product differentiation on both sides is greater than network effects on the opposing sides, the Nash equilibrium in prices is unique.

Lemma 2

Given that product differentiation is the same on the two sides of the market, the contraction mapping conditions are stronger than the second order conditions and imply the second order conditions. If product differentiation is not the same across both sides, neither the contraction mapping conditions nor the second order condition imply the other. See appendix E for proof.

Mathematically: If $t_B = t_S$, then $\alpha_B < t_S$ and $\alpha_S < t_B$ imply $(\alpha_B + \alpha_S)^2 < 4t_B t_S$.

If the platforms are equally differentiated to both sides of the markets, the fact that the best reaction functions are a contraction mapping and they cross precisely once on the set $[\underline{p}, \bar{p}]$, assures that the individual profit maximization problem has a unique interior maximum.

8.2 Simultaneous Solution of Reaction Functions

The reaction functions—one for each price on each platform—constitute a system of four linear equations in four unknowns. Therefore, I solve to get the inter-platform price differences as functions of parameters alone. For the full derivation, see appendix F. From (8) and (9) above, I solve for $p_S^{(i)}$ and $p_S^{(j)}$, subtract $p_S^{(i)}$ from $p_S^{(j)}$, and simplify to get

$$\Delta_S = \frac{1}{3}\delta_S + \left[\frac{\alpha_B}{3t_B}\right]\delta_B - \frac{2}{3}\delta_C - \left[\frac{2\alpha_S + \alpha_B}{3t_B}\right]\Delta_B. \quad (11)$$

I do the same for side B to solve for Δ_B to yield

$$\Delta_B = \left[\frac{\alpha_S}{3t_S}\right]\delta_S + \frac{1}{3}\delta_B - \left[\frac{2\alpha_B}{3t_S}\right]\delta_C - \left[\frac{\alpha_S + 2\alpha_B}{3t_S}\right]\Delta_S. \quad (12)$$

I plug Δ_B into Δ_S and simplify to solve for inter-platform price differences such that they are functions of parameters alone. Thus Δ_S is

$$\begin{aligned} \Delta_S = & \left[\frac{3t_B t_S - \alpha_S(2\alpha_S + \alpha_B)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}\right] \delta_S \\ & + \left[\frac{2t_S(\alpha_B - \alpha_S)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}\right] \delta_B \\ & + \left[\frac{2\alpha_B(2\alpha_S + \alpha_B) - 6t_B t_S}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}\right] \delta_C. \end{aligned} \quad (13)$$

Plugging Δ_S into Δ_B and simplifying yields

$$\begin{aligned} \Delta_B = & \left[\frac{2t_B(\alpha_S - \alpha_B)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}\right] \delta_S \\ & + \left[\frac{3t_B t_S - \alpha_B(\alpha_S + 2\alpha_B)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}\right] \delta_B \\ & + \left[\frac{2t_B(\alpha_S - \alpha_B)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}\right] \delta_C. \end{aligned} \quad (14)$$

Based on these expressions for inter-platform cost differences, I make several propositions about the price equilibrium.

Proposition 1

Costs of all sorts are passed to both sides of the market.

Mathematically: $\frac{\partial \Delta_S}{\partial \delta_l} \neq 0$, and $\frac{\partial \Delta_B}{\partial \delta_l} \neq 0$ for $l = S, B, C$.

The expressions for both Δ_B and Δ_S contain a term multiplied by each of the δ 's, indicating that the price the platform charges to users on both sides is a function of the marginal cost of the platform to serving both sides and the costliness of producing *on* the developer side of the market. This confirms a general prior result (Armstrong 2006, Bakos and Katsamakas 2008, Rochet and Tirole 2003) about two-sided markets—that costs of all sorts tend to be passed to all parties in the market.

Proposition 2

In pricing to consumers, cost differences on the developers' side—whether in serving developers or of the developers themselves—get passed to end-users in precisely the same manner.

Mathematically: $\frac{\partial \Delta_B}{\partial \delta_C} = \frac{\partial \Delta_B}{\partial \delta_S}$

Looking at expression (14) for the price difference to the buyers' side, it is clear that the coefficients on δ_S and δ_C are the same, indicating that differences between the two platforms in the marginal costs the platform bears to serve developers or that developers bear are passed through to consumers in exactly the same way.

Proposition 3

Platform i will pass increased advantages in costs of serving one side to that side in lower prices if network effects are higher on the other side. See proof in appendix H.

Mathematically: If $\alpha_B > \alpha_S$, then $\frac{\partial \Delta_S}{\partial \delta_S} > 0$, and if $\alpha_S > \alpha_B$, then $\frac{\partial \Delta_B}{\partial \delta_B} > 0$

If one side has relatively higher network effects, they care a lot about the presence of the other side on their platform of choice. Therefore it will be relatively easy for a platform to attract the side with high network effects indirectly by attracting members on the other side using prices. For that reason, platforms generally compete indirectly for the side with high network effects by charging low prices to the side with low network effects (See similar findings in Armstrong and Wright 2007).

Therefore, it is no surprise that when network effects are higher on the developers' side than on the buyers' side, cost savings on the buyers' side will be reflected by prices on the buyers' side in order to indirectly attract more developers. Similarly, when network effects are higher on the buyers' side, cost savings on the developers' side of one platform will cause lower prices to the developers' side of that platform to indirectly attract buyers.

Proposition 4

The impact of cross-side cost differences on equilibrium price differences is determined by the relative magnitudes of network effects between the two sides.

Mathematically: If $\alpha_B > \alpha_S$, then $\frac{\partial \Delta_S}{\partial \delta_B} > 0$, and if $\alpha_S > \alpha_B$, then $\frac{\partial \Delta_B}{\partial \delta_C} = \frac{\partial \Delta_B}{\partial \delta_S} > 0$.

The cross-side cost differences (δ_B for the seller's side and δ_C and δ_S for the buyer's side) have interesting and straightforward interpretations that provide more detail about how platforms allocate their costs amongst sides in two-sided markets. If α_S is low relative to α_B , indicating that buyers care more about the presence of developers than vice versa, it's clear that the coefficient on δ_B is positive and an advantage in marginal costs of serving the buyer's side will be passed through to developers.

As before, the platform will endeavor to pass most of the cost savings through to the side that cares more about price and less about the number of agents on the other side of the market. In this case, since α_B is large, buyers care relatively more about the presence of developers in the market. A low α_S indicates that developers are less concerned with the number of buyers and more concerned with the price they have to pay (see similar findings in Armstrong and Wright 2007, Hagiu 2009). Therefore, cost savings on the buyers' side will be strongly passed through to sellers.

As α_S starts to rise relative to α_B , the numerator on the δ_B term will decrease and the denominator will increase. Therefore, as developers care more about the number of buyers, the platform must ensure that buyers do not leave the platform to maintain developer-side membership. Thus cost savings have to be shared more equally between the two sides and cost savings on the buyers' side will not be passed through as strongly.

When α_S is large relative to α_B , indicating developers care a lot about the number of buyers on the platform compared to how much the buyers care about them, the numerator of the coefficient on δ_B is negative, indicating that prices to developers could actually increase when there are cost savings on the buyer's side.

The high α_S indicates that developers care more about the number of buyers on the platform than about the price they have to pay to join the platform and so the platform will work to indirectly attract developers by attracting buyers with low prices. Precisely symmetric logic holds for the coefficients on δ_S and δ_C in the expression for Δ_B

Proposition 5

Suppose product differentiation is stronger than network effects on the buyer's side. Then an increase in platform i 's advantage in costs to developers decreases platform i 's price advantage to developers.

Mathematically: $t_B > \alpha_B$ implies that $\frac{\partial \Delta_S}{\partial \delta_C} < 0$.

See proof in appendix I. Since the platform will work indirectly to attract the side with higher network effects through subsidizing the other side, if buyers care relatively more about product differentiation (ergo platform prices) than about network effects, any advantage in component production costs will be extracted from developers in the price to them and given to the buyers. Since product differentiation is large, buyers' behavior is qualitatively similar to in a one-sided market and they are swayed easily from one platform to the other as a result of differential pricing. Developers will indirectly be attracted to the platform with lower buyer costs as a result of the market's network effects.

8.3 Solving for Market Equilibrium

Now that price differentials are functions of market parameters alone, I solve $n_k^{(i)}$ for the equilibrium platform memberships on both sides of the market. I plug Δ_S and Δ_B , given by (11) and (12) into the formulas for platform membership, given in (3), group terms, and simplify to yield equilibrium market shares in terms of

parameters alone. A full derivation is provided in appendix G. $n_S^{(i)}$ is given by

$$\begin{aligned}
n_S^{(i)} = & \frac{1}{2} + \left[\frac{3t_B(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_S \\
& + \left[\frac{(t_B t_S - \alpha_B \alpha_S)(\alpha_S + 2\alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_B \\
& + \left[\frac{t_B}{2(t_B t_S - \alpha_B \alpha_S)} + \frac{t_B(\alpha_S^2 + 2\alpha_B^2 + \alpha_B \alpha_S - 6t_B t_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_C
\end{aligned} \tag{15}$$

and $n_B^{(i)}$ is

$$\begin{aligned}
n_B^{(i)} = & \frac{1}{2} + \left[\frac{(t_B t_S - \alpha_B \alpha_S)(2\alpha_S + \alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_S \\
& + \left[\frac{3t_S(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_B \\
& + \left[\frac{\alpha_B}{2(t_B t_S - \alpha_B \alpha_S)} + \frac{2\alpha_B^2(\alpha_S + \alpha_B) + 2t_B t_S(\alpha_S - 4\alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_C
\end{aligned} \tag{16}$$

9 Analysis of Market Equilibrium

9.1 Comparison With Armstrong 2006

The equilibrium I find is a generalized version of that which Armstrong (2006) finds in his model of two competing platforms where everyone single-homes. Armstrong assumes symmetric costs across the platform and asserts that symmetric costs, combined with the second order conditions are sufficient conditions for a unique, symmetric equilibrium to exist (674). In our notation, this translates to all of the δ 's being zero. I examine each indicator of market equilibrium and find that they all match Armstrong's formulations when all δ 's are zero.

From reaction functions (8) and (9), it is clear that when the two platforms are completely symmetric, the reaction functions are the same as those Armstrong (2006) provides in equation (10) of his work (See appendix J for proof). Examining the price differentials from (11) and (12), each term features a δ and when the cost differences represented by those δ 's are zero, both Δ 's, which represent the difference in the price chosen by the two platforms are zero. Furthermore, examining expressions (15) and (16), which determine $n_k^{(i)}$, both go to $\frac{1}{2}$ as the δ 's go to zero. This more general model confirms Armstrong's assertion that when there are no inter-platform differences in costs, there will be one symmetric market-sharing equilibrium where each platform captures half the market.

9.2 Effects of Cost Differences

A strength of the greater generality of this model is that I can examine what happens to the symmetric equilibrium in the Armstrong model when one or more of the δ 's is nonzero. To demonstrate in general what kind of movement occurs when one or more δ is nonzero, I begin by taking partial differentials of the functions determining equilibrium platform membership with respect to each δ .

9.2.1 Different Costs of Serving Developers and Buyers

Taking the partial differentials of $n_S^{(i)}$ and $n_B^{(i)}$ with respect to δ_S and δ_B and simplifying yields expressions for the change in platform membership given a change in platform i 's advantage of serving one side.

Side	Cost Difference in	Change in Side Membership
Developers	Serving Developers	$\frac{\partial n_S^{(i)}}{\partial \delta_S} = \frac{3t_B}{2(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))}$
	Serving Buyers	$\frac{\partial n_S^{(i)}}{\partial \delta_B} = \frac{(\alpha_S + 2\alpha_B)}{2(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))}$
Buyers	Serving Developers	$\frac{\partial n_B^{(i)}}{\partial \delta_S} = \frac{(2\alpha_S + \alpha_B)}{2(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))}$
	Serving Buyers	$\frac{\partial n_B^{(i)}}{\partial \delta_B} = \frac{3t_S}{2(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))}$

Proposition 6

Increasing one platform's advantage in costs of serving one side shifts the market equilibrium in its favor on both sides of the market. However, the mechanism for this change comes only through the effects of the prices charged by the platforms.

$$\text{Mathematically: } \frac{\partial n_S^{(i)}}{\partial \delta_S}, \frac{\partial n_S^{(i)}}{\partial \delta_B}, \frac{\partial n_B^{(i)}}{\partial \delta_S}, \frac{\partial n_B^{(i)}}{\partial \delta_B} > 0.$$

Since an increase in δ_B indicates that platform i gains an advantage in the marginal cost of serving buyers and δ_S indicates the same for the marginal cost of serving developers, it makes sense that the platform should be able to leverage this advantage into increasing their market share on both sides of the market.

Furthermore, recall that $\delta_k = f_k^{(j)} - f_k^{(i)}$ is platform i 's advantage in the marginal costs of serving side k . Since marginal costs to the platform have no direct effect on the utility or profits of platform members, gaining an advantage in marginal costs can only affect the market equilibrium through price changes. The effects of changing δ_S and δ_B on market equilibrium are easily rankable since all four have the same denominator. The numerators appear as below:

Side	Cost Difference in	Numerator of Change in Side Membership
Developers	Serving Developers	$3t_B$
	Serving Buyers	$(\alpha_S + 2\alpha_B)$
Buyers	Serving Developers	$(2\alpha_S + \alpha_B)$
	Serving Buyers	$3t_S$

Proposition 7

If product differentiation is stronger than network effects on both sides of the market, then an advantage in the platform's costs of serving one side has more impact on that side than an advantage in the costs of serving the other side. See proof in appendix K.

Mathematically: $t_B > \alpha_B$ implies $\frac{\partial n_S^{(i)}}{\partial \delta_S} > \frac{\partial n_S^{(i)}}{\partial \delta_B}$, and if $t_S > \alpha_S$, then $\frac{\partial n_B^{(i)}}{\partial \delta_B} > \frac{\partial n_B^{(i)}}{\partial \delta_S}$.

The logic behind this finding is straightforward. If product differentiation on one side is larger than network effects on one side, then that side of the market more closely resembles a one-sided market. If there is an increase in platform i 's advantage in marginal costs of serving developers (δ_S), the platform will pass a substantial portion of the cost savings to the buyers, who are relatively easy to influence using prices. However, the developers' side, which is relatively easy to influence using buyer membership, will increase membership even more as a result of increased buyer presence on the platform. On the other hand, if network effects are strong relative to product differentiation on the buyer's side, an increase in δ_S will largely be kept on the developers' side and will have a greater impact on the buyer's side because of the high network effects there.

9.2.1 Different Costs of Production on the Platform

Taking the partial differentials of $n_S^{(i)}$ and $n_B^{(i)}$ with respect to δ_C yields

Side	Cost Difference in	Change in Side Membership
developers	Component Production	$\frac{\partial n_S^{(i)}}{\partial \delta_C} = \frac{t_B(3(t_B t_S + (\alpha_B + \alpha_S)^2) + \alpha_B^2)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))}$
Buyers	Component Productions	$\frac{\partial n_B^{(i)}}{\partial \delta_C} = \frac{t_B t_S (\alpha_B + 2\alpha_S) + \alpha_B (2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))}$

Proposition 8

Increasing one platform's advantage in developer costs shifts the market equilibrium in their favor on both sides of the market. The mechanism for this change is complex because of the structural effects of changing developer costs.

Mathematically: $\frac{\partial n_S^{(i)}}{\partial \delta_C}, \frac{\partial n_B^{(i)}}{\partial \delta_C} > 0$.

In general, the platform gaining an advantage in costs of on-platform production attracts more agents on both sides of the market. If the platform allocates the cost savings correctly as in the functions giving the level of the price difference, (11) and (12) between the two sides, then it is to be expected that they will be able to attract more members on both sides. However, the expressions above are much more complicated than those for membership changes based on the costs of serving one side because changes in costs of component production affect both pricing *and* potential developer profits.

Proposition 8'

Combining the findings of propositions 6 and 8, it is clear that a platform gaining a cost advantage of any sort increases their equilibrium market share. However, the mechanism and strength of that effect is different depending on whether the costs are borne by the platform or by developers.

10 Tipping

The central point of this paper is to investigate when parameter values dictate that tipping will occur given some level of inter-platform cost asymmetry. I begin the investigation of tipping by examining when a single δ can tip the market (given that all the others are zero) and proceed to show how one δ can compensate for another such that the market does not tip. To that end, I provide a simple lemma describing when the market will or will not tip.

Lemma 3

A sufficient condition for the market remaining untipped is that the sum of the magnitude of all terms including a δ in the expressions for

equilibrium platform membership, (15) and (16), are less than $\frac{1}{2}$. See appendix L for proof.

Mathematically: $n_k^{(i)} \in (0, 1)$ if $\left| n_k^{(i)} - \frac{1}{2} \right| < \frac{1}{2}$.

10.1 Upper Bounds on Inter-Platform Differences

As established when comparing this model to Armstrong (2006), the value of all of the δ 's approaching zero, implies that the market moves towards the symmetric equilibrium that $n_B^{(i)} = n_S^{(i)} = \frac{1}{2}$. I now examine when the asymmetry caused by a nonzero δ is too great and the market tips fully to one platform or the other, I provide “no-compensation bounds”—upper bounds on each δ such that the market does not tip to either side given that all of the other δ 's equal zero. I first provide a corollary to simplify analysis.

Corollary to Lemma 3

If the partial derivative with respect to δ_l of the $n_k^{(i)}$ function, $l = S, B, C$, is P_l , the threshold in magnitude beyond which δ_l will tip the market is given by $\frac{1}{2|P_l|}$ given that all the other δ 's are zero. See appendix M for proof.

Mathematically: For $\frac{\partial n_k^{(i)}}{\partial \delta_l} = P_l, l = S, B, C$ and $k = S, B, n_k^{(i)} \in (0, 1)$ if $|\delta_l| < \frac{1}{2P_l}$ given $\delta_m = 0$ for $m \neq l$.

10.1.1 Difference in Costs of Serving Developers

Given that δ_B and δ_C are zero, the seller market remains untipped as long as $|\delta_S| < \frac{1}{2|P_S|}$. Therefore, the upper bound for δ_S such that it does not tip the seller market by itself is given by

$$|\delta_S| < \frac{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}{3t_B} = 3t_S + \frac{(\alpha_B + 2\alpha_S)(2\alpha_B + \alpha_S)}{3t_B}.$$

Proposition 9

Network effects have two countervailing effects in two-sided markets. They ex ante make the market more likely to tip, but also dampen the effects of price asymmetries in affecting the market equilibrium.

Mathematically: If α_k is large, $\frac{\partial|\delta_i|}{\partial n_k^{(i)}}$ is large in magnitude, but $\frac{\partial|\delta_i|}{\partial \Delta_k}$ is small in magnitude.

Proposition 9 is the main finding of this paper. From the no-compensation bound above, it's clear that the upper bound is linearly related to product differentiation (t_S). Since it is product differentiation that keeps all of the developers from jumping to the platform with more buyers it makes perfect sense that the upper bound on δ_S has a significant direct relationship to t_S . However, the second term seems more puzzling. It's generally considered that network effects make the market *more* likely to tip, not less so. It seems highly counterintuitive that the upper bound for a level of δ_S that does not tip the market is increasing in the overall level of network effects.

To find the answer to this puzzle, I go all the way back to the expressions for consumer utility and producer profit from (1). From these equations and the bound above, it's clear that as the overall level of network effects goes up, people care less about the price they have to pay relative to the number of agents on the other side of the market. Therefore, as the overall level of network effects goes up, it is harder to move people from platform to platform using price measures. As previously established, δ_S can only affect the equilibrium through its effect on prices. As the overall level of network effects goes up, a higher level of inter-platform asymmetry in marginal costs of serving one side, reflected in higher asymmetry in prices, can be tolerated before the market tips.

Similarly, the upper bound on δ_S in the buyer's market is given by

$$|\delta_S| < \frac{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}{(2\alpha_S + \alpha_B)} = \frac{9t_B t_S}{(\alpha_B + 2\alpha_S)} + 2\alpha_B + \alpha_S.$$

In the first term, it's clear that increasing product differentiation increases the bound on cost asymmetries of serving developers, while increasing network effects decreases the bound. Intuitively, this is the effect found in much of the literature—product differentiation resists tipping and network effects foster tipping. The latter two terms reflect my finding, that the relative importance of prices in determining market equilibrium decreases as the level of network effects goes up.

10.1.1 Difference in Costs of Serving Buyers

Given that δ_S and δ_C are zero, the developer side of the market remains untipped if

$$|\delta_B| < \frac{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}{(\alpha_S + 2\alpha_B)} = \frac{9t_B t_S}{(\alpha_S + 2\alpha_B)} + 2\alpha_S + \alpha_B.$$

Similarly, the upper bound on δ_B for the buyer's market to remain untipped is given by

$$|\delta_B| < \frac{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}{3t_S} = 3t_B + \frac{(\alpha_B + 2\alpha_S)(2\alpha_B + \alpha_S)}{3t_S}.$$

These upper bounds are exactly symmetric to those on δ_S and similarly expose the contrasting nature of network effects in encouraging cascading in general while lessening the importance of price asymmetries in the market.

10.1.2 Difference in On-Platform Costs

Given that δ_B and δ_S are zero, the developer market remains untipped as long as

$$|\delta_C| < \frac{(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))}{t_B(3(t_B t_S + (\alpha_B + \alpha_S)^2) + \alpha_B^2)}.$$

Similarly, the upper bound on δ_C in the buyer's market is given by

$$|\delta_C| < \frac{(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))}{t_B t_S(\alpha_B + 2\alpha_S) + \alpha_B(2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)}.$$

It is unsurprising that the upper bounds on δ_C have more complicated forms than those on δ_B and δ_S since δ_C can affect both the platform's prices and the direct profits of developers, unlike δ_B and δ_S , which only affect prices.

Proposition 10

Neither complete symmetry in costs nor full compensation of one difference by another across platforms are necessary for the market to remain untipped for any parameter values that meet the second order condition.

Mathematically: For some combinations of parameter values, $n_k^{(i)} \in (0, 1)$ for $\delta_l > 0$ even if $\delta_m \geq 0$ for $m \neq l$.

The bound provided for each δ is strictly greater than zero, given that the second order condition²¹ obtains. Therefore, for any parameter values that meet the second order conditions, there exist nonzero levels of each δ (even given that the others are zero) which will not cause either side of the market to fully tip to one platform.

²¹In particular, equation (6), which says that $t_B t_S > \alpha_B \alpha_S$.

10.2 Cost Difference Compensation

I now allow for compensation between two δ 's such that each side maintains a constant membership level. First, I derive expressions for partial effects of each δ on the others, given a fixed level of $n_k^{(i)}$. Since everything but the δ 's never change, the coefficient terms in brackets remain the same when the differential is taken. Therefore the total differentials of the $n_k^{(i)}$ functions are given by

$$\begin{aligned} dn_S^{(i)} = & \left[\frac{3t_B(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_S \\ & + \left[\frac{(t_B t_S - \alpha_B \alpha_S)(\alpha_S + 2\alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_B \\ & + \left[\frac{t_B(4\alpha_B^2 + 4\alpha_S^2 + 6\alpha_B \alpha_S + 3t_B t_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_C \end{aligned}$$

and

$$\begin{aligned} dn_B^{(i)} = & \left[\frac{(t_B t_S - \alpha_B \alpha_S)(2\alpha_S + \alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_S \\ & + \left[\frac{3t_S(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_B \\ & + \left[\frac{t_B t_S(\alpha_B + 2\alpha_S) + \alpha_B(2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_C \end{aligned}$$

Therefore, the ratio of any two of the $d\delta$'s gives comparative statics on how the δ 's can compensate for each other to maintain the same level of $n_k^{(i)}$. I begin by setting $dn_S^{(i)} = dn_B^{(i)} = 0$ such that there is perfect compensation in δ 's and a fixed level of $n_k^{(i)}$ is maintained. A full derivation of the comparative statics is provided in appendix N.

Proposition 11

A platform that gains an advantage in one type of cost can give up some advantage in another part and maintain the same level of

membership. In general, the rate at which they must trade off is determined by ratios of network effects and product differentiation.

Explanation and proof follows.

10.2.1 Cost Difference Compensation: Buyers' Side with Developers' Side

Assuming that the level of asymmetry between the platforms in component production costs is constant, when platform i gains an advantage in the cost of serving buyers, they can afford to give up some advantage of serving developers at a rate determined by

$$\left. \frac{d\delta_S}{d\delta_B} \right|_{n_S^{(i)}} = - \left[\frac{(t_B t_S - \alpha_B \alpha_S)(\alpha_S + 2\alpha_B)}{3t_B(t_B t_S - \alpha_B \alpha_S)} \right] = - \frac{\alpha_S + 2\alpha_B}{3t_B}.$$

When network effects are high relative to product differentiation on the buyer's side, the platform can afford to give up a lot of advantage in the cost of serving the developers' side when they gain an advantage in costs on the buyers' side. Since t_B is relatively low, product differentiation is low on the buyers' side, and even a small increase in platform i 's cost advantage in serving buyers will sway a lot of buyers to change sides. Since network effects are also high, price changes will not change the equilibrium on the developers' side very much and so the platform can afford to give up a lot of cost advantage on the developers' side without overshooting the attractive effects of the additional buyers who joined the platform.

Finding the same partial derivative holding the number of buyers constant yields

$$\left. \frac{d\delta_S}{d\delta_B} \right|_{n_B^{(i)}} = - \left[\frac{3t_S(t_B t_S - \alpha_B \alpha_S)}{(t_B t_S - \alpha_B \alpha_S)(2\alpha_S + \alpha_B)} \right] = - \frac{3t_S}{(\alpha_B + 2\alpha_S)}.$$

Unsurprisingly, the expression for the same comparative static on the seller's side looks very similar. When platform i gains a marginal cost advantage on the buyer's side, they can afford to give up advantage on the seller's side comparable with developers' willingness to change platforms (t_S) normalized by the network effects—indicating how strongly buyers will be induced to change sides by a change in the price to them.

10.2.2 Cost Difference Compensation: Buyers' Side with Development

Holding the number of developers on each platform constant, development costs can trade off with costs of serving buyers according to

$$\left. \frac{d\delta_C}{d\delta_B} \right|_{n_S^{(i)}} = - \left[\frac{(t_B t_S - \alpha_B \alpha_S)(\alpha_S + 2\alpha_B)}{t_B(3(t_B t_S + (\alpha_B + \alpha_S)^2) + \alpha_B^2)} \right].$$

Similarly, holding buyers constant allows the platform to trade off at the rate

$$\left. \frac{d\delta_C}{d\delta_B} \right|_{n_B^{(i)}} = - \left[\frac{3t_S(t_B t_S - \alpha_B \alpha_S)}{t_B t_S(\alpha_B + 2\alpha_S) + \alpha_B(2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)} \right].$$

10.2.3 Cost Difference Compensation: Developers' Side with Development

Given that the number of developers remains constant, differences in costs of on-platform production can compensate for developers' side costs according to

$$\left. \frac{d\delta_C}{d\delta_S} \right|_{n_S^{(i)}} = - \left[\frac{3t_B(t_B t_S - \alpha_B \alpha_S)}{t_B(3(t_B t_S + (\alpha_B + \alpha_S)^2) + \alpha_B^2)} \right].$$

Similarly, given that the number of buyers remains constant,

$$\left. \frac{d\delta_C}{d\delta_S} \right|_{n_B^{(i)}} = - \left[\frac{(t_B t_S - \alpha_B \alpha_S)(2\alpha_S + \alpha_B)}{t_B t_S(\alpha_B + 2\alpha_S) + \alpha_B(2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)} \right].$$

It is unsurprising that all of the comparative statics are negative, indicating that when one δ encourages the market to tip to one side, another δ must compensate in favor of the other side of the market in order to ensure that the number of agents on each platform remains constant. The coefficients on δ_C seem to be difficult to interpret beyond their sign. This is unsurprising considering the complex way that δ_C changes interact with the profit functions of suppliers in the market.

Part IV

Conclusion

I provide eleven propositions detailing the findings of this research. Unlike previous research on two-sided markets, I fully solve both sides of the market to find how various parameter changes can affect the market equilibrium. Propositions one through eight are largely extensions of the previous literature and flesh out more precisely the interaction between network effects, product differentiation, and asymmetric costs in two-sided markets.

Propositions 8' through 11 are new to the literature. Proposition 8' suggests that the impact of inter-platform differences in costs borne by the platform are very different from the impact of costs borne by developers. A cost borne by the platform has relatively straightforward effects on the sides of the market through prices. Costs that affect software development directly affect developers' benefit from joining the platform and have much more complicated effects. In proposition nine, I find that the importance of asymmetries in platforms' costs have a non-linear relationship with network effects. Although network effects make the market *ceteris paribus* more likely to tip, any mechanism that works through prices alone is dampened by high network effects as agents care relatively less about the prices they are charged and more about platform membership on the other side as network effects rise. Proposition ten asserts that for any set of parameters meeting the second order condition, some level of inter-platform asymmetry in costs is tolerable before the market tips fully on either side. Lastly, proposition eleven demonstrates how cost asymmetries can compensate for each other to maintain a given membership share on each platform.

Other research indicates extensions of my model that might prove to have interesting qualitative results. The model in this paper assumes a constant marginal benefit ($\alpha_k, k = B, S$) from an additional agent joining either platform. This implies that the network's overall value is quadratic in the number of members, a rule of thumb known as *Metcalf's law*. Recent research has challenged the empirical and theoretical veracity of Metcalfe's law and points out that it seems likely that network topology and composition has a larger effect on users than network size *per se*. Weitzel, Wendt, Westarp, and König (2003) develops an agent-based simulation model of technology diffusion and standardization with network effects, and hypothesize that incorporating network topology is necessary to more precisely model real-world patterns. Swann (2002) finds that for markets with direct network effects, Metcalfe's law holds only under strong conditions. One possible extension rectifying this weakness of the model would be to allow users to have different valuations of the network benefit (See one treatment in Ambrus and Argenziano 2009).

As a stronger form of membership increases having variable effects on the other side, some research has shown that "superstar" software titles can have disproportionate effects on hardware sales in two-sided markets. In fact, a superstar title for a video game console can boost *hardware* sales by 14 % over a period of five months (Binken and Stremersch 2009). Clearly, a model that could account for more than just the level of software variety could be useful. Adopting an initial model of user utility similar to the one used in Hogendorn and Yuen (2009), which models the presence of "must-have" components on a platform with a positive fixed benefit for the presence of the component on the platform, might yield interesting qualitative results.

Furthermore, bringing the choice of single versus multihoming within the scope of the model would strengthen it. Several studies show qualitative differences between markets where all users single-home and ones where they do not. Sun and Tse (2007) studies a differential games framework of a two-sided market. They find that under single-homing the market is more likely to tip and that as tendency to multi-home increases in the market, the likelihood that a smaller competitor can survive increases. Furthermore, Caillaud and Jullien (2003) finds that when all agents single-home and only membership fees are used, there exists a tipped equilibrium, but that introducing usage fees or allowing multi-homing drastically reduces the profits of the dominant firm.

My main findings, that different types of costs have different effects on the market equilibrium and that asymmetries in the costs of serving one side become less important as network effects rise are new to the literature and may have significant regulatory implications. In two-sided markets, there can be regulatory concern about one platform having access to patented technology or some sort of advantage in serving customers. This model suggests that when network effects are high anyway, these types of asymmetries may, in fact, be of limited importance and that greater weight should be given to asymmetries that might affect developers' costs after joining one platform or the other. Without keeping these results in mind when making regulatory policy, mis-regulation seems entirely possible, if not likely.

Part V

Appendices

A Derivation of First Order Conditions

Taking derivatives with respect to both of the prices the platform chooses, the first order conditions each platform must solve for their individual optimization problem are given by

$$\frac{\partial \pi^{(i)}}{\partial p_S^{(i)}} = n_S^{(i)} + (p_S^{(i)} - f_S^{(i)}) \frac{\partial n_S^{(i)}}{\partial p_S^{(i)}} + (p_B^{(i)} - f_B^{(i)}) \frac{\partial n_B^{(i)}}{\partial p_S^{(i)}} = 0 \quad (17)$$

and

$$\frac{\partial \pi^{(i)}}{\partial p_B^{(i)}} = n_B^{(i)} + (p_S^{(i)} - f_S^{(i)}) \frac{\partial n_S^{(i)}}{\partial p_B^{(i)}} + (p_B^{(i)} - f_B^{(i)}) \frac{\partial n_B^{(i)}}{\partial p_B^{(i)}} = 0. \quad (18)$$

The partial derivatives in (17) and (18) are

$$\begin{aligned} \frac{\partial n_S^{(i)}}{\partial p_B^{(i)}} &= \frac{-\alpha_S}{2(t_B t_S - \alpha_B \alpha_S)} & \frac{\partial n_B^{(i)}}{\partial p_B^{(i)}} &= \frac{-t_S}{2(t_B t_S - \alpha_B \alpha_S)} \\ \frac{\partial n_B^{(i)}}{\partial p_S^{(i)}} &= \frac{-\alpha_B}{2(t_B t_S - \alpha_B \alpha_S)} & \frac{\partial n_S^{(i)}}{\partial p_S^{(i)}} &= \frac{-t_B}{2(t_B t_S - \alpha_B \alpha_S)}. \end{aligned}$$

Taking the partial derivatives above and plugging them into (17) and (18) yields (4),

$$\begin{aligned} \frac{\partial \pi^{(i)}}{\partial p_S^{(i)}} &= n_S^{(i)} - (p_S^{(i)} - f_S^{(i)}) \left[\frac{t_B}{2(t_B t_S - \alpha_B \alpha_S)} \right] \\ &\quad - (p_B^{(i)} - f_B^{(i)}) \left[\frac{\alpha_B}{2(t_B t_S - \alpha_B \alpha_S)} \right] = 0 \end{aligned}$$

and (5)

$$\begin{aligned} \frac{\partial \pi^{(i)}}{\partial p_B^{(i)}} &= n_B^{(i)} - (p_S^{(i)} - f_S^{(i)}) \left[\frac{\alpha_S}{2(t_B t_S - \alpha_B \alpha_S)} \right] \\ &\quad - (p_B^{(i)} - f_B^{(i)}) \left[\frac{t_S}{2(t_B t_S - \alpha_B \alpha_S)} \right] = 0. \end{aligned} \quad (19)$$

B Derivation of Second Order Conditions

The extrema found by solving the first order conditions are maxima if the Hessian matrix of ordered second partial derivatives,

$$\mathcal{H} = \begin{bmatrix} \frac{\partial^2 \pi^{(i)}}{\partial p_B^{(i)2}} & \frac{\partial^2 \pi^{(i)}}{\partial p_S^{(i)} \partial p_B^{(i)}} \\ \frac{\partial^2 \pi^{(i)}}{\partial p_S^{(i)} \partial p_B^{(i)}} & \frac{\partial^2 \pi^{(i)}}{\partial p_S^{(i)2}} \end{bmatrix},$$

is negative-semidefinite. The $n \times m$ Hessian matrix is negative semidefinite if

$$(-1)^k |\mathcal{H}_k| \geq 0 \text{ for } k \leq m.$$

Thus, this Hessian is negative semidefinite if

$$-\frac{\partial^2 \pi^{(i)}}{\partial p_B^{(i)2}} \geq 0 \text{ and } |\mathcal{H}| \geq 0.$$

Where \mathcal{H}_k is the leading principal minor of order k . The second partials for the hessian are

$$\begin{aligned} \frac{\partial^2 \pi^{(i)}}{\partial p_S^{(i)2}} &= \frac{\partial n_S^{(i)}}{\partial p_S^{(i)}} - \frac{t_B}{2(t_B t_S - \alpha_B \alpha_S)} = \frac{-2t_B}{2(t_B t_S - \alpha_B \alpha_S)} \\ \frac{\partial^2 \pi^{(i)}}{\partial p_B^{(i)2}} &= \frac{\partial n_B^{(i)}}{\partial p_B^{(i)}} - \frac{t_S}{2(t_B t_S - \alpha_B \alpha_S)} = \frac{-2t_S}{2(t_B t_S - \alpha_B \alpha_S)} \\ \frac{\partial^2 \pi^{(i)}}{\partial p_S^{(i)} \partial p_B^{(i)}} &= \frac{\partial^2 \pi^{(i)}}{\partial p_B^{(i)} \partial p_S^{(i)}} = \frac{\partial n_B^{(i)}}{\partial p_S^{(i)}} - \frac{\alpha_S}{2(t_B t_S - \alpha_B \alpha_S)} = -\frac{\alpha_B + \alpha_S}{2(t_B t_S - \alpha_B \alpha_S)} \end{aligned}$$

Thus the second order conditions require that

$$-\frac{\partial^2 \pi^{(i)}}{\partial p_B^{(i)2}} = - \left[\frac{-2t_S}{2(t_B t_S - \alpha_B \alpha_S)} \right] \geq 0,$$

recalling that $t_k, \alpha_k^{(i)} > 0$, the above condition holds if and only if

$$2(t_B t_S - \alpha_B \alpha_S) > 0.$$

meaning that

$$t_B t_S > \alpha_B \alpha_S.$$

The second part of the second order conditions require that

$$|\mathcal{H}| = \frac{4t_B t_S - (\alpha_B + \alpha_S)^2}{[2(t_B t_S - \alpha_B \alpha_S)]^2} \geq 0.$$

$2(t_B t_S - \alpha_B \alpha_S) > 0$ if the second order conditions hold, so the second order conditions hold if $4t_B t_S - (\alpha_B + \alpha_S)^2 \geq 0$.

C Proof of Lemma 1

Lemma: (7) is a stricter condition than (6) such that only (7) is necessary to assure the existence of a unique maximum on the profit function of the platform.

Proof: I begin with a condition necessary later in the proof.

Subclaim: In order to prove the main claim, I will need to show that $\alpha_B^2 + \alpha_S^2 \geq 2\alpha_B \alpha_S$. Thus I define a function $f(\alpha_B, \alpha_S) = \alpha_B^2 + \alpha_S^2 - 2\alpha_B \alpha_S$ and show that it is nonnegative on its domain, implying the subclaim holds.

Proof of Subclaim: I will show that the inequality above holds by demonstrating that the function achieves a minimum at $f(\alpha_B, \alpha_S) = 0$ and thus is non-negatively valued on its whole domain. I begin by finding first partial derivatives. Extrema

of the function occur when

$$\begin{aligned}\frac{\partial f(\alpha_B, \alpha_S)}{\partial \alpha_B} &= 2\alpha_B - 2\alpha_S = 0 \\ \frac{\partial f(\alpha_B, \alpha_S)}{\partial \alpha_S} &= 2\alpha_S - 2\alpha_B = 0.\end{aligned}$$

The conditions above both solve only when $\alpha_B = \alpha_S$. Furthermore, the function is (weakly) convex if its hessian matrix of partial derivatives is positive semidefinite. A 2×2 matrix is positive semidefinite given that both the entry in the upper left corner and its determinant are non-negative. f 's hessian is given by

$$\mathcal{H} = \begin{bmatrix} \frac{\partial^2 f(\alpha_B, \alpha_S)}{\partial \alpha_B^2} & \frac{\partial^2 f(\alpha_B, \alpha_S)}{\partial \alpha_B \alpha_S} \\ \frac{\partial^2 f(\alpha_B, \alpha_S)}{\partial \alpha_S \alpha_B} & \frac{\partial^2 f(\alpha_B, \alpha_S)}{\partial \alpha_S^2} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}.$$

Clearly $\frac{\partial^2 f(\alpha_B, \alpha_S)}{\partial \alpha_B^2} \geq 0$, and $|\mathcal{H}| = 0 \geq 0$. Therefore, $f(\alpha_B, \alpha_S)$ is positive semidefinite and achieves a global minimum along the line $\alpha_B = \alpha_S$. I solve to find the value of f along that line. Let $\alpha_B = \alpha_S = \alpha$, then

$$\alpha_B^2 + \alpha_S^2 - 2\alpha_B \alpha_S = \alpha^2 + \alpha^2 - 2\alpha^2 = 0.$$

Thus, f is nonnegative on its whole domain. I can rearrange $f(\alpha_B, \alpha_S)$ to show that

$$\alpha_B^2 + \alpha_S^2 \geq 2\alpha_B \alpha_S \tag{20}$$

Proceeding with the main proof, let (7) hold such that $4t_B t_S > (\alpha_B + \alpha_S)^2$. Expanding the right side, $(\alpha_B + \alpha_S)^2 = \alpha_B^2 + \alpha_S^2 + 2\alpha_B \alpha_S$. Given that (20) holds above, $\alpha_B^2 + \alpha_S^2 \geq 2\alpha_B \alpha_S$, so

$$4t_B t_S > \alpha_B^2 + \alpha_S^2 + 2\alpha_B \alpha_S \geq 2\alpha_B \alpha_S + 2\alpha_B \alpha_S = 4\alpha_B \alpha_S.$$

Dividing the far left and the far right by 4, yields equation (6),

$$t_B t_S > \alpha_B \alpha_S.$$

Therefore, if (7) holds strictly, the second order conditions hold strictly, and there is a unique maximum on the firm's profit function. ■

D Derivation of Contraction Mapping Conditions

A Nash equilibrium is unique if the reaction functions constitute a contraction mapping. Best response functions (8) and (9) are a contraction mapping if and only if

$$\frac{\partial p_B^{(i)}}{\partial p_B^{(j)}} + \frac{\partial p_B^{(i*)}}{\partial p_S^{(j)}} < 1$$

and

$$\frac{\partial p_S^{(i)}}{\partial p_S^{(j)}} + \frac{\partial p_S^{(i*)}}{\partial p_B^{(j)}} < 1.$$

The partial derivatives above are given by

$$\begin{aligned} \frac{\partial p_B^{(i)}}{\partial p_B^{(j)}} &= \frac{1}{2} & \frac{\partial p_B^{(i*)}}{\partial p_S^{(j)}} &= \frac{\alpha_B}{2t_S} \\ \frac{\partial p_S^{(i)}}{\partial p_S^{(j)}} &= \frac{1}{2} & \frac{\partial p_S^{(i*)}}{\partial p_B^{(j)}} &= \frac{\alpha_S}{2t_B}. \end{aligned}$$

The first condition holds if

$$\frac{1}{2} + \frac{\alpha_B}{2t_S} < 1, \text{ implying that } \frac{\alpha_B}{2t_S} < \frac{1}{2} \text{ and that } \alpha_B < t_S,$$

and the second condition holds if

$$\frac{1}{2} + \frac{\alpha_S}{2t_B} < 1, \text{ thus } \frac{\alpha_S}{2t_B} < \frac{1}{2} \text{ and } \alpha_S < t_B. \blacksquare$$

E Proof of Lemma 2

Lemma: If the contraction mapping conditions hold such that $\alpha_B < t_S$, $\alpha_S < t_B$, and $t_B = t_S$ then both second order conditions hold. Otherwise neither the second order condition nor the contraction mapping conditions imply the other.

Proof: Let the contraction mapping conditions hold such that $\alpha_B < t_S$, and $\alpha_S < t_B$ and let $t_B = t_S = t$. Since both $\alpha_S < t$, and $\alpha_B < t$, it's clear that $\alpha_B + \alpha_S < 2t$. Since both sides are positive, I can square both sides and maintain the inequality, so

$$(\alpha_B + \alpha_S)^2 < (2t)^2 = 4t^2 = 4t_B t_S.$$

Thus the second order condition, (7) holds.

Now let $t_B \neq t_S$. Let both contraction mapping conditions hold such that $\alpha_B < t_S$ and $\alpha_S < t_B$. It's clear that the values $\alpha_B = 1, t_S = 2, \alpha_S = 9, t_B = 10$ meets the contraction mapping conditions. However, $(\alpha_B + \alpha_S)^2 = (9 + 1)^2 = 100 \not< 80 = 4 * 2 * 10 = 4t_B t_S$.

Similarly, for the second order conditions to obtain, it must be the case that $(\alpha_B + \alpha_S)^2 < 4t_B t_S$. It's clear that the values $\alpha_B = 5, \alpha_S = 1, t_B = 10, t_S = 1$ meet the second order condition so that $(\alpha_B + \alpha_S)^2 = (5 + 1)^2 = 36 < 40 = 1 * 10 * 4 = 4t_B t_S$, but it is obviously not the case that $\alpha_B = 5 < 1 = t_S$. \blacksquare

F Derivation of Inter-Platform Price Differences

Solution for Δ_S From (8) and (9), I solve for $p_S^{(i)}$ to get

$$p_S^{(i)} = t_S + \frac{1}{3}\delta_C + \frac{2}{3}f_S^{(i)} + \frac{1}{3}f_S^{(j)} - \frac{\alpha_B\alpha_S}{t_B} + \frac{\alpha_S\Delta_B}{3t_B} - \frac{2\alpha_B(p_B^{(i)} - f_B^{(i)})}{3t_B} - \frac{\alpha_B(p_S^{(j)} - f_S^{(j)})}{3t_B}$$

and symmetrically,

$$p_S^{(j)} = t_S - \frac{1}{3}\delta_C + \frac{1}{3}f_S^{(i)} + \frac{2}{3}f_S^{(j)} - \frac{\alpha_B\alpha_S}{t_B} - \frac{\alpha_S\Delta_B}{3t_B} - \frac{\alpha_B(p_B^{(i)} - f_B^{(i)})}{3t_B} - \frac{2\alpha_B(p_S^{(j)} - f_S^{(j)})}{3t_B}.$$

Thus, $\Delta_S = p_S^{(j)} - p_S^{(i)}$ is

$$\Delta_S = -\frac{\delta_S}{3} - \frac{2}{3}\delta_C + \frac{\alpha_B((p_B^{(i)} - f_B^{(i)}) - (p_B^{(j)} - f_B^{(j)})) - 2\alpha_S\Delta_B}{3t_B},$$

which simplifies to (11),

$$\Delta_S = \frac{1}{3}\delta_S + \left[\frac{\alpha_B}{3t_B}\right]\delta_B - \frac{2}{3}\delta_C - \left[\frac{2\alpha_S + \alpha_B}{3t_B}\right]\Delta_B.$$

Solution for Δ_B I solve for the price differential on side B by first solving for prices to side B

$$p_B^{(i)} = t_B + \frac{2}{3}f_B^{(i)} + \frac{1}{3}f_B^{(j)} - \frac{\alpha_B\alpha_S}{t_S} + \frac{\alpha_B\Delta_C}{3t_S} - \frac{2\alpha_S(p_S^{(i)} - f_S^{(i)})}{3t_S} - \frac{\alpha_S(p_S^{(j)} - f_S^{(j)})}{3t_S}$$

as before, $p_B^{(j)}$ is just a simple transformation;

$$p_B^{(j)} = t_B + \frac{1}{3}f_B^{(i)} + \frac{2}{3}f_B^{(j)} - \frac{\alpha_B\alpha_S}{t_S} - \frac{\alpha_B\Delta_C}{3t_S} - \frac{\alpha_S(p_S^{(i)} - f_S^{(i)})}{3t_S} - \frac{2\alpha_S(p_S^{(j)} - f_S^{(j)})}{3t_S}.$$

Δ_B is given by

$$\Delta_B = \frac{\delta_B}{3} + \frac{\alpha_S((p_S^{(i)} - f_S^{(i)}) - (p_S^{(j)} - f_S^{(j)})) - 2\alpha_B\Delta_C}{3t_S}.$$

I simplify to yield (12),

$$\Delta_B = \frac{1}{3}\delta_B + \left[\frac{\alpha_S}{3t_S}\right]\delta_S - \left[\frac{2\alpha_B}{3t_S}\right]\delta_C - \left[\frac{\alpha_S + 2\alpha_B}{3t_S}\right]\Delta_S.$$

Elimination of Δ_B from Δ_S Plugging Δ_B into Δ_S ,

$$\begin{aligned} \Delta_S &= \frac{1}{3}\delta_S + \left[\frac{\alpha_B}{3t_B}\right]\delta_B - \frac{2}{3}\delta_C - \\ &\left[\frac{2\alpha_S + \alpha_B}{3t_B}\right] \left[\frac{1}{3}\delta_B + \left[\frac{\alpha_S}{3t_S}\right]\delta_S - \left[\frac{2\alpha_B}{3t_S}\right]\delta_C - \left[\frac{\alpha_S + 2\alpha_B}{3t_S}\right]\Delta_S \right] \end{aligned}$$

taking everything with Δ_S to one side yields

$$\begin{aligned} \Delta_S \left[1 - \frac{(\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}{9t_B t_S}\right] &= \left[\frac{1}{3} - \frac{\alpha_S(2\alpha_S + \alpha_B)}{9t_B t_S}\right]\delta_S \\ &+ \left[\frac{\alpha_B}{3t_B} - \frac{2\alpha_S + \alpha_B}{9t_B}\right]\delta_B + \left[\frac{2\alpha_B(2\alpha_S + \alpha_B)}{9t_B t_S} - \frac{2}{3}\right]\delta_C. \end{aligned}$$

Putting everything over the same denominator yields

$$\begin{aligned} \Delta_S \left[1 - \frac{(\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}{9t_B t_S}\right] &= \left[\frac{3t_B t_S - \alpha_S(2\alpha_S + \alpha_B)}{9t_B t_S}\right]\delta_S \\ &+ \left[\frac{3t_S \alpha_B - t_S(2\alpha_S + \alpha_B)}{9t_B t_S}\right]\delta_B + \left[\frac{2\alpha_B(2\alpha_S + \alpha_B) - 6t_B t_S}{9t_B t_S}\right]\delta_C, \end{aligned}$$

and solving for Δ_S yields (13),

$$\begin{aligned} \Delta_S &= \left[\frac{3t_B t_S - \alpha_S(2\alpha_S + \alpha_B)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}\right]\delta_S \\ &+ \left[\frac{2t_S(\alpha_B - \alpha_S)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}\right]\delta_B \\ &+ \left[\frac{2\alpha_B(2\alpha_S + \alpha_B) - 6t_B t_S}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}\right]\delta_C. \end{aligned}$$

Elimination of Δ_S from Δ_B Plugging Δ_S into Δ_B ,

$$\Delta_B = \left[\frac{\alpha_S}{3t_S} \right] \delta_S + \frac{1}{3} \delta_B - \left[\frac{2\alpha_B}{3t_S} \right] \delta_C$$

$$- \left[\frac{\alpha_S + 2\alpha_B}{3t_S} \right] \left[\frac{1}{3} \delta_S + \left[\frac{\alpha_B}{3t_B} \right] \delta_B - \frac{2}{3} \delta_C - \left[\frac{2\alpha_S + \alpha_B}{3t_B} \right] \Delta_B \right]$$

reorganizing, and taking every term with Δ_B to the left side,

$$\Delta_B \left[1 + \frac{(\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}{9t_B t_S} \right] = \left[\frac{\alpha_S}{3t_S} - \frac{\alpha_S + 2\alpha_B}{9t_S} \right] \delta_S$$

$$+ \left[\frac{1}{3} - \frac{\alpha_B(\alpha_S + 2\alpha_B)}{9t_B t_S} \right] \delta_B + \left[\frac{2(\alpha_S + 2\alpha_B)}{9t_S} - \frac{2\alpha_B}{3t_S} \right] \delta_C.$$

Putting everything on the right over the same denominator yields

$$\Delta_B \left[1 + \frac{(\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)}{9t_B t_S} \right] = \left[\frac{3t_B \alpha_S - t_B(\alpha_S + 2\alpha_B)}{9t_B t_S} \right] \delta_S$$

$$+ \left[\frac{3t_B t_S - \alpha_B(\alpha_S + 2\alpha_B)}{9t_B t_S} \right] \delta_B + \left[\frac{t_B(2(\alpha_S + 2\alpha_B) - 6t_B \alpha_B)}{9t_B t_S} \right] \delta_C,$$

which can be solved for Δ_B to yield equation (14)

$$\Delta_B = \left[\frac{2t_B(\alpha_S - \alpha_B)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)} \right] \delta_S$$

$$+ \left[\frac{3t_B t_S - \alpha_B(\alpha_S + 2\alpha_B)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)} \right] \delta_B$$

$$+ \left[\frac{2t_B(\alpha_S - \alpha_B)}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)} \right] \delta_C.$$

G Solving for Market Equilibrium

From (3), $n_S^{(i)}$ is given by

$$n_S^{(i)} = \frac{1}{2} + \frac{t_B \Delta_C + \alpha_S \Delta_B}{2(t_B t_S - \alpha_B \alpha_S)}$$

also from (3), $n_B^{(i)}$ is

$$n_B^{(i)} = \frac{1}{2} + \frac{\alpha_B \Delta_C + t_S \Delta_B}{2(t_B t_S - \alpha_B \alpha_S)}$$

plugging in for Δ_B and Δ_C in $n_S^{(i)}$ yields

$$\begin{aligned} n_S^{(i)} &= \frac{1}{2} + \frac{\alpha_S \left[\frac{(2t_B(\alpha_S - \alpha_B)\delta_C + (3t_B t_S - \alpha_B(\alpha_S + 2\alpha_B))\delta_B + (2t_B(\alpha_S - \alpha_B))\delta_S}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)} \right]}{2(t_B t_S - \alpha_B \alpha_S)} \\ &+ \frac{t_B \left[\delta_C + \frac{(2\alpha_B(\alpha_S + \alpha_B) - 6t_B t_S)\delta_C + (2t_S(\alpha_B - \alpha_S))\delta_B + (3t_B t_S - \alpha_S(2\alpha_S + \alpha_B))\delta_S}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)} \right]}{2(t_B t_S - \alpha_B \alpha_S)} \end{aligned}$$

and doing the same in $n_B^{(i)}$ yields

$$\begin{aligned} n_B^{(i)} &= \frac{1}{2} + \frac{t_S \left[\frac{(2t_B(\alpha_S - \alpha_B)\delta_C + (3t_B t_S - \alpha_B(\alpha_S + 2\alpha_B))\delta_B + (2t_B(\alpha_S - \alpha_B))\delta_S}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)} \right]}{2(t_B t_S - \alpha_B \alpha_S)} \\ &+ \frac{\alpha_B \left[\delta_C + \frac{(2\alpha_B(\alpha_S + \alpha_B) - 6t_B t_S)\delta_C + (2t_S(\alpha_B - \alpha_S))\delta_B + (3t_B t_S - \alpha_S(2\alpha_S + \alpha_B))\delta_S}{9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B)} \right]}{2(t_B t_S - \alpha_B \alpha_S)} \end{aligned}$$

Grouping terms yields

$$\begin{aligned} n_S^{(i)} &= \frac{1}{2} + \left[\frac{t_B(3t_B t_S - \alpha_S(2\alpha_S + \alpha_B)) + 2t_B \alpha_S(\alpha_S - \alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_S \\ &+ \left[\frac{2t_B t_S(\alpha_B - \alpha_S) + \alpha_S(3t_B t_S - \alpha_B(\alpha_S + 2\alpha_B))}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_B \\ &+ \left[\frac{t_B}{2(t_B t_S - \alpha_B \alpha_S)} + \frac{t_B(2\alpha_B(\alpha_S + \alpha_B) - 6t_B t_S) + 2t_B \alpha_S(\alpha_S - \alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_C \end{aligned}$$

and simplifying further yields (15)

$$\begin{aligned}
n_S^{(i)} &= \frac{1}{2} + \left[\frac{3t_B(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_S \\
&+ \left[\frac{(t_B t_S - \alpha_B \alpha_S)(\alpha_S + 2\alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_B \\
&+ \left[\frac{t_B}{2(t_B t_S - \alpha_B \alpha_S)} + \frac{t_B(\alpha_S^2 + 2\alpha_B^2 + \alpha_B \alpha_S - 6t_B t_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_C.
\end{aligned}$$

Grouping terms in $n_B^{(i)}$ yields

$$\begin{aligned}
n_B^{(i)} &= \frac{1}{2} + \left[\frac{\alpha_B(3t_B t_S - \alpha_S(2\alpha_S + \alpha_B)) + 2t_B t_S(\alpha_S - \alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_S \\
&+ \left[\frac{2t_S \alpha_B(\alpha_B - \alpha_S) + t_S(3t_B t_S - \alpha_B(\alpha_S + 2\alpha_B))}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_B \\
&+ \left[\frac{\alpha_B}{2(t_B t_S - \alpha_B \alpha_S)} + \frac{2\alpha_B^2(\alpha_S + \alpha_B) - 6t_B t_S \alpha_B + 2t_B t_S(\alpha_S - \alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_C,
\end{aligned}$$

simplifying further yields (16)

$$\begin{aligned}
n_B^{(i)} &= \frac{1}{2} + \left[\frac{(t_B t_S - \alpha_B \alpha_S)(2\alpha_S + \alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_S \\
&+ \left[\frac{3t_S(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_B \\
&+ \left[\frac{\alpha_B}{2(t_B t_S - \alpha_B \alpha_S)} + \frac{2\alpha_B^2(\alpha_S + \alpha_B) + 2t_B t_S(\alpha_S - 4\alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_C
\end{aligned}$$

H Proof of Proposition 3

Claim: If $\alpha_B > \alpha_S$, then $\frac{\partial \Delta_S}{\partial \delta_S} > 0$ and if $\alpha_S > \alpha_B$, $\frac{\partial \Delta_B}{\partial \delta_B} > 0$.

Proof: Let $\alpha_B > \alpha_S$. Multiplying both sides by $2\alpha_S$, $2\alpha_B \alpha_S > 2\alpha_S^2$. Writing $2\alpha_B \alpha_S$ as $3\alpha_B \alpha_S - \alpha_B \alpha_S$, $3\alpha_B \alpha_S - \alpha_B \alpha_S > 2\alpha_S^2$. By the second order condition (6), $t_B t_S > \alpha_B \alpha_S$, so $3t_B t_S - \alpha_B \alpha_S > 2\alpha_S^2$. Subtracting $2\alpha_S^2$ from both sides and simplifying, $3t_B t_S - \alpha_S(\alpha_B + 2\alpha_S) > 0$. Therefore, the coefficient on δ_S in (13), equal to $\frac{\partial \Delta_S}{\partial \delta_S}$ is positive.

Now, let $\alpha_S > \alpha_B$. Multiplying both sides by $2\alpha_B$, $2\alpha_B\alpha_S > 2\alpha_B^2$. Writing $2\alpha_B\alpha_S$ as $3\alpha_B\alpha_S - \alpha_B\alpha_S$, $3\alpha_B\alpha_S - \alpha_B\alpha_S > 2\alpha_B^2$. As before, $3t_Bt_S - \alpha_B\alpha_S > 2\alpha_B^2$. Subtracting $2\alpha_B^2$ from both sides and simplifying, $3t_Bt_S - \alpha_S(2\alpha_B + \alpha_S) > 0$. Therefore, the coefficient on δ_S in (13), equal to $\frac{\partial\Delta_B}{\partial\delta_B}$ is positive. ■

I Coefficient on δ_C in Δ_S

Claim: If $\alpha_B < t_B$, then $2\alpha_B(2\alpha_S + \alpha_B) - 6t_Bt_S$, the coefficient on δ_C in (13) is negative.

Proof: Let $\alpha_B < t_B$. I multiply both sides by $2\alpha_B$ to yield $2\alpha_B^2 < 2t_B\alpha_B$. From (D), $\alpha_B < t_S$, so $2t_B\alpha_B < 2t_Bt_S$. I add $4\alpha_B\alpha_S$ such that $2\alpha_B^2 + 4\alpha_B\alpha_S < 2t_Bt_S + 4\alpha_B\alpha_S$, and from (6) I know that $\alpha_B\alpha_S < t_Bt_S$, so $2t_Bt_S + 4\alpha_B\alpha_S < 6t_Bt_S$. Putting the two sides of the inequality together, it is clear that $2\alpha_B^2 + 4\alpha_B\alpha_S < 6t_Bt_S$. Factoring and subtracting $6t_Bt_S$, $2\alpha_B(\alpha_B + 2\alpha_S) - 6t_Bt_S < 0$. ■

J Proof of Armstrong Reaction Functions

Claim: If the two platforms are completely symmetric such that $\delta_C = \delta_B = \delta_S = 0$, the platforms set just one price, and this price is the same as that given in Armstrong (2006, p. 674).

Proof: Let there be perfect symmetry across the two platforms such that $\delta_C = \delta_B = \delta_S = 0$. From (8) and (9) the price pair satisfies

$$p_S^{(i*)} = \frac{t_S + \delta_C + p_S^{(j)} + f_S^{(i)}}{2} + \frac{\alpha_S\Delta_B - \alpha_B\alpha_S - \alpha_B(p_B^{(i)} - f_B^{(i)})}{2t_B}$$

and

$$p_B^{(i*)} = \frac{t_B + p_B^{(j)} + f_B^{(i)}}{2} + \frac{\alpha_B\Delta_C - \alpha_B\alpha_S - \alpha_S(p_S^{(i)} - f_S^{(i)})}{2t_S}$$

applying the assertion of symmetry from above, the platforms set just one price pair, $(p_B^{(i)} = p_B^{(j)} = p_B, p_S^{(i)} = p_S^{(j)} = p_S)$

$$p_S = \frac{t_S + p_S + f_S}{2} - \frac{\alpha_B}{2t_B}(\alpha_S + p_B - f_B)$$

and

$$p_B = \frac{t_B + p_B + f_B}{2} + \frac{\alpha_S}{2t_S}(\alpha_B + p_S - f_S).$$

Solving for the prices,

$$p_S = t_S + f_S - \frac{\alpha_B}{t_B}(\alpha_S + p_B - f_B); \quad p_B = t_B + f_B - \frac{\alpha_S}{t_S}(\alpha_B + p_S - f_S).$$

Which are precisely the best response functions given by Armstrong, given our notational differences (subscript 1s in Armstrong are B s here, and 2s are S s). ■

K Proof of Proposition 7

Claim: If $t_B > \alpha_B$, then $\frac{\partial n_S^{(i)}}{\partial \delta_S} > \frac{\partial n_S^{(i)}}{\partial \delta_B}$, and if $t_S > \alpha_S$, then $\frac{\partial n_B^{(i)}}{\partial \delta_B} > \frac{\partial n_B^{(i)}}{\partial \delta_S}$.

Proof: I established that $\frac{\partial n_S^{(i)}}{\partial \delta_S}$, $\frac{\partial n_S^{(i)}}{\partial \delta_B}$, $\frac{\partial n_B^{(i)}}{\partial \delta_B}$, and $\frac{\partial n_B^{(i)}}{\partial \delta_S}$ were all rankable by their numerators, since they have the same denominator. The numerators are given as below.

Side	Cost Difference	Numerator of Change in Side Membership
Developers	Serving Developers	$3t_B$
	Serving Buyers	$(\alpha_S + 2\alpha_B)$
Buyers	Serving Developers	$(2\alpha_S + \alpha_B)$
	Serving Buyers	$3t_S$

Recall that from the contraction mapping conditions, $t_B > \alpha_S$. Therefore, if it is also the case that $t_B > \alpha_B$, then $3t_B > \alpha_B + 2\alpha_S$ and $\frac{\partial n_S^{(i)}}{\partial \delta_S} > \frac{\partial n_S^{(i)}}{\partial \delta_B}$. Similarly, the contraction mapping conditions state that that $t_S > \alpha_B$, so if $t_S > \alpha_S$, then $3t_S > \alpha_S + 2\alpha_B$ and $\frac{\partial n_B^{(i)}}{\partial \delta_B} > \frac{\partial n_B^{(i)}}{\partial \delta_S}$. ■

L Proof of Lemma 3

Claim: If all terms multiplied by a δ sum to have a magnitude of less than $\frac{1}{2}$ for either $n_B^{(i)}$ or $n_S^{(i)}$, the market remains untipped for that side.

Proof: From equations (15) and (16), $n_S^{(i)}$ and $n_B^{(i)}$ are

$$\begin{aligned} n_S^{(i)} = & \frac{1}{2} + \left[\frac{3t_B(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_S \\ & + \left[\frac{(t_B t_S - \alpha_B \alpha_S)(\alpha_S + 2\alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_B \\ & + \left[\frac{2t_B}{2(t_B t_S - \alpha_B \alpha_S)} + \frac{t_B(4\alpha_B^2 + 4\alpha_S^2 + 6\alpha_B \alpha_S + 3t_B t_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_C \end{aligned}$$

and

$$\begin{aligned} n_B^{(i)} = & \frac{1}{2} + \left[\frac{(t_B t_S - \alpha_B \alpha_S)(2\alpha_S + \alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_S \\ & + \left[\frac{3t_S(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_B \\ & + \left[\frac{\alpha_B}{2(t_B t_S - \alpha_B \alpha_S)} + \frac{t_B t_S(\alpha_B + 2\alpha_S) + \alpha_B(2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] \delta_C. \end{aligned}$$

Since the coefficients on all of the δ terms are simply linear functions of the parameters, given any set of parameter values, I can solve them for a particular number. Therefore, for $\delta_l, l = S, B, C$, take the bracketed coefficient and call it P_l . Thus the expressions for equilibrium platform membership become

$$n_k^{(i)} = \frac{1}{2} + P_S \delta_S + P_B \delta_B + P_C \delta_C$$

Now, let the sum of all terms multiplied by a δ sum to a magnitude less than $\frac{1}{2}$ such that

$$|P_S\delta_S + P_B\delta_B + P_C\delta_C| < \frac{1}{2}.$$

By the definition of the absolute value function,

$$-\frac{1}{2} < P_S\delta_S + P_B\delta_B + P_C\delta_C < \frac{1}{2},$$

and adding $\frac{1}{2}$,

$$0 < \frac{1}{2} + P_S\delta_S + P_B\delta_B + P_C\delta_C < 1$$

It was established in 6.1 that $n_B = n_S = 1$, and that market coverage holds such that $n_k^{(i)} + n_k^{(j)} = 1$, so the market remains untipped as long as

$$0 < n_k^{(i)} < 1$$

for $k = B, S$. Therefore the claim holds. ■

M Corollary to Lemma 3

Proposition: If $\frac{\partial n_k^{(i)}}{\partial \delta_l} = P_l$ where $l = S, B, C$, then the no compensation bound for $|\delta_l|$ is given by $\frac{1}{2|P_l|}$.

Proof: Since the $n_k^{(i)}$ functions are linear in all three δ s, $\frac{\partial n_k^{(i)}}{\partial \delta_l} = P_l$ is simply the coefficient on δ_l in the $n_k^{(i)}$ function. Given that all the other δ s are zero, the market remains untipped if $\frac{1}{2} > |P_l\delta_l| = |P_l||\delta_l|$ by Lemma 3. Dividing by

the coefficient, the upper bound on the magnitude of δ_i for the market to remain untipped is given by $\frac{1}{2|P_i|}$. ■

N Derivation of Comparative Statics

N.1 Comparative Statics on δ_S With Respect to δ_B

I begin with the expression for $n_S^{(i)}$ and set $d\delta_C = 0$, and subtract the whole expression multiplied by $d\delta_S$, yielding

$$- \left[\frac{3t_B(t_{Bt_S} - \alpha_B\alpha_S)}{2(t_{Bt_S} - \alpha_B\alpha_S)(9t_{Bt_S} + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_S = \left[\frac{(t_{Bt_S} - \alpha_B\alpha_S)(\alpha_S + 2\alpha_B)}{2(t_{Bt_S} - \alpha_B\alpha_S)(9t_{Bt_S} + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_B.$$

I divide by both $d\delta_B$ and the coefficient on $d\delta_S$ to yield

$$\left. \frac{d\delta_S}{d\delta_B} \right|_{n_S^{(i)}} = - \left[\frac{(t_{Bt_S} - \alpha_B\alpha_S)(\alpha_S + 2\alpha_B)}{3t_B(t_{Bt_S} - \alpha_B\alpha_S)} \right] = - \frac{\alpha_S + 2\alpha_B}{3t_B}.$$

I can do the same for $n_B^{(i)}$ to yield

$$- \left[\frac{(t_{Bt_S} - \alpha_B\alpha_S)(2\alpha_S + \alpha_B)}{2(t_{Bt_S} - \alpha_B\alpha_S)(9t_{Bt_S} + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_S = \left[\frac{3t_S(t_{Bt_S} - \alpha_B\alpha_S)}{2(t_{Bt_S} - \alpha_B\alpha_S)(9t_{Bt_S} + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_B,$$

and I divide by both $d\delta_B$ and the coefficient on $d\delta_S$ to yield

$$\left. \frac{d\delta_S}{d\delta_B} \right|_{n_B^{(i)}} = - \left[\frac{3t_S(t_{Bt_S} - \alpha_B\alpha_S)}{(t_{Bt_S} - \alpha_B\alpha_S)(2\alpha_S + \alpha_B)} \right] = - \frac{3t_S}{(\alpha_B + 2\alpha_S)}.$$

N.2 Comparative Statics on δ_C With Respect to δ_B

I begin with the expression for $n_S^{(i)}$ and set $d\delta_S = 0$, and subtract the whole expression multiplied by $d\delta_C$, yielding

$$-\left[\frac{t_B(3(t_B t_S + (\alpha_B + \alpha_S)^2) + \alpha_B^2)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_C = \left[\frac{(t_B t_S - \alpha_B \alpha_S)(\alpha_S + 2\alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_B.$$

I divide by both $d\delta_B$ and the coefficient on $d\delta_C$ to yield

$$\frac{d\delta_C}{d\delta_B} \Big|_{n_S^{(i)}} = - \left[\frac{(t_B t_S - \alpha_B \alpha_S)(\alpha_S + 2\alpha_B)}{t_B(3(t_B t_S + (\alpha_B + \alpha_S)^2) + \alpha_B^2)} \right].$$

I do the same with $n_B^{(i)}$, set $d\delta_S = 0$, and subtract the whole expression multiplied by $d\delta_C$, yielding

$$-\left[\frac{t_B t_S(\alpha_B + 2\alpha_S) + \alpha_B(2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_C = \left[\frac{3t_S(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_B.$$

I now divide by both $d\delta_B$ and the coefficient on $d\delta_C$ to yield

$$\frac{d\delta_C}{d\delta_B} \Big|_{n_B^{(i)}} = - \left[\frac{3t_S(t_B t_S - \alpha_B \alpha_S)}{t_B t_S(\alpha_B + 2\alpha_S) + \alpha_B(2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)} \right].$$

N.3 Comparative Statics on δ_C With Respect to δ_S

I begin with the expression for $n_S^{(i)}$ and set $d\delta_B = 0$, and subtract the whole expression multiplied by $d\delta_C$, yielding

$$- \left[\frac{t_B(3(t_B t_S + (\alpha_B + \alpha_S)^2) + \alpha_B^2)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_C = \left[\frac{3t_B(t_B t_S - \alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_S.$$

I now divide by both $d\delta_B$ and the coefficient on $d\delta_C$ to yield

$$\frac{d\delta_C}{d\delta_S} \Big|_{n_S^{(i)}} = - \left[\frac{3t_B(t_B t_S - \alpha_B \alpha_S)}{t_B(3(t_B t_S + (\alpha_B + \alpha_S)^2) + \alpha_B^2)} \right].$$

I do the same with $n_B^{(i)}$, set $d\delta_S = 0$, and subtract the whole expression multiplied by $d\delta_C$, yielding

$$- \left[\frac{t_B t_S(\alpha_B + 2\alpha_S) + \alpha_B(2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_C = \left[\frac{(t_B t_S - \alpha_B \alpha_S)(2\alpha_S + \alpha_B)}{2(t_B t_S - \alpha_B \alpha_S)(9t_B t_S + (\alpha_S + 2\alpha_B)(2\alpha_S + \alpha_B))} \right] d\delta_S.$$

I now divide by both $d\delta_B$ and the coefficient on $d\delta_C$ to yield

$$\frac{d\delta_C}{d\delta_S} \Big|_{n_B^{(i)}} = - \left[\frac{(t_B t_S - \alpha_B \alpha_S)(2\alpha_S + \alpha_B)}{t_B t_S(\alpha_B + 2\alpha_S) + \alpha_B(2\alpha_S^2 + 4\alpha_B^2 + 7\alpha_B \alpha_S)} \right].$$

References

- AMBRUS, A., AND R. ARGENZIANO (2009): “Asymmetric Networks in Two-Sided Markets,” *American Economic Journal: Microeconomics*, 1(1), 17–52.
- ARMSTRONG, M. (2006): “Competition in two-sided markets,” *RAND JOURNAL OF ECONOMICS*, 37(3), 668–691.
- (2007): “Two-Sided Markets: Economic Theory and Policy Implications,” *Recent Developments in Antitrust: Theory and Evidence. 2007*, pp. 39–59.
- ARMSTRONG, M., AND J. WRIGHT (2007): “Two-Sided Markets, Competitive Bottlenecks and Exclusive Contracts,” *Economic Theory*, 32(2), 353–80.
- BAKOS, Y., AND E. KATSAMAKAS (2008): “Design and ownership of two-sided networks: Implications for Internet platforms,” *JOURNAL OF MANAGEMENT INFORMATION SYSTEMS*, 25(2), 171–202.
- BECK, R., D. BEIMBORN, T. WEITZEL, AND W. KÖNIG (2008): “Network effects as drivers of individual technology adoption: Analyzing adoption and diffusion of mobile communication services,” *Information Systems Frontiers*, 10(4), 415–429.
- BINKEN, J. L., AND S. STREMERSCHE (2009): “The Effects of Superstar Software on Hardware Sales in System Markets,” *American Marketing Association*, 73(2), 88–104.
- BIRKE, D. (2009): “The Economics of Networks: A Survey of the Empirical Literature,” *Journal of Economic Surveys*, 23(4), 762–93.
- CAILLAUD, B., AND B. JULLIEN (2003): “Chicken & Egg: Competition among Intermediation Service Providers,” *RAND Journal of Economics*, 34(2), 309–28.
- CARNOY, D. (2008): “Warner goes Blu-ray exclusively, delivering crushing blow to HD DVD,” *Cnet News*.
- CHEN, J. (2009): “Switching Costs in Network Industries,” *SSRN eLibrary*.
- CHOU, C., AND O. SHY (1990): “Network Effects without Network Externalities,” *International Journal of Industrial Organization*, 8(1990), 259–70.
- CHURCH, J., AND N. GANDAL (1992): “Network Effects, Software Provision, and Standardization,” *Journal of Industrial Economics*, 40(1), 85–103.
- (1993): “Complementary Network Externalities and Technological Adoption,” *International Journal of Industrial Organization*, 11(1993), 239–60.

- (2000): “Systems Competition, Vertical Merger, and Foreclosure,” *Journal of Economics and Management Strategy*, 9(2000), 25–51.
- CORTS, K., AND M. LEDERMAN (2009): “Software exclusivity and the scope of indirect network effects in the U.S. home video game market,” *International Journal of Industrial Organization*, 27(2), 121–136.
- CREMER, J., P. REY, AND J. TIROLE (2000): “Connectivity in the Commercial Internet,” *Journal of Industrial Economics*, 48(4), 433–472.
- DUBE, J. H., G. J. HITSCH, AND P. K. CHINTAGUNTA (2008): “Tipping and Concentration in Markets with Indirect Network Effects,” *SSRN eLibrary*.
- ECONOMIDES, N., AND F. FLYER (1997): “Compatibility and Market Structure for Network Goods,” *SSRN eLibrary*.
- GANDAL, N. (1995): “Competing Compatibility Standards and Network Externalities in the PC Software Market,” *Review of Economics and Statistics*, 1995(4), 599–608.
- GOOLSBEE, A., AND P. KLENOW (2002): “Evidence on learning and network externalities in the diffusion of home computers,” *JOURNAL OF LAW & ECONOMICS*, 45(2), 317–343.
- HAGIU, A. (2009): “Two-Sided Platforms: Product Variety and Pricing Structures,” *Journal of Economics and Management Strategy*, 18(2009), 1011–43.
- HEAL, G., AND H. KUNREUTHER (2010): “Social Reinforcement: Cascades, Entrapment, and Tipping,” *American Economic Journal: Microeconomics*, 2(1), 86–99.
- HOGENDORN, C., AND S. K. Y. YUEN (2009): “Platform Competition with ‘Must Have’ Components,” *The Journal of Industrial Economics*, 57(2), 294–318.
- JULLIEN, B. (2006): “Two-Sided Markets and Electronic Intermediaries,” *Industrial Organization and the Digital Economy. 2006*, pp. 273–302.
- KATZ, M., AND C. SHAPIRO (1985): “Network Externalities, Competition, and Compatibility,” *The American Economic Review*, 75(3), 424–440.
- (1986): “Technology Adoption in the Presence of Network Externalities,” *Journal of Political Economy*, 94(4), 822.
- (1994): “Systems Competition and Network Effects,” *The Journal of Economic Perspectives*, 8(2), 93–115.
- KLEMPERER, P. (1987): “Markets with Consumer Switching Costs,” *The Quarterly Journal of Economics*, 102(2), 375–394.

- KWOKA, J. E., AND L. WHITE (2004): *The Antitrust Revolution: Economics, Competition, and Policy*. Oxford University Press, New York, NY.
- LEE, D., AND H. MENDELSON (2007): “Adoption of information technology under network effects,” *INFORMATION SYSTEMS RESEARCH*, 18(4), 395–413.
- MATTUCK, A. P. (1998): *Introduction to Analysis*. Prentice Hall, united states ed edn.
- PARKER, G., AND M. W. V. ALSTYNE (2005): “Two-Sided Network Effects: A Theory of Information Product Design,” *Manag*, 51(10), 1494–1504.
- ROCHET, J., AND J. TIROLE (2003): “Platform Competition in Two-Sided Markets,” *Journal of the European Economic Association*, 1(2003), 990–1029.
- (2006): “Two-sided markets: a progress report,” *The RAND Journal of Economics*, 37(3), 645–667.
- ROHLFS, J. H. (2003): *Bandwagon effects in high-technology industries*. MIT Press.
- ROMAN, B., B. DANIEL, W. TIM, K. WOLFGANG, AND NIG (2008): “Network effects as drivers of individual technology adoption: Analyzing adoption and diffusion of mobile communication services,” *Information Systems Frontiers*, 10(4), 415–429.
- RUBINFELD, D., J. J. E. KWOKA, AND L. WHITE (2004): “Maintenance of Monopoly: U.S. v. Microsoft (2001),” in *The Antitrust Revolution: Economics, Competition, and Policy*, vol. 4, pp. 476–501. Oxford University Press, New York, NY.
- RYSMAN, M. (2009): “The Economics of Two-Sided Markets,” *Journal of Economic Perspectives*, 23(3), 125–43.
- SHAPIRO, C., AND H. VARIAN (1999): *Information Rules*. Harvard Business School Press, Boston, Massachusetts.
- STREMERSCH, S., G. TELLIS, P. FRANSES, AND J. BINKEN (2007): “Indirect network effects in new product growth,” *JOURNAL OF MARKETING*, 71(3), 52–74.
- SUN, M., AND E. TSE (2007): “When Does the Winner Take All in Two-Sided Markets?,” *Review of Network Economics*, 6(1), 16–41.
- SWANN, G. M. P. (2002): “The functional form of network effects,” *Information Economics and Policy*, 14(3), 417–429.

WEITZEL, T., O. WENDT, F. WESTARP, AND W. KÖNIG (2003): “Network Effects and Diffusion Theory: Network Analysis in Economics,” *Int. J. IT Standards and Standardization*, 1(2), 1–21.

WRIGHT, J. (2003): “One-Sided Logic in Two-Sided Markets,” *SSRN eLibrary*.