

Central Bank Ambiguity

by

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Abstract

Prevailing contemporary wisdom is that transparency in monetary policy-making is beneficial, but this thesis ventures to provide evidence that limited transparency may be desirable. A New-Keynesian model of monetary policy-making is augmented with time-varying employment objectives, and given this it is shown that a monetary authority has some incentive to “lie” to the public. Then, after two forms of credibility are identified, it is demonstrated in a “baseline” case that “ambiguity” is preferable to “lying” in that it avoids deceit’s associated credibility problems while achieving similar benefits to moderating inflation. Next, a counterargument for transparency is admitted, as a risk-averse monetary authority may desire total transparency as a consequence of the Characterization Theorem for second-order stochastic dominance. (Hadar and Russell, 1971) However, opacity is supported by three additional arguments: (1) When the public has its own private information available to all agents, some decrease in the accuracy of the monetary authority’s signal may be beneficial as in Geraats (2007). (2) When individual private agents have their own estimates of varying accuracy of the natural rate of unemployment, opacity of the monetary authority’s estimate may be desirable as a consequence of a Keynesian beauty-contest phenomenon seen in Morris and Shin (2001). (3) In a Markov perfect equilibria model of persistent inflation similar to Ball (1995), opacity may provide an accommodative regime the means to avoid an expectations trap.

Preface and Acknowledgements

Central bank transparency is now a particularly popular topic in academic and policy discussion. Unavoidably, the more it is publicly discussed by policy-makers, the more it is facilitated. Ironically, it would be one extreme instance of transparency which led me to a study against it.

A co-intern and friend of mine at the Bureau of Economic Analysis in Washington D.C., Jordan Boslego (Harvard '08), was lucky enough to attend a series of conferences in Washington for Harvard students with famous and noteworthy individuals. When I learned he would be attending a small, private meeting with Federal Reserve Chairman Ben Bernanke in July, I gave him a simple mission: ask for a thesis topic. When the question was asked (by another individual), the answer was clear: Bernanke's suggestion would be a study of the degree transparency in central banks worldwide.

There was a publication on the "amount" of transparency in many central banks released by Geraats (2002) prior to Bernanke's meeting with Harvard students. Within this paper Geraats defines several types of transparency and makes use of self-guided surveys.¹ However, empirical studies of transparency are complicated and necessarily intricate, since even the color of Greenspan's tie does not go unnoticed.² Due to such complications in empirical studies of transparency, theory pieces have proved more numerous in the literature.

The first instance in which I came to question the benefits of transparency was when I attended Chairman Bernanke's Humphrey-Hawkins testimony to the Senate in

¹ The types are: political, procedural, economic, policy, and operational transparency; to be defined.

² See Murdzhev and Tomljanovich (2006).

July. I was astounded by the claims Senator Bunning (R-KY) made to the Chairman. The most resounding of these was when Bunning implicitly contended that the Federal Reserve should aim to keep the Dow Jones and other industry averages high. Bernanke simply replied that the Federal Reserve's mission is the dual-mandate of minimum inflation and sustainable unemployment. It seemed as if Senator Bunning was not exactly clear as to the Federal Reserve's mission, and I was led to consider the pressures the Fed must receive from short-horizon politicians.

My topic truly solidified in meetings with my advisor, Professor Hanson. In conversation with him I began to see that my question of "how much transparency?" was in fact a question of "how little." When recently attending multiple interviews at the Board of Governors in March 2007, I was reaffirmed in my topic choice by the responses I received from many of the thirty-to-forty Fed economists I met. Among the comments I heard were (paraphrased), "transparency is Bernanke's pet topic," "we believe in transparency, so it'd be interesting to read your paper," and, "such an argument would be especially relevant in the move to inflation targeting."³

Among the many people I have to thank for their contributions to this paper, Professor Hanson stands out as a shining example of a dedicated tutor and academic guide. Professor Hanson provided my first opportunity to experience academic research when I worked as a research assistant for him and Jayson Whitehorn ('05) after my sophomore year in the summer of 2005. From there, he has served as my academic advisor and assisted me in completing research projects on the Phillips curve in his Monetary Economics course and on the Kalman filter method of estimating the NAIRU in his Econometrics course. The sophistication with which I

³ Interestingly, this last comment took a "matter-of-fact" tone.

have begun to study economics is largely due to him and I am thankful for his faith in my ability; without his continued guidance over the course of my studies at Wesleyan, I surely would not have been able to complete a thesis of this caliber. My choice to pursue a PhD in economics may be accredited to the excitement for academic research I have inherited from him.

Professor Skillman has also been extremely helpful in my progress toward more advanced economic thought. I feel that his Mathematical Economics course is one of the highest-quality courses I have been fortunate enough to take at Wesleyan, and it has encouraged me to undertake a mathematically complicated theoretic approach to this thesis. The fact that his course was extremely difficult forced me to learn and think at a higher level, and instilled a confidence in my ability in economics that I feel is essential for undertaking a thesis such as this. He was additionally available on several occasions to speak with me specifically about my independent research, and options for graduate school, for which I am very grateful.

Among those others I have to thank are the Economics faculty of Wesleyan University and the staff economists at the Federal Reserve of New York City and the Federal Reserve Board in Washington, D.C. for the willingness to hear my project in person and providing helpful suggestions. Additional thanks are due to Jordan Boslego, for his assistance in meeting with Chairman Bernanke, and Ellen Werble for introducing me to Mathematica which proved to be immensely useful in the more intricate computation of this thesis. Thanks to friends and family for unwavering moral support.

Dedicated to Stephanie O'B

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“A firm belief in the miraculous healing power of central bank transparency is a core tenet of the new religion that the economics of monetary policy has somehow become. [as to whether less transparency could be better than more,] ...the question is plainly worth consideration.”

~Benjamin Friedman, 2005
(Comments to Morris and Shin, 2005)

1. Introduction

a. Inflation expectations and monetary policy

Public expectations of inflation are of great concern to monetary policy-makers. Michael Woodford, a foremost monetary economist has written, “For not only do expectations about policy matter [for effective monetary policy], but, at least under current conditions, very little *else* matters.” (Woodford, 2005) However, inflation expectations have not always been central to monetary policy-making. Despite playing an important role in the theory of Pigou, Hicks, and Keynes, (Sargent, 2002) it was not until the time of the critiques of Phelps (1967) and Friedman (1968) that expectations, along with the natural rate of unemployment, would become of obvious concern to central bankers. Inflation expectations were explicitly mentioned in the minutes of the Federal Reserve Board for the first time in 1968 (Romer and Romer 2002):

“[E]xpectations of continued inflationary pressures appeared to be widespread, and the Committee referred to the persistence of inflationary pressures and expectations and the prevailing inflationary psychology.”

(US FRB *Minutes*, 1968)

Friedman and Phelps rebuffed the prevailing wisdom that the Phillips curve's negative relationship between inflation and unemployment (or positive relationship with output) could be exploited to stimulate the economy. They contested that in the long-run, inflationary policy could not permanently maintain unemployment below its natural rate. However, even for short-run trade-offs to be made, Friedman and Phelps believed that inflation would need to be set unequal to private agent expectations, possibly having in mind a Phillips curve not dissimilar to:

$$\pi_t = E_{t-1}\pi_t + (u_t - u_t^n) + e_t \quad (1.1)$$

where the term π_t is inflation, $E_{t-1}\pi_t$ is the public's estimate of concurrent inflation last period, u_t is the unemployment rate, u_t^n is the natural rate of unemployment, and e_t is a white-noise macroeconomic shock.

Friedman and Phelps' analyses concluded that the attempts of a central bank to push inflation above expectations in order to achieve unemployment below the natural rate would be thwarted by adaptive expectation-forming private agents.¹ In the long run, unemployment could not deviate from its natural rate permanently.²

Since Friedman and Phelps' seminal work, there has been a particularly large volume of studies of expectation formation. (Sheffrin, 1983) It was not until the theory of *rational* expectation formation, first proposed by Muth (1960), was developed by Lucas (1976), and later Sargent (1986), that a near-contemporary understanding of expectations was achieved. The theory of rational expectations asserts that people do not make systematic errors when predicting the coming (i.e.,

¹ Adaptive private agents form expectations as: $E_{t-1}\pi_t = \pi_{t-1}$

² Friedman was awarded the Nobel Prize for his contributions to this topic in 1976 and recently Phelps in 2006.

next period) level of inflation. To be precise, rational expectations are formed using as much relevant information as is available, although there may be unavailable information relevant to inflation formation. Furthermore, increased expectations may themselves cause actual inflation to rise, if an “accommodative” monetary authority is in power and would characteristically rather endure inflation than a recession (equation (1.1) implies these dynamics). Thus, expectations play an active role in the state of the economy rather than simply a predictive one.

The concerns of the Fed documented in their 1968 *Minutes* came at the beginning of a decade of stagflation, or high inflation and unemployment, lasting throughout the 1970’s and ending only in a significant recession in the early 80’s. Indeed, these central bankers may have suspected, as theory would suggest, that rising expectations were a sign of oncoming inflation. However, the Fed’s apprehension apparent from their 1968 *Minutes* does not necessarily imply that it had the means or the know-how to prevent the oncoming decade of inflation.

One possible explanation for why the Federal Reserve was pressured into high money creation throughout the 1970’s is the “expectations trap” hypothesis suggested by Chari, Christiano, and Eichenbaum (1998). The theory posits that the concurrent oil supply crisis served as a macroeconomic shock which cause expectations of inflation to increase. The theory suggests that the accommodative Federal Reserve under Arthur Burns (1970-78) was forced to deliver higher inflation so as not to incur very high unemployment via an employment-inflation relationship like the Phillips curve (1.1). Such a predicament induces a self-fulfilling cycle in which expectations continue to rise, and the central bank, fearing very high unemployment, continually

validates (at the very least, partially) skyward expectations with inflationary policy. The only way for such an expectations trap to be appeased is at the hands of a recession-inducing, non-accommodative central bank such as Paul Volcker's Fed (1979-87).

A depiction of the upward-sloping inflation through Burns' "accommodative" tenure and downward-sloping through Volcker's "hawkish" tenure is seen in *Figure 6*. Christiano and Gust (2000) have shown that the expectations trap hypothesis quantitatively explains the take-off and persistently high level of inflation in the early 1970's. The key in their result is that an exogenous transitory shock, specifically shocks relating to the 1970's oil supply crisis, is the push-start to pendulum-like interaction between private agent heightened expectation formation and Federal Reserve verification.

An alternative explanation for why inflation may be persistently high, as it was in the 1970's, is given by Barro and Gordon (1983), motivated by the Nobel-prize winning work of Kydland and Prescott (1977). Kydland and Prescott show that a discretionary policy-maker will not be able to maximize the intertemporal social welfare function it shares with the public without turning to inflationary policy, provided market imperfections such as unemployment compensation cause the natural rate of unemployment to be higher than without these distortions. Such imperfections induce the central bank to target unemployment below the artificially high natural rate.

In order to create this negative unemployment gap, the central bank must somehow create unexpectedly high inflation, so that unemployment falls below the

artificially high natural rate to the undistorted level. As rational agents may not be repeatedly fooled, from the one time they are tricked onward, their expectations will rise. Thus, inflation policy will need to respond by increasing, if the desired unemployment gap is still negative. Rational expectations thereby prevent the central bank from duping the public and maximizing its social welfare function over two periods. Thus, given market imperfections create incentives for employment stimulation, Kydland and Prescott conclude that discretionary policy is suboptimal to a policy rule.

Barro and Gordon arrive at a similar result, citing a politically-motivated incentive to push unemployment below the natural rate. They reiterate a result of Kydland and Prescott that a policy rule is a Pareto-superior alternative to discretion for keeping inflation policies, and expectations, low. They go on to say, however, that reputational equilibria³ may replace the need for a formal policy rule and grant time-consistent policy-making to a discretionary central bank. Ireland (1999) shows that the Barro-Gordon time-inconsistency model quantitatively explains the dynamics of inflation in the United States from 1960 to 1997.

The expectations trap and Kydland-Prescott / Barro-Gordon explanations for the Great Inflation of the 1970's bring into focus why public expectations of inflation are such an essential topic in the process of monetary policy-making. To complicate things further, expectations of inflation may even become unbounded without any fault of the central bank whatsoever; Ball (1991) has shown that inflation may arise out of self-fulfilling prophecies in Barro-Gordon models. Thus, the Federal Reserve

³ "Reputation" is often, and in this particular case, used synonymously with "credibility."

should be, and is, concerned with keeping expectations of inflation well-anchored.⁴

In the June 2006 Semiannual Monetary Policy Report to Congress presented by Chairman Ben Bernanke, inflation expectations are mentioned with great importance on two separate occasions. In the February 2006 Report, they are mentioned on five separate occasions.

b. Credibility

"If they [expectations of inflation] begin to move upward, it's a sign that people are losing confidence in the Fed's commitment to price stability... They are, in a sense, a report card on Fed credibility."

~ Laurence Meyer,
Former Governor of the FRB
(Conkey, June 12, 2006)

One way central banks attempt to keep expectations anchored is by establishing credibility for delivering low inflation. The credibility of a central bank, as suggested by Meyer, is often defined by the public's inference of future inflation given its current relevant information set; however, a universally agreed-upon definition does not exist. Blinder (2000) cites that in the academic literature, credibility is associated with one of three things: strong aversion to inflation, incentive compatibility, or precommitment. However, these three definitions are regrettably vague, and potentially at odds with one another. For example, in the Barro-Gordon framework, credibility is a product of a reputation for delivering low inflation in the past (i.e., a strong aversion to inflation) is an alternative to formal

⁴ "Anchored expectations" is the Fed's term of choice for "low expectations."

precommitment, or policy rules. That is, although credibility is achievable through precommitment, it is not a necessary condition for achieving optimal inflation levels. Secondly, incentive compatibility in no way guarantees aversion to inflation; it is conceivable that the public and central bank could both enjoy high inflation and employment, and the bank could be perfectly credible for delivering this.

Alternatively, credibility is implicitly defined by Faust and Svensson's (2001) theoretic framework as the central bank's past ability to achieve their inflation announcement. This definition implicitly defines credibility in terms of achieving stated goals, and inherits precommitment as a case rather than a defining factor. The Federal Reserve has an official dual mandate for low inflation and maximum sustainable employment, i.e., maximum long-run employment. Therefore in the Fed's case, achieving a single objective of minimum inflation may not be seen as fully credible. Finally, as credibility is here defined in terms of what the central bank communicates, it provides a link to transparency. These two concepts will be shown to be, in fact, inextricably linked.

Blinder (2000) polls 84 heads of central banks around the world, asking if credibility is "unimportant" (score=1) up to "of the utmost importance" (score=5) in monetary policy-making. As might be expected, their responses average a score of 4.83. Given these observations and the implicit definition of Faust and Svensson, it is clear a central bank might like to achieve credibility for fighting inflation so that inflation expectations remain anchored. Achieving credibility could be achieved by adhering to a low-inflation rule, or in a discretionary framework, achieving a target rate in many consecutive periods and building a favorable reputation. However, it

must be stressed neither of these possibilities is a necessary condition; the German Bundesbank, once widely considered one of the most credible banks in the world, missed its target rate 50% of the time. (Blinder, 2000) The Bundesbank was considered credible because it delivered consistently low inflation and was communicative about its goals with the public.

c. Transparency

When a central bank is very clear to the public about its practices and goals, it is said to be transparent. Transparency can aid credibility by reducing a source of the public's uncertainty regarding what monetary policy will be. One definition of central bank transparency is given by William Poole (2001):

“Transparency in a general sense simply means providing the fullest explanation possible of policy actions and the considerations underlying them, in as timely a manner as possible.”

Many central banks have trended towards being more communicative, or transparent, with the public. Theoretically, transparency can help a central bank manage the economy by keeping the public's expectations of inflation more closely aligned with its own planned policy. (Woodford, 2005) Also, transparency is potentially beneficial if the central bank has some insider information, facilitation of this knowledge increases private agent forecast accuracy, and thereby reduces inflation and output variability. (Carpenter, 2004)

Petra Geraats (2002) has given more structure to these issues by defining five different types of transparency. She defines a monetary authority as being

“politically” transparent if it communicates its objectives and targets to the public, and as being “economically” transparent if it makes available its data, models and forecasts. A monetary authority is “procedurally” transparent in proportion to the amount of information it releases on its strategy, minutes, and voting, and transparent in the “policy” sense if it releases its policy decision and explanation. Finally, a monetary authority is “operationally” transparent if it communicates its control error, or, shocks in the monetary transmission mechanism, which it does not know itself until actual inflation is realized. All of these types of transparency except for “procedural” are explicitly identified and examined in this thesis, and they are pointed out where they occur. “Procedural” transparency may be considered an operational case of how communication is facilitated when mentioned abstractly. This thesis attempts to address all feasible definitions of transparency, and will show the detriments to increased transparency in all cases.

Ben Bernanke, an advocate of increased transparency, has said:

“In the long run, communicating the central bank’s objectives and policy strategies can help to anchor the public’s long-term expectations – most importantly, its expectations of inflation.”

(Bernanke, 2004)

In the near-century history of the Federal Reserve, its transparency has increased drastically, as may be seen in the evolution of Federal Reserve communication extensively documented in *Table 1*. In a striking example of the progress that has made towards transparency, consider the following comments of then-Chairman Alan Greenspan:

1987: “Since I have become a central banker I have learned to mumble with great incoherence.”

2002: “Openness is an obligation of a central bank in a free and democratic society.”

Although Greenspan is famous for his often cryptic language, known by Fed watchers as “Greenspeak,” he certainly became more deliberate and clear in his speech over his tenure. Indeed, many of the quotations in this thesis are his own. The second of his statements noted above additionally brings to attention the fact that transparency enhances central bank accountability to the government. In a democratic society such as the United States, politicians and the public may push for information to be made public. However, increased accountability is not necessarily in the central bank’s best interest.⁵ For example, in the Barro-Gordon model, political incentives cause the central bank to enact a policy that leads to greater inflation in equilibrium than if it were not subject to political pressure. Thus, the central bank may want to avoid accountability to the Congress so that they are not pressured to create unexpected inflation and push unemployment below its natural rate. As Milton Friedman said (quoted by Fischer (1990)),

“From revealed preference, I suspect that by far and away the two most important variables in their loss functions are avoiding accountability on the one hand and achieving public prestige on the other.”

⁵ See the analysis by Chant and Acheson (1986), who show that increased accountability may be bad for central bank initiatives.

As it has been described, central bank transparency may aid the formation of expectations, and increase the central bank's credibility. However, transparency is not necessarily a "cure-all" for monetary policy-making as some modern analysis might seem to suggest. As my subsequent analysis will demonstrate, despite the importance of the public's inflation expectations, there are situations in which a certain degree of opacity may help the central bank achieve its goals.

d. Opacity

"People sometimes wonder why central bank transparency has evolved as slowly as it has and why some central banks do not take additional steps to talk more... The answer, I believe, is that more is not necessarily always better, and at each step of the way central banks have needed to take account of the potential costs as well as the benefits of greater transparency."

~ Donald L. Kohn,
Former Vice-Chairman of the FRB
(Kohn, 2005)

As shown by *Table 1*, the Federal Reserve has become more transparent to the public over time, but it is still behind countries announcing an inflation target such as England, Canada, and New Zealand.⁶ Some academics and central bankers alike seem to suggest that transparency is limitlessly beneficial for the conduct of monetary policy; that is, in order to conduct monetary policy optimally, the central bank should not keep *any* information from the public. Blinder (2000) suggests that as of 2000, *all* central banks worldwide should work to increase their transparency. Svensson

⁶ New Zealand is widely considered the world's most transparent central bank.

(2002) advocates publication of output and inflation forecasts, projections of future policy, and parameters of the central bank's objective function of inflation and unemployment.⁷

Problems with suggestions of complete transparency such as these are almost immediate. First, no central bank – not even the world's "most transparent" bank, The Bank of New Zealand – publishes an exact objective function. (Mishkin, 2004) John Vickers, while in 1998 acting as Executive Director and Chief Economist of the now inflation-targeting Bank of England, remarked that announcing the parameters of an objective function is "infeasible." (Cukierman, 2006) For example, the Federal Reserve Board makes policy based on the decision of its seven governors; surely, not all of these governors have the same objective function, if it even makes sense to speak of objective functions in practice in the first place.

Secondly, although increased transparency may theoretically increase credibility, if the economy operates according to a Phillips curve such as (1.1), increased transparency can mean the central bank might have less leeway to respond to macroeconomic shocks. If the central bank is perfectly transparent about its policy, then expectations of inflation will always equal actual policy. If this is always true in (1.1), then a shock in e_t cannot be addressed by monetary policy alone.

Next, if the focus of transparency is for short-run goals, it could be detrimental to the long-run objectives of a bank. The threat of political intervention reiterates the concerns brought forth in the Barro-Gordon model, in which a central bank is given the politically-motivated incentive to target unemployment below the

⁷ However, in 2002 Svensson (Faust and Svensson, 2002) went on to show that under certain conditions, a central bank may choose limited transparency, unambiguously.

natural rate. Echoing the words of Friedman on avoiding accountability, a *Wall Street Journal* article from 1984 reads, “Secrecy is designed to shield the Fed from political oversight... most politicians have a shorter time horizon than is optimal for monetary policy.” (*WSJ Editors*, 1984) The words of this *Journal* article came at a critical time following the recession induced by the Great Inflation of the 1970’s and early 80’s. The article shows an early awareness of how political oversight might interfere with the Fed’s goals.

If there is politically-motivated inflationary bias explicit in the Barro-Gordon model, a central bank may not want to be completely transparent, since this could lead to credibility problems depending on the policy practices revealed. Cukierman and Meltzer (1986) consider a model inspired by the Kydland-Prescott and Barro-Gordon time inconsistency models in which the central bank has a time-varying incentive to create employment by inflationary monetary policy. The central bank takes the time-varying magnitude of this incentive as given, and may decide with what clarity to reveal its preferences to reveal to the public in any given period. The result of Cukierman and Meltzer’s study is that the central bank has an incentive to disguise its true preference in order to maximize its objective function. The central bank is able to make its preferences “opaque” by communicating it with a degree of stochastic noise. The degree of opacity which is optimal is called by Cukierman and Meltzer the “creative ambiguity” of the central bank, so to separate it from outright “lying.” This is an important distinction in the context of the model to come. Backus and Driffill (1985) come to a similar conclusion where the monetary regime type is

not continuously stochastically varying, but stochastically changing infrequently and discretely.

Arguments against results of Cukierman and Meltzer (1986) and Backus and Driffill (1985) have been made by Blinder (1997) and McCallum (1995). In particular, Backus and Driffill have been criticized because the finite-regime structure of the model entices a regime nearing the end of its term to sacrifice long-term social welfare to minimize its own loss function in the short-run. Such behavior is not consistent with the practice of exiting central bank heads. Thus, it is desirable to investigate other reasons why transparency may be limited optimally.

One possible argument for opacity arises out of an interesting concept out of Keynes. When he was writing his *General Theory* (1936), English newspapers printed pictures of beauty pageant contestants asking people to pick the “most beautiful” woman; whoever picked the woman who received the most votes would win. The result was that people would pick whoever was likely to be picked by other people, not necessarily who they individually thought was the most beautiful. Likewise it might be thought that economic agents care more about relative expected price rather than only their own expectation.

Morris⁸ and Shin (2002) consider a model inspired by Keynes’ beauty contest example in which public agents rely more on the estimates of the central bank than their own individual estimates. Private agents behave in this manner since relative prices matter, and they believe all other agents behave according to the public signal whereas their individual private estimates are all different. Thus if the information the central bank gives is faulty, any errors may be augmented further. The central

⁸ Stephen Morris of Princeton University, not the author of this thesis.

bank is therefore given the incentive not to release too much information if there is uncertainty in their estimates.

Uncertainty is prevalent in monetary policy-making, and thus the worries brought about by Morris and Shin are well-founded. As then-Chairman Alan Greenspan notably said at the 2003 Jackson Hole Symposium,

“Uncertainty is not just an important feature of the monetary policy landscape; it is the defining characteristic of that landscape.”

Greenspan’s statement represents the fact that econometricians often face great estimation challenges. For example, estimates of unobservable variables cannot be exact. Staiger, Stock, and Watson (1997) find that estimates of the natural rate of unemployment since 1960 have large error bands, up to 3 percentage points. Thus, if the central bank’s private information is sufficiently imprecise, they may choose not to release their estimates for fear of them drowning out potentially more accurate private estimates.

A final study of limited transparency for optimal monetary policy is delivered in a recent paper by Geraats (2007). She finds that when the public has its own inference of certain parameters of the monetary authority’s loss function, it may help the monetary authority achieve less volatile expectations, and hence inflation, if they make their signal relatively less accurate. This result relies on the fact that private agents form their expectation of parameters of the model using a weighted average of their own signal and the monetary authority’s, weighted according to relative accuracy.

e. This thesis

The intent of this thesis is to study why and how limited transparency may prove superior to complete transparency for monetary policy-making in a theoretic model. It comes at a time when studies of transparency, and its limits, are starting to become particularly popular in the literature. The *International Journal of Central Banking* recently released its March 2007 publication entitled “Special Issue: Transparency, Communication, and Commitment.” Within this edition are various studies of the limits of transparency including works by a foremost monetary economist in Carl Walsh (2007) and Geraats (2007), the latter study of which is used for various parts of this thesis.

Many key results of this thesis I believe to be unique, and I have ventured to point out where they appear. Additionally, this thesis presents a wide range of viewpoints through which the benefits of opacity are elucidated, and is thereby more robust than most papers that consider only one channel. This work resides squarely in the middle of an emerging literature while at the same time presenting new ideas that could pave the way for future research.

Additionally, this thesis stands alone in that it addresses two types of credibility implicit of the literature, and the five types of transparency defined by Geraats’ (2002) thorough research. Most studies on the topic are highly dependent on the definitions of credibility and transparency defined within the model at hand and are thus not robust to alternative specifications.

The current framework builds off of the New-Keynesian model of Clarida, Gali, and Gertler (1999), and incorporates time-varying preferences for the monetary

authority's objectives, similar to those proposed by Cukierman and Meltzer (1986).

The study begins in *Chapter 2* by showing the unique result that with only the alteration of time-varying preferences, there is not a short-run trade-off between inflation and real activity⁹ variability for all levels of preferences in the New-Keynesian framework. Given this result, which is antithetical to *Result 1* of Clarida et. al., there is the incentive for the monetary authority to “lie” about its preferences.

The distinction between the monetary authority “lying” to the public and being “creatively ambiguous,”¹⁰ or opaque, will be made in some detail to clarify the present theory is in support of communicative ambiguity and *not* deception. In *Chapter 3* it is shown that although there are incentives for “lying,” it is undesirable because of the credibility problems that result. Opacity, on the other hand, can achieve similar benefits to moderation of inflation possible from “lying” without inducing the same credibility problems. Thus, the unique distinction of “lies” from ambiguity in this paper demonstrates the most fundamental reason why opacity may be beneficial, which, to the best of my knowledge, is unanimously overlooked by the current literature.

In practice, opacity of the monetary authority could mean one of several different things, including withholding information from the public, or failing to give testimony such as Humphrey-Hawkins.¹¹ In this paper as in other theoretic pieces, the amount of opacity is distinguished by the amount of stochastic “noise” in a signal

⁹ Here, the unemployment gap; in Clarida, Gali, and Gertler, the output gap.

¹⁰ This convenient terminology is accredited to Cukierman and Meltzer (1986).

¹¹ As of 2000, Humphrey-Hawkins testimony was no longer required by law, though the Fed continues to give the testimony biannually.

given by the monetary authority to the public. With increased noise, the actual value of a given parameter is harder to extract.

Following the so-called baseline rationale for limited transparency in *Chapter 4*, possible counter-arguments of transparency advocates are summarized in *Chapter 5*, beginning with an overview of points commonly made. It is then shown that transparency may be beneficial to a risk-averse monetary authority in the sense of second-order stochastic dominance.

Credibility is described in *Chapter 3* in two unique ways. First, credibility is defined in terms of honesty. Where the monetary authority has the incentive to “lie,” the public will naturally want to measure its reputation of doing so in the past to gauge its probability of doing so in the future. This definition of credibility is consistent with that of Faust and Svensson (2002) described earlier in that it measures the monetary authority’s ability to match words with actions. An additional type of credibility – for accuracy of signals – is defined by this study. Thus, credibility is measured not only in terms of lying, but also in terms of relative correctness so that there is allowed to be some costs to communicative ambiguity in terms of credibility.

The model of Geraats (2007) is then adapted to this framework in *Chapter 6*, and it is shown that her basic conclusions are robust to the New-Keynesian model; the variability of expectations is decreased with some degree of transparency for certain accuracies of the monetary authority and private agents’ own collective signal. Concerns of relative variability of inflation are particularly relevant to New-Keynesian monetary policy rather than older studies considering relative levels.

Uncertainty, of the public *and* the monetary authority, will take a major role in this study, since it may prove to be one of the strongest arguments for limited transparency. As portrayed by Greenspan's earlier quotation, uncertainty is clearly at the forefront of monetary policy decisions. In *Chapter 7* a public-private information model similar to that of Morris and Shin (2001) is considered and it is shown that less transparency may be beneficial to anchoring inflation expectations if the monetary authority and the public are similarly uncertain about the true value of the natural rate of unemployment. Whereas the Morris-Shin model is abstract, this paper becomes one of few-to-none to apply a more realistic policy-making dilemma to the public-private information Keynesian beauty contest dilemma. To the author's knowledge, this is the only study to utilize uncertainty in the estimate of the natural rate of unemployment (or output).

Finally, using Markov perfect equilibria as in Ball (1995), in *Chapter 8* it is shown that increased ambiguity in the public signal released by the monetary authority may help prevent an expectations trap. The result is particularly elegant in its simplicity stemming from the use of Markov perfect equilibria, and thereby addresses Occam's Razor. Chari et. al.'s (1998) construction of an expectations trap included more detailed construction of the preferences and actions of the private sector which addresses the Lucas critique of the insufficiency of reduced-form models, but complicates the analysis. Using Markov perfect equilibria is also convenient in that it makes explicit the game implicitly played between the monetary authority and the public in all parts of this study. To the author's knowledge, the

specificity of the result that opacity may help avert an expectations trap is unique to the literature.

The Lucas critique may seem to suggest that the theoretic reduced-form model studied here should not be necessarily be taken as necessarily relevant to the implementation of monetary policy. However this caution is uncommon of the literature, most of which seems to ignore it completely. Furthermore, most of the components seen here are broadly consistent with microfoundations and thus the Lucas critique has little power against its conclusions.

This thesis provides intuition as to how much private information a monetary authority should reveal. It is its conclusion that some secrecy of the central bank may, at least theoretically, be a good thing, and there exists a multitude of explanations for why.¹² Although continued increases to transparency may seem beneficial in the current context, this thesis attests that there is a limit to its benefits; it maintains that there is, indeed, no such thing as a free lunch.

¹² A glossary of the propositions and corollaries developed in this paper are presented in the last chapter.

2. New-Keynesian monetary policy with time-varying employment objectives

a. Set-up of the model and the dynamics of expectation formation

Consider a discretionary monetary authority like the Federal Reserve which each period sets monetary policy in order to achieve economic goals defined exogenously, such as by a governing body. Consistent with the Congressional dual-mandate of minimum inflation and sustainable unemployment adhered to by the Fed, here the monetary authority wishes to minimize the present discounted value of a linear combination of inflation, and the deviation of unemployment from its natural rate.¹³ As in Clarida, Gali, and Gertler (1999) (henceforth, CGG):

$$\text{Min}_{\pi_t, \tilde{u}_t} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[s_{t+i} \tilde{u}_{t+i}^2 + \pi_{t+i}^2 \right] \right\} \quad (2.1)$$

$0 < \beta < 1$ is the discount factor, π_t is inflation in period t , and monetary authority relative weight on unemployment gap (\tilde{u}_t) minimization as opposed to inflation are given by s_t . The unemployment gap is defined as the deviation of unemployment from its natural rate, which is itself a random-walk with white-noise (Gaussian) error

ω_t :

$$\tilde{u}_t = u_t - u_t^n \quad (2.2)$$

¹³ Formally, the unemployment rate consistent with flexible-price equilibrium in the labor market, subject to possible real distortions (taxation, imperfect competition) that may lead to a socially optimal unemployment rate that is even lower than the natural rate.

$$u_t^n = u_{t-1}^n + \omega_t \quad (2.3)$$

It is assumed, as is the case in practice, that there is a large degree of uncertainty surrounding the actual value of u_t^n at any point in time. Staiger et. al. (1996) find a typical 95% confidence interval for the concurrent NAIRU is 5.1-7.7%, with past NAIRU estimates having similarly broad confidence intervals.

The objective function (2.1) is minimized when equal to zero, implying zero-inflation and unemployment gap are optimal targets, though whether this is optimal in the long or immediately in the short run is yet indeterminate. The zero target for inflation is simply a normalization of a “realistic” target, for instance, a low but positive value.

Preferences that represent the relative weight given to unemployment gap stabilization over inflation stabilization, s_t , are composed of constant and time-varying parts:

$$s_t = A + a_t \quad (2.4)$$

$$a_t = \tau a_{t-1} + \nu_t \quad (2.5)$$

The parameter $A \geq 0$ is a constant, $0 \leq \tau \leq 1$, and ν_t is a white-noise shock to preferences.¹⁴ Thus, the time-varying portion of preferences, a_t , is either white-noise, or any covariance-stationary first-order autoregressive process up to a non-covariance-stationary random-walk where $\tau = 1$. It is realistic to posit that preferences may vary in a persistent manner such as this in the Federal Reserve Board of Governors, which changes an incomplete portion of its membership multi-annually.

¹⁴ This time-varying description of preferences is similar to that of Cukierman and Meltzer (1986), an influence of many contemporary studies of oblique monetary policy.

Preferences are taken *as given* in any monetary policy decision-making. Furthermore, preferences are inherently private information of the monetary authority which it may choose to reveal, or to obscure, from the public.

For ease of exposition, in the upcoming analysis the constant component of preferences, A , is normalized to equal zero. However, it should not be misconceived that the weight on the unemployment gap is necessarily functionally zero when a_t is zero in the mathematics forthcoming. Rather, in the case that a_t is equal to zero, the weight on the unemployment gap functionally used by the monetary authority in deciding policy is A .

In the long run, given $0 \leq \tau < 1$, preferences are expected to revert to their underlying constant, A :

$$\lim_{i \rightarrow \infty} E_t s_i = \lim_{i \rightarrow \infty} (A + \tau^i a_t) = A \quad (2.6)$$

Thus, this model captures the idea that although individual membership of the Federal Reserve Board may have short-run affects on monetary policy, in the long-run, inflation and unemployment preferences will be representative of the monetary authority as a whole.

Inflation and the unemployment gap are related by a Phillips curve incorporating forward-looking private agent¹⁵ expectations; this is known as a New-Keynesian specification:

$$\pi_t = \chi E_t \pi_{t+1} - \lambda \tilde{u}_t + \varepsilon_t \quad (2.7)$$

¹⁵ “Private agents” is used synonymously with “the public.”

where the relative weight on expectations and the unemployment gap are greater than zero, $\chi, \lambda > 0$, respectively. ε_t is a macroeconomic disturbance called a cost-push shock by CGG having some persistence:

$$\varepsilon_t = \rho\varepsilon_{t-1} + \zeta_t \quad (2.8)$$

where $0 \leq \rho \leq 1$ and ζ_t is a white-noise shock term.

Given this set-up, in each period the monetary authority chooses π_t and \tilde{u}_t to minimize:

$$a_t \tilde{u}_t^2 + \pi_t^2 + F_t \quad (2.9)$$

where:

$$F_t = E_t \left\{ \sum_{i=1}^{\infty} \beta^i [a_{t+i} \tilde{u}_{t+i}^2 + \pi_{t+i}^2] \right\} \quad (2.10)$$

F_t is separated to make clear that current decisions on unemployment and inflation will not affect future decisions. This is equivalent to the analysis of CGG. Optimal inflation and unemployment are chosen by the monetary authority for (2.9) subject to the Phillips curve constraint (2.7) which may be rewritten:

$$\tilde{u}_t = \frac{1}{\lambda} [\chi E_t \pi_{t+1} - \pi_t] + \frac{\varepsilon_t}{\lambda} \quad (2.11)$$

or,

$$\tilde{u}_t = -\frac{1}{\lambda} \pi_t + G_t \quad (2.12)$$

where:

$$G_t = \frac{1}{\lambda} [\chi E_t \pi_{t+1} + \varepsilon_t] \quad (2.13)$$

The separation of G_t shows that the monetary authority takes private agent expectations and the cost-push shock as given in their monetary policy decisions, as in CGG. So, in each period the monetary authority chooses only π_t to minimize:

$$a_t \left(-\frac{1}{\lambda} \pi_t + G_t \right)^2 + \pi_t^2 + F_t \quad (2.14)$$

The corresponding optimality condition for inflation and the unemployment gap is¹⁶:

$$\pi_t^p = \frac{a_t}{\lambda} (\tilde{u}_t) \quad (2.15)$$

In choosing only inflation, the monetary authority must estimate the natural rate of unemployment, but does not set unemployment directly. Instead, they choose inflation which affects the unemployment via the Phillips curve. Thus, the unemployment rate is set implicitly by the *de facto* inflation policy choice.

Private agents form expectations rationally by assumption; that is, their expectations of inflation policy will be perfectly well-specified up to the amount of information available to them and have only transitory errors. The amount of information available to rational agents need not be complete but may be. Private agents' own preferences for inflation or unemployment do not matter presently, as agents exist in this model having only the desire to form expectations of inflation accurately. It is assumed accurate expectation formation in itself maximizes their utility, whether by direct or indirect means. Given the setup of this model, it can be shown that expectations are set rationally for specific values of c and r_t :

$$E_t \pi_{t+1}^p = c r_t \pi_t^p \quad (2.16)$$

¹⁶ see Appendix proof A (2.15)

The parameter c is a constant and r_t is a general time-varying variable of immediately unobvious form. These expectations are taken *as given* by the monetary authority.

The set-up of the model is thus far reminiscent of CGG (p. 1672), aside only from the fact that preferences are taken in their model as a constant. A constant implies one unchanging monetary regime, so that the game played between the public and the monetary authority is infinite-horizon and has a multitude of possible Nash equilibria. In this case, the choice of any one equilibrium is an ambiguous choice. (Ball, 1995) In the following, it is shown that simply adding the assumption of time-varying preferences changes the characteristics of this model significantly.

Inserting expectations (2.16) and the optimality condition (2.15) into the Phillips curve, (2.7), the inflation policy rule for each period t is optimally¹⁷:

$$\pi_t^p = a_t q_t \varepsilon_t \tag{2.17}$$

where:

$$q_t = \frac{1}{a_t (1 - \chi c r_t) + \lambda^2} \tag{2.18}$$

However, due to error inherent in the monetary policy transmission, realized inflation will equal intended inflation policy plus some white-noise *control error*, η_t . This control error is unpredictable to the monetary authority and private agents and when actual inflation policy π_t^p is unannounced, control error is visible only to the monetary authority, and not until actual inflation is realized:

$$\pi_t = \pi_t^p + \eta_t \tag{2.19}$$

¹⁷ See Appendix proof A (2.17)

This stochastic control error is exogenous to actual monetary policy-making decisions, and is thus uncorrelated with parameters of the existing model, including inflation preferences. While not used in CGG, such a specification of control error is used in major studies of transparency in monetary policy including but not limited to Cukierman and Meltzer (1986) and Faust and Svensson (2002).

Given the specification of optimal policy for period t , inflation policy in period $t+1$ will be:

$$\pi_{t+1}^p = a_{t+1} q_{t+1} \varepsilon_{t+1} \quad (2.20)$$

Since a_t and ε_t have persistence, their expectations are $E_t a_{t+1} = \tau a_t$ and

$E_t \varepsilon_{t+1} = \rho \varepsilon_t$. Then, period t expected value of π_{t+1}^p is rationally:

$$E_t \pi_{t+1}^p = E_t \pi_{t+1} = \frac{\tau a_t \rho \varepsilon_t}{\tau a_t (1 - \chi c r_t) + \lambda^2} \quad (2.21)$$

Expectations of policy equal expectations of realized inflation, $E_t \pi_{t+1}^p = E_t \pi_{t+1}$, because control error is mean-zero. The form of (2.21) verifies that expectations written in (2.16) are indeed rational, where the parameters c and r_t are accordingly specified as¹⁸:

$$E_t \pi_{t+1}^p = c r_t \pi_t^p \quad (2.22)$$

$$c = \tau \rho \quad (2.23)$$

$$r_t = \left(a_t + \lambda^2 \right) \left[\left(\frac{f(a_t)}{g(a_t)} \right) \pm \frac{([f(a_t)]^2 - [g(a_t)]^2)}{f(a_t) g(a_t)} \right] \quad (2.24)$$

¹⁸ See Appendix proof A(2.22)

$$f(a_t) = \tau(a_t + \rho\chi + \lambda^2) \quad (2.25)$$

$$g(a_t) = 2\chi\tau^2\rho(a_t + \lambda^2) \quad (2.26)$$

The pair of solutions to (2.24) emerge from the solution of a quadratic equation. It is implicitly assumed here that the public observes a_t perfectly, or in other words, there is complete transparency of preferences and thereby perfect information. Let us assume this value of preferences can only be communicated by announcement, whether formal or informal.¹⁹ Then, the fact that public expectations are rational does not in itself require the value of a_t it uses in its expectation formation to be entirely *accurate*. Rather, the assumption of rational expectations implies that expectation formation uses all relevant information available to private agents that they deem as accurate *as possible*. The current case just happens to be the particular instance in which the relevant private information of the monetary authority, its preferences, are revealed completely and thus rationally what the public uses. It will be shown later how the public deals with the possibility that the announcement of preferences is not accurate, namely, by evaluating credibility. The public uses credibility as its evaluation of the relative accuracy of the monetary authority's signal.

Optimal inflation policy, and the optimal unemployment gap implicit of the Phillips curve, (2.17) and (2.18), may be rewritten via (2.15):

$$\pi_t^p = a_t q_t \varepsilon_t \quad (2.27)$$

$$\tilde{u}_t^p = \lambda q_t \varepsilon_t \quad (2.28)$$

where:

¹⁹ An “informal” statement occurred when Chairman Ben Bernanke told a reporter at a social function that his April 27, 2006 Congressional hearing speech had been “misunderstood.”

$$q_t = \frac{1}{a_t(1 - \chi\tau\rho r\{a_t\}) + \lambda^2} \quad (2.29)$$

The parameter r_t , having a form described by (2.24), is in (2.29) written explicitly as a function of a_t , $r\{a_t\}$, for emphasis of its dependency on the monetary authority's employment objectives. Note, $r\{a_t\}$ is the only part of inflation and unemployment gap policy which incorporates the value of preferences, a_t , that the public believes characterizes the monetary authority; all other a_t are always actual preferences of the monetary authority.

b. Replicated and unique characteristics of the model

This model is consistent with CGG in that optimal policy is expected to converge to zero inflation in the long-run, and thus incorporate some version of inflation targeting. Recall, zero inflation is necessary (along with zero unemployment-gap) for the objective function (2.1) to be absolutely minimized. Since ε_t and a_t are autoregressive with order one having persistence quantifiable by ρ and τ , respectively:

$$\lim_{i \rightarrow \infty} E_t \pi_{t+i}^p = \lim_{i \rightarrow \infty} \tau^i \rho^i E_t r_{t+i-1} \quad (2.30)$$

$$= \lim_{i \rightarrow \infty} \tau^i \rho^i (a_t + \lambda^2) \left[\left(\frac{f^i(a_t)}{g^i(a_t)} \right) \pm \frac{(f^i(a_t))^2 - g^i(a_t)^2}{f^i(a_t) g^i(a_t)} \right] \quad (2.31)$$

where:

$$f^i(a_t) = \tau^i (a_t + \rho^i \chi + \lambda^2) \quad (2.32)$$

$$g^i(a_t) = 2\chi\tau^{i+1}\rho^i (a_t + \lambda^2) \quad (2.33)$$

Thus,

$$\lim_{i \rightarrow \infty} E_t \pi_{t+i}^p = \lim_{i \rightarrow \infty} \tau^i \rho^i (a_t + \lambda^2) \left[\left(\frac{f^i(a_t)}{g^i(a_t)} \right) \pm \frac{(f^i(a_t))^2 - g^i(a_t)^2}{f^i(a_t) g^i(a_t)} \right] = 0 \quad (2.34)$$

So, zero-inflation targeting, representing absolute minimization of objective function (2.1), is the ultimate objective in the long-run. Zero-inflation targeting is alternatively immediately optimal in the absence of cost-push shocks ($\varepsilon_t = 0$), or if a_t is zero, as in CGG.

The second consequence of the time-varying preference set-up is also the most striking. As in CGG, the time-varying employment preferences (a_t) of the monetary authority have a direct effect on the variability of the unemployment gap and inflation. However, here the effects are different. Under the assumption that cost-push shocks are present, $\varepsilon_t \neq 0$, it follows from (2.27) and (2.28) that²⁰:

$$\text{As } a_t \rightarrow 0: \sigma_{\tilde{u}} = \frac{\sigma_{\varepsilon}}{\lambda}; \sigma_{\pi} = 0 \quad (2.35)$$

$$\text{As } a_t \rightarrow \infty: \sigma_{\tilde{u}} = 0; \sigma_{\pi} = 0 \quad (2.36)$$

Recall that a_t is the normalized ($A = 0$) stand-in for the value of s_t described in (2.1), (2.4), so that $a_t = 0$ does not imply no relative weight allocated to unemployment gap stabilization. In CGG, σ_{π} approaches a strictly positive constant to be described temporarily.

²⁰ See, Mathematica notebook (1) in the Mathematica appendix.

Despite the outcome of (2.35) and (2.36), the standard deviation of π_t is not zero throughout all possible values of a_t . For $0 < \alpha \ll \infty$,

$$\text{As } a_t \rightarrow \alpha: \sigma_{\tilde{u}} = D\lambda\sigma_\varepsilon; \sigma_\pi = D\alpha\sigma_\varepsilon \quad (2.37)$$

where D is the following strictly positive constant²¹:

$$D = \frac{(\alpha + \lambda^2 + \chi\rho)}{(\alpha + \lambda^2)(\alpha + \lambda^2 + \chi\rho + 2(\alpha^2 + \alpha)\chi^2\rho^2\tau^2)} \quad (2.38)$$

This value of D is consistent with the more plausible of two roots.

Specifically, D takes this value when the “*minus* solution” is used over *plus* in the quadratic solutions presented in (2.24). When *plus* is used over *minus* in (2.24), D may take the value of a negative constant, which clearly would not make sense in the comparison of standard deviations in (2.37).²² Therefore, from here on only the strictly positive root of r_t is used.

For their case of time-invariant preferences, CGG show that in the short-run as a approaches zero, the standard deviation of the unemployment gap converges to $\sigma_\varepsilon/\lambda$ and the variance of inflation heads to zero. The time-variant preference result of (2.35) is consistent with this finding. However, as a approaches infinity in CGG, the standard deviation of the output gap heads to zero and the variability of inflation approaches a positive constant.²³ This is clearly not consistent with (2.36), and (2.37). A graph reproduced from CGG representing their dynamics, and two supplemental graphs produced here for clarity, are given in *Figures 1 (a-c)*. The supplemental graphs make clear the intuitive dynamics that as the relative weight place on

²¹ See, Mathematica notebook (2) in the Mathematica appendix.

²² See Appendix proof A(2.38) for negative solution of D ; see Mathematica notebook (2) for proof.

²³ An equivalent of this positive constant in the present specification is $\sigma_\varepsilon/(1 - \chi\rho\tau r\{a_t\})$.

unemployment stabilization increases, the variability of the unemployment gap decreases and the variability of inflation increases.

Figures 2 (a-c) present short-run dynamics of inflation and unemployment gap variability with the added assumption of time-variant preferences. In contrast with *Figure 1 (b)* corresponding to CGG, which shows inflation variability approaching a positive asymptote as a increases, *Figure 2 (b)* shows inflation variability reaching a maximum and then converging to zero as a_t continues to increase.²⁴ Although *Figure 1 (c)* and *Figure 2 (c)* show a similarity between unemployment gap variability with time-varying or stagnant preferences, *Figure 1 (a)* and *Figure 2 (a)* make clear the affect time-varying preferences have made: the first major result of CGG – indeed, “Result 1” on p. 1672 that there is a trade-off between inflation and output variability in the presence of cost-push shocks – is rebuked simply by adding time-varying preferences. *Figure 2 (a)* furthermore suggests that for certain parameters, unemployment variance may be unambiguously decreased while keeping inflation variance the same.

The model and resulting limits of CGG are not entirely distinct from the time-variant preference case presented here; their results are just a special case of the current specification. Recall that from the original set-up of the monetary authority’s objective function, (2.6), that in the long-run, $\lim_{i \rightarrow \infty} E_t s_i = \lim_{i \rightarrow \infty} (A + \tau^i a_i) = A$. In this case, as in CGG, rational expectations are set as:

$$E_t \pi_{t+1}^p = d \pi_t^p \tag{2.39}$$

²⁴ *Figures 2(b)* through *Figure 3(e)* created in Mathematica. Sample code for creation of figures in Mathematica notebook (3) in the Mathematica appendix.

where d is a constant. Given expectations are set as such, inflation policy is set as a function of the cost-push shock:

$$\pi_t^p = Aw\varepsilon_t \quad (2.40)$$

$$w = \frac{1}{A(1-\chi d) + \lambda^2} \quad (2.41)$$

The result follows directly from proof A(2.40) in the *Appendix*. Thus, inflation policy next period will be:

$$\pi_{t+1}^p = Aw\varepsilon_{t+1} \quad (2.42)$$

Since the cost-push shock has persistence so that $E_t\varepsilon_{t+1} = \rho\varepsilon_t$, private agent's expectations, (2.39), are rational when $d = \rho$. In the long-run case of time-invariant preferences, there is a trade-off between unemployment gap and inflation variability as pictured in *Figure 1(a)*. In limits,

$$\text{As } t \rightarrow 0, E_t s_t = A. \text{ As } A \rightarrow 0: \sigma_{\tilde{u}} = \frac{\sigma_\varepsilon}{\lambda}; \sigma_\pi = 0 \quad (2.43)$$

$$\text{As } t \rightarrow \infty, E_t s_t = A. \text{ As } A \rightarrow 0: \sigma_{\tilde{u}} = 0; \sigma_\pi = \frac{\sigma_\varepsilon}{1-\chi\rho} \quad (2.44)$$

So in the long-run, the current specification and that of CGG are similar. However, in direct contrast with CGG, given preferences are time-varying, standard deviation of inflation does not approach a positive constant ($\sigma_\varepsilon/(1-\chi\rho)$) as a_t approaches infinity in the short-run. Instead, as shown by (2.36), as time-varying preferences a_t increase to infinity, the standard deviation of inflation peaks at a positive constant and then converges to zero. Thus, given time-variant preferences, there is not an unambiguous trade-off between inflation and unemployment gap

variability in the short-run, as they find with time-homogenous preferences. Rather, this is a long-run effect.

Figure(s) 3 show the partial effects of changing multiple parameters of the model. *Figure 3(a)* shows that as cost-push variability increases, the curves for inflation and unemployment gap variability shift outward for each value of a_t . When cost-push shocks are absent, the curves do not exist, but are instead a point at the origin.

The case in which the weight on the unemployment gap is zero in *Figure 3(b)*, $\lambda = 0$, results in the standard deviation of inflation being maximized at $a_t = 0$ and decreasing linearly as a_t increases. For any non-zero value of λ , the standard deviation of inflation converges to zero as preferences do. Lower values of λ result in maximum standard deviation of inflation being higher and occurring with lower values of a_t . For varying levels of λ , the curves of unemployment gap variability against preferences make a saddle point which do not seem to make immediately obvious any inferences about trade-offs.

Although the standard deviation of the unemployment gap varies very little with varying values of χ , when expectations have no weight in the Phillips curve ($\chi = 0$), the inflation gap converges to a non-zero number in the short-run as time-varying preferences increase, $a_t \rightarrow \infty$; as seen in *Figure 3(c)*, the number it converges to is exactly the standard deviation of the cost-push shock. In CGG's specification, or in the long-run case of the current model, as a approaches infinity the standard deviation of inflation approaches $\sigma_\varepsilon / (1 - \chi\rho)$. Thus, it seems to make sense

that the variability of inflation approaches the same limit in both the preferences-invariant and preferences-variant cases in the short-run if and only if the weight of expectations in the Phillips curve is zero.

Figures 3(d) and (e) imply that as ρ and τ increase, curves representing the variability of inflation against preferences shift outward. However, the effect on variability on output is marginal.

Given these observations of the dynamics of inflation and unemployment gap variability against time-varying preferences, the following postulate, refuting “*Result I*” (p.1672) of CGG that there is a trade-off between inflation and output gap variability, may be made:

Proposition 2.1: When preferences a_t are time-varying (specifically, $AR(1)$), and additionally cost-push inflation is present ($\sigma_\varepsilon > 0$) and expectations matter in the Phillips curve ($\chi > 0$), the variability of the unemployment gap may, in the short-run, be unambiguously decreased when a_t increases at no expense of variability of inflation.

However, *Proposition 2.1* is potentially not as powerful a statement as it may seem. Notice first that when a_t is taken as given, which is assumed, that such an unambiguous decrease in unemployment gap variability is not a choice of the monetary authority. An investigation of how the power of *Proposition 2.1* may be utilized with respect to private agent expectations is to be undertaken *Chapter 3*’s first sub-section, *Incentives for “lying”*.

Also, a question that is begged of *Proposition 2.1*, and *Figure 2 (a)*, is for what parameter values is such an unambiguous improvement possible. In particular, *Figure 2 (a)* might seem to suggest that only for unreasonably high values of a_t is such an improvement possible.

However, for arguably reasonable values of the parameters, the relationship still holds. Walsh (2003) posits that reasonable values for λ are between .01 and .2 and for a_t between .1 and .5 (though on some other accounts, this value is far lower, in the region of .05-.25). Additionally, a recent *Wall Street Journal Article* (Ip, Feb. 7, 2007) suggests that the value of χ may be relatively much greater than λ so that expectations matter more than the unemployment gap. Also, it is reasonable to assume that time-variation in preferences is small, as Board composition changes infrequently and only partially at any one time.

Assuming cost-push shocks are small and highly persistent, reasonable values may be $\lambda = .1$, $\rho = \tau = \chi = 1$, and $\sigma_\varepsilon = 2$. In this case, the maximum value of the standard deviation, approximately 1.8, is reached when a_t is approximately 0.14. A graphical representation of this may be seen in *Figure 4*. Varying the standard deviation of the cost-push shock does not have affect this inference, so the arbitrary choice of $\sigma_\varepsilon = 2$ is not problematic. However, raising λ , or decreasing ρ , τ , or χ , will cause the “peak” value of a_t to increase. Furthermore, it is important to note that increases and decreases in inflation and unemployment gap variability vary directly with their levels, since both are distinguished by (2.27) and (2.28). These observations lead to the following corollary which seems paradoxical to the existing

literature on the benefits of a conservative central banker. However, note that the corollary posits only short-run dynamics.

Corollary 2.1: Given time-varying preferences, a relatively less-conservative (i.e., high a_t) monetary authority may, in the short-run, achieve lower variability and level of the unemployment gap than a more conservative (low a_t) monetary authority, while also delivering equivalent or less inflation variability and level, for realistic values of the parameters of the model.

Although this result is highly counter-intuitive in the long-run, it captures the idea that the benefits of a conservative central banker are not unambiguous for a reasonable specification of the model in the short-run. This uniquely captures the idea that a liberal (expectation-appeasing) central banker may be beneficial in the short-run but not for long-run goals.

3. Credibility and the infeasibility of “lying”

a. Incentives for “lying”

Proposition 2.1 and its tributary *Corollary 2.1* seem to imply that at least in the short-run, a relatively less conservative stance on monetary policy may be preferable for moderating unemployment gap and inflation variability and level. However, altering actual preferences may not be possible, particularly if they are assumed to be taken as given as they are realistically postulated to be here.²⁵ Furthermore, the benefits of conservative central bankers have been bolstered in the last decades and thus it may be assumed that most central bankers appointed are relatively conservative.

Even so, there is potentially room for maneuverability; under *Section 2*'s implicit assumption of perfect information and rationality of private agents, preferences a_t also enter policy formulation (2.27)-(2.29) through the public's inflation expectations. The value private agents believe, which is correct under the assumption of rational expectations and perfect information, materializes in and only in $r\{a_t\}$, in equations (2.27)-(2.29). Conceivably, if the monetary authority could convince the public that their preferences were otherwise than in reality, they could alter the value of a_t entering policy through $r\{a_t\}$. In the terminology of Geraats (2002), the monetary authority may wish to be less “politically” transparent. Since expectations are taken as given, the value of a_t entering policy *not* through r would

²⁵ *De gustibus non est disputandum.*

be unaltered. Given the potential ability to alter the public's perception of preferences, it may be possible for the same employment and inflation objectives to be achieved, but at a different (preferably lower) level, and variability, of inflation.

Stabilization preferences are private information of the monetary authority which it may choose to share (as in *Section 2*), or not. Let a_t^* be the value of a_t the public believes is accurate and uses in the formation of their inflation expectations, and thus which is incorporated into $r\{a_t^*\}$ in (2.27) and (2.28). Then, by authority of the envelope theorem²⁶:

$$\frac{\partial \pi_t^p}{\partial a_t^*} = -a_t \theta_t \quad (3.1)$$

and,

$$\frac{\partial \tilde{u}_t^p}{\partial a_t^*} = -\lambda \theta_t \quad (3.2)$$

where:

$$\theta_t = \frac{\left[2a_t \chi_t^2 \sigma_\varepsilon \rho^2 (a_t^* + \lambda^2) (a_t^* + \lambda^2 + 2\chi\rho) \tau^2 \right]}{\left[\lambda^2 (a_t^* + \lambda^2 + \chi\rho) + a_t (a_t^* + \lambda^2 + \chi\rho + (1 + 2\lambda^2 + \lambda^4) 2a_t^* \chi^2 \rho^2 \tau^2) \right]^2} \quad (3.3)$$

These partials are both unambiguously negative since all parameters in (3.3) are nonnegative, and therefore θ_t is positive. They represent the effect on the optimal choices of inflation and the unemployment gap when the public's perception of a_t , in particular, a_t^* , moves away from the actual value.

²⁶ See, Mathematica notebook (4) in the Mathematica appendix.

Let us assume for the time-being that the value of a_t^* is the exact value communicated to the public by the monetary authority. That is, the monetary authority makes an announcement about its preferences, accurate or otherwise, which is “believed” by the public and used in the formation of their expectations. Then, when inflation and the unemployment gap are both positive, an announcement of preferences which is less conservative than in reality, $a_t^* > a_t$, will result in an unambiguously lower unemployment gap and inflation variability, and levels, than with truth-telling; the magnitude of this decrease is indicated by the values of the parameters of the model as in (3.1)-(3.3). When inflation and the unemployment gap are both negative, a façade of conservativeness, $a_t^* < a_t$, will result in inflation and the unemployment gap increasing towards (or without restraint, beyond) their implicit targets, zero. These dynamics are consistent with the existing literature of the effect of a conservative central banker on market expectations. When one of inflation or the unemployment gap is positive and the other is negative, there are opposing effects on either when a_t^* is either above or below actual preferences a_t .

From (3.1)-(3.3), the effect on monetary authority utility of announcing $a_t^* \neq a_t$ when both inflation and the unemployment gap are simultaneously above or below zero is clear. However, the effect on monetary authority utility of announcing $a_t^* \neq a_t$ when one of inflation or the unemployment gap is above zero and the other below depends on the proportion of their relative deviations from unity. Using (2.1), (2.9), (2.11), (3.1), and (3.2), and invoking the envelope theorem:

$$\frac{\partial U}{\partial a_t^*} = -2a_t \tilde{u}_t^p \frac{\partial \tilde{u}_t^p}{\partial a_t^*} - 2\pi_t^p \frac{\partial \pi_t^p}{\partial a_t^*} \quad (3.4)$$

$$= 2a_t (\lambda \tilde{u}_t^p + \pi_t^p) \theta_t \quad (3.5)$$

θ_t is strictly greater than zero and previously defined by (3.3).

Equation (3.5) verifies the utility effects stated previously; when $\lambda \tilde{u}_t^p + \pi_t^p > 0$ utility increases when the public believes $a_t^* > a_t$ and vice-versa for $\lambda \tilde{u}_t^p + \pi_t^p < 0$. Note, \tilde{u}_t^p and π_t^p are themselves functions of a_t^* and a_t . However, the actual form is assumed away for simplicity and does not effect any inferences presented here, as only the cumulative sign of $\lambda \tilde{u}_t^p + \pi_t^p$ matters for inferences of the partial effect of setting a_t^* differently than a_t .

*State of inflation
and unemployment gap:*

$$\lambda \tilde{u}_t^p + \pi_t^p > 0$$

$$\lambda \tilde{u}_t^p + \pi_t^p < 0$$

$$\lambda \tilde{u}_t^p + \pi_t^p = 0$$

Temptation to “lie”:

$a_t^* > a_t$ unambiguously increases utility.

$a_t^* < a_t$ unambiguously increases utility.

Any $a_t^* \neq a_t$ leaves utility unaltered.

For varying states of inflation and the unemployment gap, the monetary authority *always* has the temptation to lie about its preferences thereby increasing utility if $\lambda \tilde{u}_t^p + \pi_t^p \neq 0$. This sum is equal to zero if the economy is in optimal equilibrium, $\pi_t^p = \tilde{u}_t^p = 0$, or by coincidence. Zero-inflation targeting may be one reason for $\pi_t^p = \tilde{u}_t^p = 0$ in the short-run. Thus when preferences do not matter due to a formal inflation target, there is no incentive to “lie.” This leads to the following proposition, and associated corollary considering the benefit of inflation targeting:

Proposition 3.1: Given $\lambda \tilde{u}_i^p + \pi_i^p \neq 0$, a monetary authority may achieve unambiguously lower inflation and unemployment gap variability and level, thereby increasing utility, for *any* values of the parameters of the model, if they can convince private agents that their preferences are not what they actually are.

Corollary 3.1: Given a formal inflation target of zero, the monetary authority has no incentive to “lie.”

b. “Honesty” and credibility

Proposition 3.1 shows the monetary authority may have an *a-priori* incentive to “lie” about its preferences in the given model. A key assumption of the success of the monetary authority’s deceit, however, is that its announcements are believed by private agents and are used verbatim in the formation of their expectations. The public, although effectively functioning as an automaton existing only to form expectations, is rational and thereby not naïve to the monetary authority’s incentive to overstate their unemployment gap stabilization preferences. Private agents wish to correctly specify expectations and must take the monetary authority’s deceitful incentives into account. This suspicion of the public is reminiscent of the Barro-Gordon (1983) framework in which rational private expectation-formers realize the monetary authority’s inflationary (economic stimulation) incentives and therefore set expectations rationally higher, resulting in an inflationary bias. Indeed, it will be

shown that the dynamic interaction of the distrusting public and incentive-driven monetary authority here shares some of the same characteristics, though draws important distinctions.

Given private agents are wary of the monetary authority, they will rationally attempt to deduce whether they have been “lied” to or not each period. Private agents know the structure of policy formulation (2.27) and (2.28), and if they knew the exact value of inflation policy (or implicit unemployment gap policy) each period, they would have one variable and one unknown for each equation. The unknown is the monetary authority’s actual preferences last period which could easily be backed-out mathematically. Thus, if the public may view inflation policy directly, they will know whether they have been deceived the moment policy is made visible to them. However, inflation policy is only visible under the assumption that there is no control error, or that it is directly visible to the public, which it is not. With control error, the public’s view of inflation policy through their observation of inflation is obstructed.

If the monetary authority “lies” and there is no control error, or control error is revealed, their lie will be realized the moment actual inflation is realized. It is assumed that if the public has such indisputable proof that they have been lied to, they lose all faith in the monetary authority; from then on they play the grim trigger strategy of expecting ambiguously high inflation. Such a strategy may be conceived to be relatively realistic from the quotation of L. Meyer in the *Introduction*. "If [expectations] begin to move upward, it's a sign that people are losing confidence in the Fed's commitment to price stability... They are, in a sense, a report card on Fed credibility." Thus, a complete loss in the credibility from a policy “lie” could result

in ascending expectations. Whether such a strategy is “rational” is regrettably debatable. Furthermore, whether the monetary authority believes the public is credible for enacting a trigger strategy such as this is unclear. However, if it is accepted that a “lie” may be a window into the potential poor performance of a monetary authority in the future, then expectations of very high inflation are rational in that mismanaged economies do traditionally experience hyperinflation.²⁷ It is furthermore assumed that this trigger strategy makes “lying” infeasible with no or observable control error.

The monetary authority on the other hand *does* have a veil to hide its lies behind, as inflation policy and realized inflation are not necessarily the same. Control error induced by the monetary transmission mechanism (2.19) is not directly observable to the public when policy is not announced explicitly. Therefore, since only realized inflation is directly viewed by the public, the monetary authority can make preference “lying” go unnoticed (that is, $a_t^* = a_t$ when backed out by private agents) if they purposefully misstate control error η_t as η_t^* such that actual preferences apparently equal communicated preferences. When control error is not made available publicly, in the terminology of Geraats (2002), the monetary authority is not transparent in the “operational” sense.

It would seem as if the public has one other way of establishing what true preferences are, namely, in the equation for specification of the optimal unemployment gap, (2.28). However, the value of the natural rate, with all of its

²⁷ For example, many Latin American countries experienced a financial crisis in the late 1990’s. In particular, Venezuela was incurring 29.9% inflation as of 3/14/07. This is approximately a ten-year *low*. (Banco Central De Venezuela: <http://www.bcv.org.ve/>)

inherent uncertainty, is also effectively an unknown in the equation. Thus given the public knows the formulation of (2.28), they have one equation and two unknowns, u_t^n and a_t , so either value is intractable.

Given the monetary authority who lies about preferences will always misstate control error purposefully so that $a_t^* = a_t$ when backed-out of (2.27) and (2.28), private agents must find a more indirect approach of detecting “lying.” One of the only feasible ways for them to do this is to compare the preferences and control error that the monetary authority has announced to them directly against one another. In fact, given the set-up of the model, no other method is immediately obvious.

Control error is exogenous, and so as mentioned in *Section 2*, it is expected that the stochastic component of preferences should be uncorrelated with it. However, if the monetary authority has in fact lied on one or more occasions, there may appear to be some correlation between the two. That is, each time a “lie” is made about one parameter, a “lie” necessarily must be made about the other to avoid the public’s grim trigger strategy. Thus, using the entire history of past announcements, the public could hypothetically test for past “lying” by calculating sample correlation between announced control error and the announced²⁸ stochastic component of preferences, v_t^* .

The correlation sample statistic incorporates data from the entire span of time the monetary authority has been announcing preferences. It is defined as one form of monetary authority *credibility*, labeled $C_{H,t}$ for credibility arising out of “honesty.”

²⁸ Implicitly announced by announcements of a_t^* , as $v_t^* = a_t^* - \tau a_{t-1}^*$.

Private agents calculate it as one minus the square²⁹ of the Pearson correlation coefficient of the two shock terms:

$$C_{H,t} = 1 - \left(\frac{\sum_{i=1}^t v_i^* \eta_i^*}{(t-1) \sqrt{\frac{1}{t} \sum_{i=1}^t \{v_i^*\}^2} \sqrt{\frac{1}{t} \sum_{i=1}^t \{\eta_i^*\}^2}} \right)^2 \quad (3.6)$$

So, $C_{H,t}=1$ implies perfect credibility and $C_{H,t}=0$ no credibility. This definition of credibility is consistent with that of Faust and Svensson (2001) referred to in the *Introduction* in that it measures the monetary authority's ability to match its announcements with its deeds, in this case, their employment stabilization preference announcements and actions. "Credibility for honesty" is maximized when $C_{H,t} = 1$ and decreasing as $C_{H,t}$ approaches its absolute minimum, zero. Given less-than complete credibility of a monetary authority may be some indication of its tendency to lie in the past, in other words its *reputation*, the public may want to use $C_{H,t}$ to gauge how they should interpret announcements in the future.

Since the partial effect of a_t^* on expectations is exactly equivalent to the partial effect on inflation policy in (3.1),³⁰ private agents will augment their expectations accordingly with respect to the state of the economy. In particular, if the monetary authority is not credible and additionally $\lambda \tilde{u}_t^p + \pi_t^p > 0$, the public should, with respect to the monetary authority's incentives outlined earlier, expect that $a_t^* > a_t$, and thus by (3.1) that inflation will in reality be higher than their

²⁹ The importance of calculating correlation here is not that the sign, but that there is any.

³⁰ See, Mathematica notebook (5) in the Mathematica appendix.

expectations. If $\lambda\tilde{u}_t^p + \pi_t^p < 0$, the public should expect that $a_t^* < a_t$ and via (3.1) that inflation will be below their expectations. Thus, private agents should bias their inflation expectations accordingly – upwards in the former case and downwards in the latter – according to how much credibility the monetary authority has.

In the Barro-Gordon model, private agents know exactly to what extent the monetary authority wishes to bias expectations upwards, a single parameter k . However, in this framework the public (potentially more realistically) does not know how far any “lie” will be from the truthful value. Additionally, the value of preferences communicated by the monetary authority is not always above or below a_t , but depends on the value of $\lambda\tilde{u}_t^p + \pi_t^p$ with respect to the monetary authority’s corresponding incentives. So, unlike the Barro-Gordon model in which expectations are always biased upwards to account for k , there is not a specific value here that the public knows represents the monetary authority’s lie.

Assume for simplicity that in each period private agents form expectations still using whatever announcements on preferences and control error are given them, exactly as would be implied by (2.22), but then scale these expectations upwards or downwards depending on credibility and the sign of $\lambda\tilde{u}_t^p + \pi_t^p$. Where

$m = \pm 1$ corresponds to the sign of $\lambda\tilde{u}_t^p + \pi_t^p$, and $\pi_t - \eta_t^*$ is perceived policy:

$$E_t \pi_{t+1}^p = \left(1 + (m)(1 - C_{H,t})\right) cr \{a_t^*\} (\pi_t - \eta_t^*) \quad (3.7)$$

Thus, in the case that the monetary authority is completely credible, $C_{H,t} = 1$, and in the present period $a_t^* = a_t$ and $\eta_t^* = \eta_t$, expectations collapse to their original

full-information specification in *Section 2*, (2.22). In the case that credibility is not complete, $C_{H,t} < 1$, expectations will be biased accordingly.

Proposition 3.2: Given even one preference “lie” is detectable to the public (that is, reveals some correlation between announced signals of independent variables), expectations will be biased upwards or downwards according to the monetary authority’s deceitful incentives for the current level of inflation and the unemployment gap.

This result is dissimilar from the Barro-Gordon model in that there is not a distinct value of the inflation bias, but similar in that credibility issues will play a role in determining what the public’s expectations will be. As in the Barro-Gordon model, the monetary authority can here keep expectations anchored by increasing their credibility, or potentially, by introducing a zero-inflation target.

There are additional potential problems with lying when the public takes account of credibility for “honesty.” First, notice that due to the formulation of credibility, even if a deceitful monetary authority attempts to reform its ways, it will still experience low credibility for a time proportional to that which it lied for. Thus, the longer a monetary authority lies, the less willing it may be to stop and suffer the consequences of low credibility without the profits of lying. This is consistent with the idea that credibility is earned through consistent duty and is not immediate any single period. Also, the lingering effect of correlation represents the idea that suspicious correlations in the past are never completely forgotten, though they can be made less important with consistent highly credible action.

Secondly, the actual benefit from lying may be completely offset by losses to credibility. The actual gains to decreasing inflation and the unemployment gap described by (3.1) and (3.2) may be completely offset by credibility if the combination of lying necessary to achieve these goals, and the unpredictable stochastic nature of control error and preferences, result in high sample correlation.

Thirdly, consider the dilemma of a monetary authority with relatively poor credibility that experiences a “perfect storm” of seemingly correlated η_t and ν_t . This would be unpredictable and unstoppable by the monetary authority, and could result in skyrocketing or plummeting inflation expectations if the monetary authority were not able to convince the public this correlation was in fact an act of God and not of man. With low credibility, there would be no way for a monetary authority to credibly “cry wolf” and prevent expectations, and hence inflation, from increasing.

This method of measuring credibility is a double-edged sword; even with accurate communication, small spurious correlations in ν_t and η_t may fold into the public’s perception of credibility, and so expectations may inherently be inaccurate even if the monetary authority is relatively “honest.” The driving force behind this inefficiency is the simple fact that the monetary authority has the incentive to “lie.” The public, having no other way of monitoring the monetary authority, has no choice but to judge credibility as such.

If the monetary authority were able to guarantee that it were not lying somehow, possibly by making visible what its policy choice was each period, there would be no need to calculate this kind of credibility, as the public would simply know what the monetary authority’s preferences were. Thus at the time-being, one

may be led to believe it might help the monetary authority to be transparent about its undertakings. However, this would inevitably make impossible the potential benefits of ambiguity stated in *Proposition 3.1*.

c. “Accuracy” and credibility

Private agents understand that the “honesty” measure of credibility is not exact, and may want to try to improve their estimate. In particular, they realize that stochastic variables may exhibit some spurious correlation that may bias their estimates of credibility via sample correlation such that they are inconsistent with the actual “truthfulness” of the monetary authority, and thus the double-edged sword. In order to account for this, the public may try to estimate the average accuracy of the communicated values of preferences and control error, a_t^* and η_t^* . By doing so, they would be relatively more inclined to guess when correlations were man-made rather than incidental.

One feasible method by which the public may attempt to estimate the accuracy of the signals a_t^* and η_t^* is possible if the signals’ deviations from the actual values of preferences and control error, a_t and η_t , are Gaussian shocks, and additionally the variances of control error and preferences, σ_η^2 and σ_a^2 , are known to (or accurately estimated by) the public. However, private agents do not assume these shocks are independent of one another, since they expect correlation if the monetary authority lies. Ultimately, the former assumption of normal stochastic deviations would arise out of the history of the sign of $\lambda \tilde{u}_t^p + \pi_t^p$ and the monetary authority’s

corresponding incentives to “lie” as in the previous tables. Since incentives of how to lie are exactly correspondent to the sign of $\lambda\tilde{u}_t^p + \pi_t^p$, if the value of $\lambda\tilde{u}_t^p + \pi_t^p$ over time is mean-zero and normally distributed around zero, the assumption is viable. The latter assumption of public knowledge (or accurate estimates) of the variances of preferences and control error may be made if the public either has trustworthy economic priors for what these values should be, or if the monetary authority decides to reveal these values.³¹ The public’s knowledge of variances does not imply their knowledge of period-by-period values.

If these assumptions may be made, the signals for preferences and control error may be written:

$$a_t^* = a_t + X_t \tag{3.8}$$

and

$$\eta_t^* = \eta_t + Y_t \tag{3.9}$$

where $X \sim N(0, \sigma_X^2)$ and $Y \sim N(0, \sigma_Y^2)$. Correspondingly, the normal distribution allows the public to infer the accuracy of each of these signals is³²:

$$A_{a^*} = \frac{\bar{\sigma}_a^2}{\bar{\sigma}_a^2 + s_X^2} \tag{3.10}$$

and:

$$A_{\eta^*} = \frac{\bar{\sigma}_\eta^2}{\bar{\sigma}_\eta^2 + s_Y^2} \tag{3.11}$$

³¹ Potentially, these values could be forcibly revealed by mandate such as the Freedom of Information Act.

³² Geraats (2007) provides the inspiration for measuring accuracy of public signals as such.

The terms $\bar{\sigma}_a^2$ and $\bar{\sigma}_\eta^2$ represent the public's estimates of the actual values of the variances of a_t and η_t , σ_a^2 and σ_η^2 , the public brings to the problem as an economic prior; it is possible that $\bar{\sigma}_a^2 = \sigma_a^2$ and $\bar{\sigma}_\eta^2 = \sigma_\eta^2$. The terms s_X^2 and s_Y^2 represent the sample variances of X and Y . Appealing to (3.8) and (3.9) and the fact that variances may be added, these sample statistics may be estimated by the public as:

$$s_X^2 = s_{a^*}^2 - \bar{\sigma}_a^2 \quad (3.12)$$

and

$$s_Y^2 = s_{\eta^*}^2 - \bar{\sigma}_\eta^2 \quad (3.13)$$

The sample variances of a_t^* and η_t^* , $s_{a^*}^2$ and $s_{\eta^*}^2$, are directly calculated by the public in order to construct estimates of the variances of X and Y , s_X^2 and s_Y^2 . It is possible that sample variances may equal the actual variances of X and Y , σ_X^2 and σ_Y^2 in any given period. Thus, A_{a^*} and A_{η^*} , with $0 \leq A_{a^*}, A_{\eta^*} \leq 1$, represent the accuracy the public perceives of the public signals for preferences and control error; $A_{(\bullet)} = 1$ represents perfect accuracy and thereby complete transparency and $A_{(\bullet)} = 0$ the limit of inaccuracy and complete opacity. This judgment of credibility is in a sense measuring a *second order of transparency*; instead of transparency of the value of a given parameter, “credibility for accuracy” measures the relative transparency of the accuracy of the signal. This concept will be revisited in the later investigation of optimal transparency.

Accordingly, the public may augment their original credibility estimate $C_{H,t}$ by taking account of how accurate they believe the signals are. Specifically, the more

accurate the signals are, the less attention the public should pay to sample correlation between ν_t^* and η_t^* since it is more likely spurious. Thus, a new credibility term for accuracy, $C_{A,t}$, is computed as a weighted average of A_{a^*} and A_{η^*} :

$$C_{A,t} = \frac{A_{a^*} + A_{\eta^*}}{2} \quad (3.14)$$

Since $0 \leq A_{a^*}, A_{\eta^*} \leq 1$, then $0 \leq C_{A,t} \leq 1$ with $C_{A,t} = 1$ representing perfectly accurate signals (perfect transparency) and $C_{A,t} = 0$ representing the limit of inaccuracy. Thus, expectations (3.7) may be further augmented beyond credibility for honesty, $C_{H,t}$, for credibility for accuracy, $C_{A,t}$:

$$E_t \pi_{t+1}^p = (1 + (m)(1 - C_{H,t})(1 - C_{A,t})) cr \{a_t^*\} (\pi_t - \eta_t^*) \quad (3.15)$$

When the signals are perfectly accurate, $C_{A,t} = 1$, $a_t^* = a_t$, and $\eta_t^* = \eta_t$, expectations reduce to their original perfect-information formulation (2.22), even if there are spurious correlations in ν_t^* and η_t^* present. However, as signals approach precision, the gains from lying described by *Proposition 3.1* diminish as expectations collapse to their perfect-information specification. If signals are less accurate, correlations have greater effects on credibility-induced inflation bias. Thus, there is allowed to be detractions from opacity in the sense of credibility, but these are smaller than the effects of outright deception. The ‘‘accuracy’’ method of calculating credibility, it should be stressed once more, is only possible under the assumptions of normal distributions acting as preference and control error signals, and that the actual variances of preferences and control error are known to the public.

Proposition 3.3: Under the assumption of normally distributed signals for control error and preferences, losses to credibility from spurious *or* man-made correlations in v_t^* and η_t^* may be avoided the more accurate a_t^* and η_t^* are. However, gains similar to “lying” at the same time diminish with added accuracy.

“Credibility for honesty” and “credibility for accuracy” together make up a fruitful method by which the public can attempt to judge the true credibility of the monetary authority as inaccuracy in one may be offset by accuracy in the other. The monetary authority on the other hand only loses by accurate judgments of its credibility, and might thereby be tempted to simply “be truthful” and forgo the potential benefits of lying. However, there are yet still methods by which the benefits of lying can be achieved to some degree without the same credibility problems.

4. “Baseline” case for opacity of preferences

a. “Lying” versus “ambiguity”

“Lying” seems to have inevitable detriments. Just the potential to lie will result in credibility-induced inaccurate expectations if there is some extrinsic spurious correlation in the stochastic component of preferences and control error. Additionally “credibility for accuracy” is not a perfect measure, and necessitates some assumptions be made about the distribution of lies around the actual values. Even if credibility for accuracy is high and mutes most losses to credibility from spurious correlation, the benefits from lying will diminish as the accuracy of the signals improve. However, remembering *Proposition 3.1*, there are potential benefits to lying, were credibility not affected so adversely.

In place of making the binary decision to “lie” or “not lie,” the monetary authority may consider instead releasing truthful information with a degree of noise, which may be thought of as “creative ambiguity.”³³ Here, this will be modeled as revealing a normally distributed signal around an accurate mean, the realization of the assumption made by private agents in *Section 3.3*. When the monetary authority decides to reveal all information on preferences and control error, as was implicitly assumed when the model was developed in *Section 2*, they are said to be *transparent*. The amount of information withheld by communicating ambiguously with a degree of noise makes the monetary authority proportionally more *opaque*.

³³ This term is credited to Cukierman and Meltzer (1986), but is also reminiscent of Alan Greenspan’s use of the term “constructive ambiguity.” (Woodward, 2000, p.147)

Instead of “lying” the monetary authority decides to reveal a public signal each period, a_t^s , of its preferences:

$$a_t^s = a_t + \kappa_t \tag{4.1}$$

The error term κ_t is a white-noise shock. This public signal is distinct from a lie in that the monetary authority does not distinguish the value of κ_t itself, but chooses only the variance of the normal distribution, σ_κ^2 , from which it is randomly chosen; it is not disallowed that this would result in an a_t^s that would be detrimental to, instead of assisting, the monetary authority’s goals. The smaller the variance of κ_t , the more transparent the monetary authority is in the “political” sense in Geraats’ (2002) terminology. The signal a_t^s is equivalently a distribution around a mean of actual preferences, a_t . Note, (4.1) is functionally equivalent to (3.8), but written with different variables to differentiate between “lying,” a_t^* , and “opaquely signaling,” a_t^s .

In order to make sure the value of a_t can not be mathematically backed-out by the public, the monetary authority must also announce control error as the mean of a normal distribution:

$$\eta_t^s = \eta_t + \gamma_t \tag{4.2}$$

The term γ_t is a white-noise shock with variance chosen by the monetary authority, but the specific value is *not* chosen directly each period; additionally, this random value of γ_t is selected at random so that concurrent η_t^s is independent of it. This is crucial, as it will result in the main difference between “lying” where the choices of a_t^* and η_t^* were expected to be correlated when a “lie” was told. The

smaller the variance of γ_t , the more transparent the monetary authority is “operationally” in Geraats’ (2002) terminology. Also, “policy” transparency is implicitly decreasing in the variance of γ_t , since in this model availability of control error implies availability of the policy decision. Notice that (4.1) and (4.2) represent the realization of the assumption of a normal distribution necessitated in the calculation of “credibility for accuracy.” However, the essential difference here is that the shocks are independent, which was not necessarily true in the scenario of “lying.”

b. Noisy signaling and comparative statics

The monetary authority need wait until after inflation policy is set each period to release its signals. Since control error is not known until inflation is realized, the monetary authority can not release γ_t until then. Additionally, to ensure κ_t is set independently of γ_t and avoid correlation-induced credibility problems, it must wait until inflation is realized to release both signals. This is no problem however, since when policy is invisible to the public, expectations themselves must wait for inflation to be realized, as portrayed by the upcoming equation (4.3).

Thus, when policy is set each period, the monetary authority assumes the expected value $\kappa_t = 0$. To avoid complications with time-inconsistency deliberations, it is assumed that once the monetary authority selects a κ_t , they may not decide to

choose again, or announce a different value.³⁴ Most importantly, γ_t and κ_t are selected at random, for better or worse so that the monetary authority's only choice is the variance of the distributions from which they are chosen. This avoids correlation and/or lying.

For the time-being putting aside credibility issues, rational expectations will be:

$$E_t \pi_{t+1}^p = cr \{a_t^s\} (\pi_t - \eta_t^s) \quad (4.3)$$

So, each period the monetary authority will set inflation policy as in (2.27), but with r as a function of a_t^s :³⁵

$$\pi_t^p = a_t q_t \varepsilon_t \quad (4.4)$$

$$q_t = \frac{1}{a_t (1 - \chi cr \{a_t^s\}) + \lambda^2} \quad (4.5)$$

The function r is described by (2.24). First, note that $\kappa_t = a_t^s - a_t$ by (4.1), so the variance of κ_t , σ_κ^2 , may be written³⁶:

$$\sigma_\kappa^2 = E \left[(a_t^s - a_t)^2 \right] \quad (4.6)$$

and clearly, by rearranging (4.1) and squaring both sides,

$$\kappa_t^2 = (a_t^s - a_t)^2 \quad (4.7)$$

³⁴ To avoid the problem in practice, all of the values for κ_t and γ_t could be selected ahead of time by a random-number generator. Then, they necessarily may not be viewed until the given period so that they enter policy formulation as an expected value.

³⁵ See Appendix proof A (4.4).

³⁶ See Appendix proof A (4.6).

If a_t^s is substituted with $a_t + \kappa_t$ via (4.1), inflation policy (4.4) may be rewritten as a function of κ_t and κ_t^2 .³⁷ When it is taken into account that policy is made before the value of κ_t is known, the expected value $E_t \kappa_t = 0$ must be used. Additionally, by (4.6) and (4.7), $E[\kappa_t^2] = E[(a_t^s - a_t)^2] = \sigma_\kappa^2$ also enters policy formation. Thus, inflation policy is a function of the variance of the stochastic portion of the public signal for inflation, σ_κ^2 .

The partial effect on inflation and implicit unemployment policy of this variance is:

$$\frac{\partial \pi_t^p}{\partial \sigma_\kappa^2} = -a_t \theta_{1,t} \quad (4.8)$$

and

$$\frac{\partial \tilde{u}_t^p}{\partial \sigma_\kappa^2} = -\lambda \theta_{1,t} \quad (4.9)$$

where:

$$\theta_{1,t} = \frac{[2a_t \chi_t^2 \rho^2 \tau^2 \sigma_\varepsilon (a_t + \lambda^2 + \chi \rho)]}{[\lambda^4 + \chi \lambda^2 \rho + 2a_t^3 (\chi \rho \tau)^2 + a_t^2 (1 + 4(\chi \lambda \rho \tau)^2) + a_t \theta_{2,t}]^2} \quad (4.10)$$

and:

$$\theta_{2,t} = 2\lambda^2 + 2(\chi \lambda^2 \rho \tau)^2 + \chi \rho (1 + 2\chi \rho \tau^2 \sigma_\kappa^2) \quad (4.11)$$

Thus the comparative static of the variance of the stochastic portion of the signal on inflation and unemployment gap policy is negative; this suggests that more variability in the public signal, in other words, more opacity of preferences, has a

³⁷ This and all following steps up to (4.11) in Mathematica notebook (6) in the Mathematica appendix.

similar beneficial effect as “lying” in (3.1). Additionally, the amount of opacity desired is strictly finite since zero inflation is desired, not absolute minimum in the negative realm. The partial effect on utility may be written:

$$\frac{\partial U}{\partial \sigma_\kappa^2} = -2a_t \tilde{u}_t^p \frac{\partial \tilde{u}_t^p}{\partial \sigma_\kappa^2} - 2\pi_t^p \frac{\partial \pi_t^p}{\partial \sigma_\kappa^2} \quad (4.12)$$

$$= 2a_t (\lambda \tilde{u}_t^p + \pi_t^p) \theta_{1,t} \quad (4.13)$$

Somewhat similar to the “lying” solution, when $\lambda \tilde{u}_t^p + \pi_t^p > 0$, additional noise, or opacity, in the public signal of preferences may indeed lead to increases in utility. However, zero noise, or perfect transparency, is optimal if $\lambda \tilde{u}_t^p + \pi_t^p < 0$; any noise above zero in this case will lead to unambiguous losses in utility. If $\lambda \tilde{u}_t^p + \pi_t^p = 0$, changes in the relative noise of the signal do not effect utility.

*State of inflation
and unemployment gap:*

$$\lambda \tilde{u}_t^p + \pi_t^p > 0$$

$$\lambda \tilde{u}_t^p + \pi_t^p < 0$$

$$\lambda \tilde{u}_t^p + \pi_t^p = 0$$

Amount of noise in signal:

Increasing σ_ε^2 increases utility.

Only $\sigma_\varepsilon^2 = 0$ avoids utility loss.

Changing σ_ε^2 does not affect utility.

c. Effects on credibility

The last result portrayed by the above table seems to imply that opacity is preferable to transparency “some of the time,” but inferior to perfect transparency other times. Indeed, given the long-run optimal targets of inflation and the unemployment gap are both zero, opacity might be expected to be superior about

“half of the time.” It is unimportant whether there is any noise in the long-run, since then $\lambda \tilde{u}_t^p + \pi_t^p = 0$ by default and the presence of noise does not affect utility.

However, consider if the monetary authority must in a hypothetical first period choose the amount of noise in the signal for preferences and leave it. This seems potentially necessary since choosing a new variance each period, or each few periods, might seem more like “lying” than being “ambiguous.” For example, if the monetary authority were to choose new variances of the distributions from which to randomly choose κ_t and γ_t each period, and the public could tell this was taking place, they may have the incentive to play the same grim-trigger strategy as when lying was directly visible in *Section 3*. If not told values explicitly, private agents may still be able to detect heteroskedasticity in the inflation signal by calculating the sample statistic for variance, s_κ^2 . However, even though variances were changing, there would still be no apparent correlation between ν_t^s and η_t^s as long as they are chosen to be independent of one another. Thus, credibility for honesty $C_{H,t}$ will always be one, up to the amount of spurious correlation in ν_t^s and η_t^s .

If the monetary authority were to continually change the variance of κ , there would inevitably be great variation in the public’s calculation of credibility for accurate signaling, $C_{A,t}$. Given this section’s current specification of parameters:

$$C_{A,t} = \frac{\frac{\sigma_a^2}{\sigma_a^2 + s_\kappa^2} + \frac{\sigma_\eta^2}{\sigma_\eta^2 + s_\gamma^2}}{2} \quad (4.14)$$

Thus, if s_κ^2 or s_γ^2 were very volatile, credibility for accuracy could fluctuate wildly, potentially increasing to values that would make creative ambiguity inferior to

transparency, thus forgoing the potential benefits to the monetary authority from revealing less-than-complete information.

If the monetary authority were to choose σ_{κ}^2 in the first period and leave it for the foreseeable future, given their incentives with varying states of the unemployment gap and inflation, they might rationally want to choose its value based on the expected future values of $\lambda\tilde{u}_t^p + \pi_t^p$. If they expected this sum to be above zero about half of the time, and below zero the other half, they might want to choose a weighted average of a “high” variance choice (high opacity), and a zero-variance choice (complete transparency). Given the choice of σ_{κ}^2 is made according to some weighted average, if $\lambda\tilde{u}_t^p + \pi_t^p$ is expected to be above zero half of the time, the choice of σ_{κ}^2 will be positive, if even slightly so.

Even if the expected future stream of $\lambda\tilde{u}_t^p + \pi_t^p$ were not half above-zero and half below, a weighted-average choice of σ_{κ}^2 could still be positive. In fact, if it is assumed that σ_{κ}^2 is chosen according to a weighted average of the signs of expected future $\lambda\tilde{u}_t^p + \pi_t^p$, the *only* case in which σ_{κ}^2 would not be even slightly positive is if $\lambda\tilde{u}_t^p + \pi_t^p$ is expected to be *always* less than or equal to zero. In the long-run when $\lambda\tilde{u}_t^p + \pi_t^p = 0$, this zero-variance would have no averse or beneficial effects. These observations lead to the following proposition:

Proposition 4.1: If the monetary authority must choose a permanent value of σ_κ^2 to avoid credibility problems, and this choice is made according to a weighted average of expected future $\lambda \tilde{u}_t^p + \pi_t^p$, if $\lambda \tilde{u}_t^p + \pi_t^p$ is expected to be positive *but one time* in the future, a choice of $\sigma_\kappa^2 > 0$ is made, even if only marginally greater than zero.

Under the stated conditions, a choice of $\sigma_\kappa^2 > 0$ is likely, meaning some degree of opacity may indeed be preferable to full transparency.

The one vital difference between this opacity result and that of lying is that credibility is not affected the same way. In particular, since the stochastic portion of the preference and control error signals is chosen so that the signals are uncorrelated, the only losses to credibility for honesty as defined by (3.6) will occur purely out of luck-of-the-draw.

Proposition 4.2: Releasing a noisy public signal of preferences (and control error) may result in the same minimizing effects to inflation as “lying,” but without the associated “credibility for honesty” problems.

However, if the public has economic priors or factual information on the variances of control error and preferences, they will be able to calculate “credibility for accuracy.” Given there is even some variance in the stochastic component of the signal, this will result in some biasing of expectations, unless “credibility for honesty” is exactly unity, which it theoretically would be if the signals released were completely independent of one another. The benefit from opacity in this case is then a trade-off between the benefits to opacity inherited from “lying” and the drawbacks from inaccuracy of the signal.

Proposition 4.3: Opacity achieves the utility-increasing benefits of lying without suffering the credibility problems from dishonesty. However, there exists a trade-off between the benefits from opacity and its detractions from credibility for accuracy *if* the public has an idea (accurate or otherwise) of the actual variances of control error and preferences.

Thus, if the monetary authority is able to keep private enough of its own private information on the actual variances of control error and preferences, a degree of opacity (noise) in their public signal of either may result in the same beneficial effects as lying, but without the associated credibility problems. Even if the public can estimate credibility for accuracy, opacity may still be preferable to transparency depending on the magnitude of resultant credibility loss.

5. Potential counterarguments for transparency

a. Common wisdom

The case made in the past chapters suggests opacity, or, “creative ambiguity,” may be beneficial as it may accomplish some of the benefits “lying” has over “truth-telling,” or, complete transparency, while avoiding credibility problems. This opacity has been modeled as the degree of noise in signals from the monetary given to the public. However, prevailing wisdom seems to be that increased transparency is beneficial, as suggested by the quotation of B. Friedman at the beginning of the *Introduction*. This does not seem an egregious claim by any means, as transparency may reduce at least one portion of private agents’ uncertainty, that of the central banker preferences or other private information of the monetary authority. The purpose of this section is to elucidate some of the reasons why transparency may seem superior to opacity from the standpoint of the monetary authority.³⁸

Transparency is in this model the intuitive answer, to some degree. Effectively, the only reason for opacity is to distort private agent expectations, which are otherwise well-specified with complete information as in *Section 2*. Choosing to hide information does not seem to be consistent with the idea of aligning private agent expectations with those of the monetary authority, or necessarily “anchoring” expectations as referred to by Chairman Bernanke in his quote in *Section 1 (c)*. Chairman Bernanke is an advocate of making himself clear, which Alan Blinder explains by quirky example in his personal online commentary:

³⁸ From the standpoint of the public is trivial given the framework at hand.

“Remember Horton [from Dr. Suess], the elephant who hatched the egg? He meant what he said, and he said what he meant. So does Bernanke.”

~Alan Blinder³⁹

And so says Grep Ip in a similar thought in a later Wall Street Journal article:

“Mr. Bernanke feels the Fed's words should mean what they say, and not be shaded to manage market expectations, as was sometimes the case under his predecessor, Alan Greenspan.”

(Ip, March 27, 2007[a])

Particularly Ip's sentence suggests that Bernanke might not behave according to the expectation-anchoring masking of information presented in this paper.

Additionally, Bernanke is a supporter of inflation targeting, a dramatic move towards transparency that has relatively recently been adopted by the central banks of England, Germany, Canada, and New Zealand. Recall from *Corollary 3.1* that in the present model, given a formal inflation target of zero, the monetary authority has no incentive to “lie” and thereby would make the completely credible if it truly could not deviate from its target.⁴⁰

Any opacity naturally incurs some degree of “credibility for accuracy” problems if the term for “credibility for honesty” is above-zero, as described in *Section 4*. Given the public knows, or has some priors of, the variances for control error and preferences, no variance in the noise terms X_t in (3.8) and Y_t in (3.9) or κ_t in (4.1) and γ_t in (4.2) is the only way to achieve complete credibility so that

³⁹ Availability: <http://www.princeton.edu/~blinder/commentaries.htm>

⁴⁰ The inflation-targeting Bundesbank of Germany was a good example of how this is not always the case; it missed its money growth target 50% of the time. (Blinder, 2000)

expectations are not biased upwards or downwards.⁴¹ However, opacity is still sometimes optimal regardless of these detractions from credibility depending on the magnitude of the trade-off with the benefits to opacity inherited from “lying.”

b. Risk aversion

If the monetary authority is risk-averse, it might prefer to keep the public’s expectations of inflation closely aligned with its own; this may prevent expectations from becoming unanchored. Such a claim of practical risk-avoidance seems to characterize the intuition of the Federal Reserve under Greenspan; as he said at the 2003 Jackson Hole Symposium⁴²:

“Monetary policy based on risk management appears to be the most useful regime by which to conduct policy.”

Equation (3.5) described the partial effect of a preference “lie” on utility.

Assuming instead of a “lie” the central bank releases a signal as the public posited in (4.1), the equation may be rewritten:

$$\frac{\partial U}{\partial a_t^s} = 2a_t (\lambda \tilde{u}_t^p + \pi_t^p) \theta_t \quad (5.1)$$

$$\theta_t = \frac{\left[2a_t \chi_t^2 \sigma_\varepsilon \rho^2 (a_t^s + \lambda^2) (a_t^s + \lambda^2 + 2\chi\rho) \tau^2 \right]}{\left[\lambda^2 (a_t^s + \lambda^2 + \chi\rho) + a_t (a_t^s + \lambda^2 + \chi\rho + (1 + 2\lambda^2 + \lambda^4) 2a_t^s \chi^2 \rho^2 \tau^2) \right]^2} \quad (5.2)$$

⁴¹ Assuming of course, $\lambda \tilde{u}_t^p + \pi_t^p \neq 0$ and $C_{H,t} > 0$ if even marginally so.

⁴² Availability: <http://www.federalreserve.gov/boarddocs/Speeches/2003/20030829/default.htm>

As a product of positive terms over a square, θ_t is unambiguously positive.

The similarity between (4.13) and (5.2) illustrates the partial effect on utility of the level and the variance of the signal are alike.

So, (5.1) is positive or negative according to the sign of $\lambda\tilde{u}_t^p + \pi_t^p$. In *Section 4*, it was shown that increasing the variance of the signal would unambiguously increase the utility of the monetary authority. However, relative risk-avoidance, as characterized by the curvature of the utility function, was not explicitly investigated. In order to take account of this and draw further inferences as to the optimal degree of noise with respect to risk, the curvature must be taken into account. The second-order partial of utility with respect to announced preferences a_t^d is⁴³:

$$\frac{\partial^2 U}{(\partial a_t^d)^2} = \left(\lambda \frac{\partial \tilde{u}_t^p}{\partial a_t^d} + \frac{\partial \pi_t^p}{\partial a_t^d} \right) 2a_t \theta_t + (\lambda \tilde{u}_t^p + \pi_t^p) \left(2a_t \frac{\partial \theta_t}{\partial a_t^d} \right) \quad (5.3)$$

$$= -2a_t (\lambda^2 + a_t) \theta_t^2 + (\lambda \tilde{u}_t^p + \pi_t^p) \left(2a_t \frac{\partial \theta_t}{\partial a_t^d} \right) \quad (5.4)$$

where:

$$\frac{\partial \theta_t}{\partial a_t^d} = \frac{(4a_t \chi^2 \rho^2 \Delta_4) \Delta_1}{\left(2\Delta_4 a_t^d + a_t (\lambda^2 + \chi \rho + a_t^d) + \lambda^4 (1 + 2\Delta_4 a_t^d) + \lambda^2 (\chi \rho + a_t^d + 4\Delta_4) a_t^d \right)^3} \quad (5.5)$$

$$\Delta_1 = \left(a_t - 2\lambda^8 \tau^2 - 2\chi \rho \tau^2 a_t^d - 2\lambda^6 \tau^2 (2 + 2\chi \rho + a_t^d) - \lambda^2 \Delta_2 - 2\lambda^4 \tau^2 \Delta_3 \right) \quad (5.6)$$

$$\Delta_2 = \left(-1 + 2\tau^2 a_t^d + 4\chi \rho \tau^2 (1 + a_t^d) \right) \quad (5.7)$$

$$\Delta_3 = \left(1 + 2a_t^d + \chi \rho (4 + a_t^d) \right) \quad (5.8)$$

$$\Delta_4 = \chi^2 \rho^2 \tau^2 \quad (5.9)$$

⁴³ See Mathematica notebook 7 in the Mathematica appendix.

The denominator of (5.5) is unambiguously positive because all terms within it are positive and there is no subtraction. However, it is immediately unclear whether the numerator is positive as Δ_1 is positive or negative depending on specific values of the parameters. In order to draw any inferences, it makes sense to use numerical values of the parameters. When the parameters take on the values referred to as “feasible” earlier, namely, $\lambda = .1$, $\rho = \tau = \chi = 1$, and $\sigma_\varepsilon = 2$, then⁴⁴:

$$\frac{\partial \theta_t}{\partial a_t^d} = \frac{4a_t(-.031008 + a_t - 2.0606a_t^d)}{(.0101 + 2.0502a_t^d + a_t(1.01 + a_t^d))^3} \quad (5.10)$$

Let $-.031008 + a_t - 2.0606a_t^d = \Lambda$. Then $\partial \theta_t / \partial a_t^d$ is negative if and only if $\Lambda < 0$. If $a_t = a_t^*$, then Λ is negative for all non-zero values of a_t . If $a_t < a_t^d$, Λ is again negative unambiguously for non-zero values of a_t and a_t^d . If $a_t > a_t^d$, Λ is nonnegative if and only if $-\Lambda \leq a_t$, in other words, if a_t is about twice as great as a_t^d .

Thus, with the reasonable parameters $\lambda = .1$, $\rho = \tau = \chi = 1$, and $\sigma_\varepsilon = 2$, as long as a_t^d is at least half of a_t , $\partial \theta_t / \partial a_t^d$ is negative. If in addition it is known that $\lambda \tilde{u}_t^p + \pi_t^p > 0$, then the first-order partial $\partial U / \partial a_t^d$ is positive (U is strictly increasing in a_t^d), and $\partial^2 U / (\partial a_t^d)^2$ is negative (U is strictly concave in a_t^d), representing relative risk aversion of the monetary authority. Thus in this case utility is increasing in the relative size of the announced preference parameter but less and less more so for higher values. This makes sense in the context that a small value of a_t^d allows more control over expectations than $a_t^d = 0$, but for high values of a_t^d , the control

⁴⁴ See Mathematica notebook 8 in the Mathematica appendix.

diminishes as agents believe the monetary authority weights inflation relatively less.

For the next step, a definition from Hadar and Russell (1971), is used⁴⁵:

Definition: The density g is said to dominate f in the sense of *second-order stochastic dominance* (or to *second-order stochastically*

dominate f (denoted $g \succeq_s f$) if and only if $\int_a^z G(t)dt \leq \int_a^z F(t)dt$

$\forall z \in Z$. The density g is said to *strictly* dominate f in the sense of second-order stochastic dominance (denoted $g \succ_s f$) if in addition

$\int_a^z G(t)dt < \int_a^z F(t)dt$ for some subset of Z .

Note, a_i^d without noise, $a_i = a_i^d$, is strictly dominant to a_i^d with any degree of noise in an extreme case of second-order stochastic dominance. Additionally, in the case that $\lambda \tilde{u}_i^p + \pi_i^p$ is positive and $\partial \theta_i / \partial a_i^d$ is negative, then $\partial U / \partial a_i^d$ is positive and $\partial^2 U / (\partial a_i^d)^2$ is negative. Now, another definition and a theorem also from Hadar and Russell (1971) is needed:

Definition: W_2 is the set of all C^2 functions that are bounded and strictly increasing in the domain such that the first derivative is finite and the second derivative is negative everywhere in the domain.

⁴⁵ Provided by Gil Skillman in his course, Econ 380: Mathematical Economics

Characterization Theorem for Second-Order Stochastic

Dominance: For any two densities f and g with finite means defined

on Z , $E_g \{u(z)\} \geq (>) E_f \{u(z)\} \quad \forall u \in W_2$ if and only if $g \succeq_s f$

$(g \succ_s f)$.

Therefore, by the Characterization Theorem of second-order stochastic dominance, the expected value of utility arising from announcing a signal without noise is strictly greater than utility arising from announcing a signal with any degree of noise.

Proposition 5.1: When the monetary authority is risk-averse and

$\lambda \tilde{u}_t^p + \pi_t^p$ is positive, the expected value of utility is always greater

with lower variance in the signal for preferences due to a second-order stochastically dominating shift.

There are indeed several strong arguments for transparency which have been supported by Bernanke, Svensson, Blinder, and many others. However, aside from the reasons outlined, the present time-varying New Keynesian framework is capable of presenting several other reasons why limited transparency may be beneficial for monetary policy-making.

6. Opacity decreases variability of expectations

a. “Credibility for accuracy” revisited

In most previous sections it has been assumed that the public’s only guaranteed source of information on the monetary authority’s preferences were given by the willful communication of the monetary authority. The notable tangent from this rule was in the hypothetical case the public brings economic priors to the problem about what the variances of control error and preferences might be in the calculation of “credibility for accuracy,” *Section 3(c)*. “Credibility for accuracy” ventured to measure how accurate the signals (specifically, lies) were on average. Thus, in some sense, credibility for accuracy may serve to measure how transparent the monetary authority is as well. However, this type of transparency is different from previously when transparency was measured as only the noise in the monetary authority’s signal. Where the accuracy of announcements is compared against the accuracy of the public’s own signal, “credibility for accuracy” may alternatively be thought of as “second-order” transparency.”

Assume again that private agents bring economic priors of the variances of preferences and control error, but now additionally that they have intuition prior to any monetary authority credibility judgments as to what the actual *values* may be. The public’s no-communication estimate of the actual values of a_t and η_t are \tilde{a}_t and $\tilde{\eta}_t$, respectively. These inferences are not necessarily accurate but are the public’s “best guess” without any communication. This differs from all proceeding analysis in

which it was assumed the only information the public has on the value of preferences and control error came directly from the monetary authority's signal.

Let the monetary authority communicate its preferences and control error as the signals a_t^s and η_t^s with Gaussian noise as in (4.1) and (4.2). Then as in (3.10) and (3.11), and as in Geraats (2007), the relative accuracy of these signals are:

$$A_{a^s} = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_\kappa^2} \quad (6.1)$$

and,

$$A_{\eta^s} = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\gamma^2} \quad (6.2)$$

In Geraats' (2002) terminology, the more accurate the preference signal is, the more "politically" transparent the monetary authority is, and the more accurate the control error is, the more "operationally" and "policy" transparent. So, when the signals are completely accurate ($\sigma_\kappa^2 = \sigma_\gamma^2 = 0$), there is complete second-order transparency, and $A_{a^s} = A_{\eta^s} = 1$. As the variance of the noise approaches infinity, A_{a^s} and A_{η^s} approach zero. Thus, $0 < A_{a^s}, A_{\eta^s} \leq 1$ where monetary authority signal accuracy, and thus transparency are increasing as A_{a^s} and A_{η^s} increase, and the variance of noise is finite. The condition $\sigma_\kappa^2 = \sigma_\gamma^2 = 0$ does not guarantee in itself that κ_t and γ_t are not non-zero constants, which Geraats points out. However, in the present model this solution falls conveniently into the "lying" category rather than opacity, and is a scenario in which the public could discern the monetary authority discreditable.

It is possible the public does not have complete information on the relevant variances in (6.1) and (6.2), in which case there is an additional layer of informational asymmetry. Whereas before this second order of transparency – transparency of the relative accuracy of the signal – was built into “credibility for accuracy,” here it will be built into private agent expectations explicitly. Let $\tilde{\sigma}_a^2$ and $\tilde{\sigma}_\eta^2$ be the public’s pre-communication estimates or economic priors of σ_a^2 and σ_η^2 . Also, let $s_{a^s}^2$ and $s_{\eta^s}^2$ be the sample variances of the signals a_t^s and η_t^s which are clearly estimable by private agents. Since variances may be added, appealing to (4.1) and (4.2), σ_κ^2 and σ_γ^2 may be estimated by the public as:

$$s_\kappa^2 = s_{a^s}^2 - \tilde{\sigma}_a^2 \quad (6.3)$$

and

$$s_\gamma^2 = s_{\eta^s}^2 - \tilde{\sigma}_\eta^2 \quad (6.4)$$

Thus the public’s rational estimates of the accuracy of the monetary authority’s signals, or “perceived” estimates, are:

$$\tilde{A}_{a^s} = \frac{\tilde{\sigma}_a^2}{\tilde{\sigma}_a^2 + s_\kappa^2} \quad (6.5)$$

and,

$$\tilde{A}_{\eta^s} = \frac{\tilde{\sigma}_\eta^2}{\tilde{\sigma}_\eta^2 + s_\gamma^2} \quad (6.6)$$

Similarly to the perfect information case, $0 < \tilde{A}_{a^s}, \tilde{A}_{\eta^s} \leq 1$ where transparency is increasing (in the eyes of the public) as \tilde{A}_{a^s} and \tilde{A}_{η^s} approach one.

For two jointly normally distributed variables x and z :

$$E[x|z] = E[x] + \frac{Cov(x,z)}{Var(z)}(z - E[z]) \quad (6.7)$$

Thus similar to Geraats (2007), appealing to (6.5) - (6.7), the conditional estimates of the values of a_t and η_t of private agents with prior estimates of their values and variances may be written:

$$E[a_t | a_t^s] = \tilde{a}_t + \frac{\tilde{\sigma}_a^2}{\tilde{\sigma}_a^2 + s_\kappa^2} (a_t^s - \tilde{a}_t) \quad (6.8)$$

$$= (1 - \tilde{A}_{a^s}) \tilde{a}_t + \tilde{A}_{a^s} a_t^s \quad (6.9)$$

and,

$$E[\eta_t | \eta_t^s] = \tilde{\eta}_t + \frac{\tilde{\sigma}_\eta^2}{\tilde{\sigma}_\eta^2 + s_\gamma^2} (\eta_t^s - \tilde{\eta}_t) \quad (6.10)$$

$$= (1 - \tilde{A}_{\eta^s}) \tilde{\eta}_t + \tilde{A}_{\eta^s} \eta_t^s \quad (6.11)$$

Thus, private agents' estimates of a_t and η_t are weighted averages of their own no-communication estimates, \tilde{a}_t and $\tilde{\eta}_t$, and the signals of the monetary authority, depending on the *perceived* accuracy of the monetary authority's signals, \tilde{A}_{a^s} and \tilde{A}_{η^s} .

b. Accuracies of signals are known to the public

Consider the case in which the public knows the actual variances, though not necessarily the values, of a_t , η_t , κ_t , and γ_t . Specifically, A_{a^s} and A_{η^s} , are known explicitly to private agents. Then, rational expectations from (2.22) - (2.26) will become:

$$E_t \pi_{t+1}^p = cr_t \pi_t^p \quad (6.12)$$

$$= cr \left\{ E[a_t | a_t^s] \right\} \left(\pi_t - E[\eta_t | \eta_t^s] \right) \quad (6.13)$$

where

$$E[a_t | a_t^s] = (1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s \quad (6.14)$$

and

$$E[\eta_t | \eta_t^s] = (1 - A_{\eta^s}) \tilde{\eta}_t + A_{\eta^s} \eta_t^s \quad (6.15)$$

Equation (6.13) is written to stress that expectations are a function of the public's conditional expected values of preferences and control error. The conditional expectations are themselves functions of the public's inference of the accuracy of the signals so that in this case of perfect information, they use A_{a^s} and A_{η^s} . Accordingly with rational expectations as derived earlier, (6.13) is specified as:

$$c = \tau \rho \quad (6.16)$$

$$r_t = \left(E[a_t | a_t^s] + \lambda^2 \right) \left[\left(\frac{f}{g} \right) \pm \frac{([f]^2 - [g]^2)}{fg} \right] \quad (6.17)$$

where:

$$f \equiv f \left(E[a_t | a_t^s] \right) = \tau \left((1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s + \rho \chi + \lambda^2 \right) \quad (6.18)$$

$$g \equiv g \left(E[a_t | a_t^s] \right) = 2 \chi \tau^2 \rho \left((1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s + \lambda^2 \right) \quad (6.19)$$

The variance of expectations may be calculated using the delta approximation method which uses second-order Taylor expansions.⁴⁶ Provided a function $f(x)$ is

⁴⁶ The intricacy of the current model seems to necessitate this method. Geraats (2007) uses a much simpler model.

twice-differentiable and the mean and variance of a stochastic variable x are finite, the delta method approximates the variance of $f(x)$ as:

$$\text{Var}[f(x)] \approx (f'(E[x]))^2 \text{Var}[x] \quad (6.20)$$

Thus, where expectations are a function of $E[\eta_t | \eta_t^s]$ as made apparent by (6.13), they are also a function of the stochastic variable η_t^s by (6.11). Given expectations are specified as a product of squares and are thus twice-differentiable, and that expectations' mean and variance are finite since they are composed of finite terms, the variance of expectations may then be approximated as:

$$\text{Var}[E_t \pi_{t+1} \{\eta_t^s\}] \approx \left(\frac{\partial E_t \pi_{t+1} \{E[\eta_t^s]\}}{\partial \eta_t^s} \right)^2 \left(\frac{\sigma_\eta^2}{A_{\eta^s}} \right) \quad (6.21)$$

The expression $E_t \pi_{t+1} \{\eta_t^s\}$ clarifies that expectations can be written as a function of the control error signal, and $\frac{\partial E_t \pi_{t+1} \{E[\eta_t^s]\}}{\partial \eta_t^s}$ is the partial derivative of expectations with respect to the control error signal at the control error signal's expected value (zero). Additionally, since variances are additive, $\text{Var}[\eta_t^s] = \sigma_\eta^2 + \sigma_\gamma^2$ by (4.2), so $\text{Var}[\eta_t^s] = 1 / (A_{\eta^s} / \sigma_\eta^2)$ by (6.2) and inversion.

The delta method estimate of the variance of expectations may be expressed as⁴⁷:

$$\text{Var}[E_t \pi_{t+1} \{\eta_t^s\}] \approx A_{\eta^s} \sigma_\eta^2 (\rho\tau)^2 \left[(1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s \right]^2 \quad (6.22)$$

⁴⁷ See Appendix proof A (6.22).

Thus, as in Geraats (2007), the variance of expectations is smallest when the signals are very unreliable ($A_{\eta^s}, A_{a^s} \rightarrow 0$). The reason for this is that with decreased accuracy of the signals, private agents depend more on their own no-communication inferences. Note, $A_{\eta^s} = 0$ is not an absolute solution for zero variance of expectations for any value of A_{a^s} since σ_γ^2 can not have infinite variance and thereby A_{η^s} is never equal to zero.

The partial derivatives of the variance of expectations $Var[E_t \pi_{t+1}] = \sigma_{E_t \pi_{t+1}}^2$ with respect to both the accuracy of the control error signal A_{η^s} and the preference signal A_{a^s} are⁴⁸:

$$\frac{\partial \sigma_{E_t \pi_{t+1}}^2}{\partial A_{\eta^s}} = \sigma_\eta^2 (\rho\tau)^2 \left[(1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s \right]^2 \quad (6.23)$$

and,

$$\frac{\partial \sigma_{E_t \pi_{t+1}}^2}{\partial A_{a^s}} = 2 \left[(A_{a^s} - 1) \tilde{a}_t^2 + A_{a^s} a_t^{s2} + \tilde{a}_t a_t^s (1 - 2A_{a^s}) \right] \tilde{a}_t^2 A_{\eta^s} \sigma_\eta^2 (\rho\tau)^2 \quad (6.24)$$

Thus, since the partial with respect to the accuracy of control error, A_{η^s} , is strictly positive, increasing the accuracy of the signal for control error leads to an unambiguous increase in the variance of expectations.⁴⁹ The comparative static with respect to the accuracy of the preference signal, A_{a^s} , is positive or negative corresponding to the sign of $(A_{a^s} - 1) \tilde{a}_t^2 + \tilde{a}_t a_t^s (1 - 2A_{a^s}) + A_{a^s} (a_t^s)^2$. For example, if

⁴⁸ See Appendix proof A (6.23).

⁴⁹ Increasing accuracy implies $A_{\eta^s} < 1$ originally and thereby $\sigma_\eta^2 > 0$ by (6.2) so the partial is non-zero.

$\tilde{a}_t = a_t^s = 1/10$ and $A_{a^s} = 1/2$, the partial derivative is equal to zero and the accuracy of the signal for preferences has no effect. If $\tilde{a}_t = a_t^s = 1/10$ and $A_{a^s} < 1/2$, the partial derivative is positive and if $A_{a^s} > 1/2$ the partial derivative is negative. Alternatively, if $A_{a^s} = 1/2$ and $\tilde{a}_t < a_t^s$, the partial is positive, and if $A_{a^s} = 1/2$ and $\tilde{a}_t > a_t^s$, the partial is negative. Thus the variance of expectations with respect to the accuracy of the signal depends largely on the values of the parameters of the model.

Proposition 6.1 Given the public has its own estimate of preferences and control error, and knows the accuracy of public signals, a decrease in the accuracy of the control error signal always decreases the variability of expectations. A decrease in the accuracy of the preference signal does the same for certain values of the parameters of the model.

By the Phillips curve, inflation is written as a function of expectations the variance of inflation necessarily varies with the variance of expectations. In particular, from equation (2.7) the exact relationship is:

$$Var[\pi_t] = \chi^2 Var[E_t \pi_{t+1}] \quad (6.25)$$

It may then be posited:

Corollary 6.1 Under the assumptions of *Proposition 7.1*, a decrease in the accuracy of the control error signal decreases the variability of inflation, while a decrease in the decrease in the accuracy of the preference signal does the same for certain values of the parameters of the model.

In conclusion, under the assumption that the public has its own no-communication prior of the values of preferences and control error, and that it knows the average accuracy of the signals the monetary authority releases, a decrease in the accuracy of the control error signal will unambiguously decrease the variability of inflation, and a decrease in the accuracy for preferences will do the same for certain values of the parameters of the model.

c. Accuracies of signals are *not* known to the public

Now the case is considered in which the average accuracy of each of the signals is not known by private agents, but that they have their own priors for what these values may be, namely, \tilde{A}_{a^s} and \tilde{A}_{η^s} . Thus, conditional expectations no longer take the form of (6.14) and (6.15), but of (6.9) and (6.11). Additionally, the variances of a_t , η_t , κ_t , and γ_t are not known to, but estimated by the public. Then, by using the formula of (6.20), the variance of expectations may again be estimated. Since the variance of η_t must be estimated or brought as a prior, (6.21) is altered:

$$\text{Var} \left[E_t \pi_{t+1} \{ \eta_t^s \} \right] \approx \left(\frac{\partial E_t \pi_{t+1} \{ \eta_t^s \}}{\partial \eta_t^s} \left\{ E \left[\eta_t^s \right] \right\} \right)^2 \left(\frac{\tilde{\sigma}_\eta^2}{\tilde{A}_{\eta^s}} \right) \quad (6.26)$$

Thus, private agents' expectations will have the variance⁵⁰:

$$\text{Var} \left[E_t \pi_{t+1} \right] \approx \tilde{A}_{\eta^s} \tilde{\sigma}_\eta^2 (\rho\tau)^2 \left[(1 - \tilde{A}_{a^s}) \tilde{a}_t + \tilde{A}_{a^s} a_t^s \right]^2 \quad (6.27)$$

Then, using equations (6.3)-(6.6), and rearranging:

⁵⁰ Follows trivially from Appendix proof A(0.104) when $\tilde{A}_{a^s} \neq A_{a^s}$ and $\tilde{A}_{\eta^s} \neq A_{\eta^s}$.

$$\text{Var}[E_t \pi_{t+1}] = \sigma_{E_t \pi_{t+1}}^2 \approx \frac{\tilde{\sigma}_\eta^2}{s_{\eta^s}^2} \tilde{\sigma}_\eta^2 (\rho\tau)^2 \left[\tilde{a}_t + \frac{\tilde{\sigma}_a^2}{s_{a^s}^2} (a_t^s - \tilde{a}_t) \right]^2 \quad (6.28)$$

Therefore, the partial effects of the public's no-communication estimates of the variance of control error and preferences and the signals for control error and preferences are:

$$\frac{\partial \sigma_{E_t \pi_{t+1}}^2}{\partial \tilde{\sigma}_\eta^2} = 2 \frac{\tilde{\sigma}_\eta^2}{s_{\eta^s}^2} (\rho\tau)^2 \left[\tilde{a}_t + \frac{\tilde{\sigma}_a^2}{s_{a^s}^2} (a_t^s - \tilde{a}_t) \right]^2 \quad (6.29)$$

$$\frac{\partial \sigma_{E_t \pi_{t+1}}^2}{\partial \tilde{\sigma}_a^2} = 2 \left(\frac{\tilde{\sigma}_\eta^2}{s_{\eta^s}^2} \right) (\rho\tau)^2 \left[\tilde{a}_t \left(\frac{(a_t^s - \tilde{a}_t)}{s_{a^s}^2} \right) + \tilde{\sigma}_a^2 \left(\frac{(a_t^s - \tilde{a}_t)}{s_{a^s}^2} \right)^2 \right] \quad (6.30)$$

$$\frac{\partial \sigma_{E_t \pi_{t+1}}^2}{\partial s_\eta^2} = - \left(\frac{\tilde{\sigma}_\eta^2 \rho\tau}{s_\eta^2} \right)^2 \left[\tilde{a}_t + \frac{\tilde{\sigma}_a^2}{s_{a^s}^2} (a_t^s - \tilde{a}_t) \right]^2 \quad (6.31)$$

$$\frac{\partial \sigma_{E_t \pi_{t+1}}^2}{\partial s_a^2} = - \left(2 \frac{\tilde{a}_t (\tilde{\sigma}_\eta^2)^2 \tilde{\sigma}_a^2}{s_{\eta^s}^2} (\rho\tau)^2 \right) \left[\left(\frac{(a_t^s - \tilde{a}_t)}{(s_{a^s}^2)^2} \right) + \left(\frac{\tilde{\sigma}_a^2 (a_t^s - \tilde{a}_t)^2}{(s_{a^s}^2)^3} \right) \right] \quad (6.32)$$

So clearly, the variance of expectations is increasing in the public's no-communication estimates of the variances of control error $\tilde{\sigma}_\eta^2$ (6.29), but decreasing through the sample variances of its signal (6.31). The decrease in the volatility of expectations from increases in the variance of the signal may be explained by the public's increasing willingness to use their own no-communication estimate and place relatively less weight on the inaccurate varying signal. Additionally, if $a_t^s > \tilde{a}_t$ then the volatility of expectations is increasing in the public's no-communication estimate of the variance of preferences, $\tilde{\sigma}_a^2$ (6.30), and decreasing through the noisiness of the sample variance of the signal, $s_{a^s}^2$ (6.32).

If $a_t^s < \tilde{a}_t$, then the volatility of expectations is decreasing in the public's no-communication estimate of the variance of preferences, $\tilde{\sigma}_a^2$ if and only if

$$-\tilde{a}_t \underset{(+)}{(a_t^s - \tilde{a}_t)} / s_{a^s}^2 > \tilde{\sigma}_a^2 \underset{(+)}{\left((a_t^s - \tilde{a}_t) / s_{a^s}^2 \right)^2};$$

if the inequality switches, the volatility of expectations is decreasing in the increasing variance of the public's no-communication estimate. If there exists an equality, (which is true if $a_t^s = \tilde{a}_t$), then there is no trade-off. Additionally if $a_t^s < \tilde{a}_t$, then the volatility of expectations is increasing in the variance of the public signal of preferences, s_a^2 , if and only if

$$-\left(a_t^s - \tilde{a}_t \right) / \left(s_{a^s}^2 \right)^2 > \tilde{\sigma}_a^2 \left(a_t^s - \tilde{a}_t \right)^2 / \left(s_{a^s}^2 \right)^3;$$

a reversed inequality means decreasing variance of expectations in an increase of the public signal. Equality (which results if $a_t^s = \tilde{a}_t$) leads to no trade-off.

Assuming sample variance is somewhat representative of actual variance with enough periods' data, these dynamics suggests that the monetary authority can moderate the volatility of inflation expectations by increasing the actual noise in the signal $\sigma_{\eta^s}^2$, and thus, decreasing "second-order" transparency. If $a_t^s > \tilde{a}_t$ or if $a_t^s < \tilde{a}_t$ and $-\left(a_t^s - \tilde{a}_t \right) / \left(s_{a^s}^2 \right)^2 > \tilde{\sigma}_a^2 \left(a_t^s - \tilde{a}_t \right)^2 / \left(s_{a^s}^2 \right)^3$, then an increase in $\sigma_{a^s}^2$ decreases the volatility of expectations as well. This analysis is summarized by the following proposition:

Proposition 6.2: Given private agents have no-communication estimates of the variances and values of the parameters of the model which are inaccurate, the monetary authority may moderate the volatility of inflation expectations by increasing the variance of the control error signal for any values of the parameters of the model, and/or by increasing the variance of the preference signal for certain values of the parameters of the model.

The negative effect on the variance of expectations of increased variance in the signal of control error, is unambiguous; decreased “second-order” transparency in control error results in less volatile expectations, and thus, less volatile inflation by (6.25). However, the case for the decreased transparency in the preference parameter is contingent upon the monetary authority’s conservativeness being overestimated, $a_t^s > \tilde{a}_t$, or if $a_t^s < \tilde{a}_t$, that in addition $-(a_t^s - \tilde{a}_t)/(s_{a^s}^2)^2 > \tilde{\sigma}_a^2 (a_t^s - \tilde{a}_t)^2 / (s_{a^s}^2)^3$. In the case that $a_t^s = \tilde{a}_t$, increasing the volatility of the preference signal has no effect on the volatility of expectations.

Thus, the preferred amount of noise in the signal for preferences depends upon the values of the parameters of the model, which is not in itself a particularly strong argument for opacity. However, this is not to say that the variance of the signal is the only mechanism through which transparency is facilitated; the release of the value of a_t^s itself is a form of transparency. Additionally, the effects of opacity of preferences may be particularly strong in the certain circumstances it is beneficial.

Note from (6.28) that if $a_t^s = \tilde{a}_t$, then :

$$Var[E_t \pi_{t+1}] \approx \frac{\tilde{\sigma}_\eta^2}{s_{\eta^s}^2} \tilde{\sigma}_\eta^2 (\rho\tau)^2 \tilde{a}_t^2 \quad (6.33)$$

$$\frac{\partial \sigma_{E_t \pi_{t+1}}^2}{\partial s_a^2} = 0 \quad (6.34)$$

However, if $a_t^s \neq \tilde{a}_t$, the variance of expectations described by (6.28), and expectations' derivative with respect to the sample variance of the preference signal, are rewritten here:

$$\text{Var}[E_t \pi_{t+1}] \approx \frac{\tilde{\sigma}_\eta^2}{s_{\eta^s}^2} \tilde{\sigma}_\eta^2 (\rho\tau)^2 \left[\tilde{a}_t + \frac{\tilde{\sigma}_a^2}{s_{a^s}^2} (a_t^s - \tilde{a}_t) \right]^2 \quad (6.35)$$

$$\frac{\partial \sigma_{E_t \pi_{t+1}}^2}{\partial s_a^2} = - \left(2 \frac{\tilde{a}_t (\tilde{\sigma}_\eta^2)^2 \tilde{\sigma}_a^2}{s_{\eta^s}^2} (\rho\tau)^2 \right) \left[\left(\frac{a_t^s - \tilde{a}_t}{(s_{a^s}^2)^2} \right) + \left(\frac{\tilde{\sigma}_a^2 (a_t^s - \tilde{a}_t)^2}{(s_{a^s}^2)^3} \right) \right] \quad (6.36)$$

The difference between (6.34) and (6.36) is obvious; for a given variance of expectations, it is only possible to decrease this variance by manipulating the variance of the preference signal if $a_t^s \neq \tilde{a}_t$. It is likewise apparent that if agents' no-communication estimate of preferences is greater than the actual value, $a_t^s < \tilde{a}_t$, the variance of expectations (6.35) will be unambiguously lower than with perfect information (6.33). Additionally, in the admittedly special case that

$$-(a_t^s - \tilde{a}_t) / (s_{a^s}^2)^2 > \tilde{\sigma}_a^2 (a_t^s - \tilde{a}_t)^2 / (s_{a^s}^2)^3, \text{ increased variability of the preference}$$

signal will lead to further decreases in volatility. Thus, if the monetary authority is thought to be less conservative than it actually is, $a_t^s < \tilde{a}_t$, it will incur unambiguously lower inflation variability than with perfect information. This inference is potentially motivation for the monetary authority to withhold the actual value of its preferences if the public has their own (inaccurate) guess. Under certain conditions, further variability in its signal of its preferences will result in yet lower variability. This

result suggests a potentially extreme form of opacity may decrease the variability of expectations, and thus inflation, greatly.

Proposition 6.3: If the monetary authority is thought to be less conservative than it actually is ($a_t^s < \tilde{a}_t$), it will encounter unambiguously lower expectation (and hence inflation) variability than with perfect information. Thus, there is potentially the motivation to withhold actual preferences.

Corollary 6.3: The assumption of an inaccurately high no-communication estimate of a_t^s makes possible further moderation of expectations (and hence inflation) variability with increases in the variability of the preference signal, for certain values of the parameters of the model: $-(a_t^s - \tilde{a}_t) / (s_{a^s}^2)^2 > \tilde{\sigma}_a^2 (a_t^s - \tilde{a}_t)^2 / (s_{a^s}^2)^3$.

It is not to be ignored that such a benefit to opacity of the preference signal is not unambiguous. When $a_t^s > \tilde{a}_t$, the variance of expectations (6.35) is unambiguously higher than with perfect information (6.33). However, when $a_t^s > \tilde{a}_t$ there is opportunity for the monetary authority to curb volatility by increasing the variability in the preference signal via (6.36).

Thus, when private agents form their own (no-communication) estimates of the values and variances of preferences and control error, the monetary authority may moderate expectations, and hence inflation, by allowing some degree of noise in their control error signal. They may further moderate expectations, and hence inflation, if the public believes they are less conservative than they are in reality, $a_t^s < \tilde{a}_t$, and by then announcing a preference signal with some degree of noise.

7. Public-private natural rate of unemployment estimates

a. Monetary authority and individual agent signals

Morris and Shin (2002) develop a framework in which each member of a continuum of private agents receives two noisy signals of the true value of an economic parameter; the resulting estimate of this parameter plays a role in each agent's decision of what economic action to take. One of the noisy signals is available publicly, and is used by each agent conditionally upon its relative accuracy as measured by its distribution. The other signal available to each agent is its own signal which is developed internally and is strictly private information of each individual private agent. Note that in previous sections, the "public signal" has been available to all private agents, but here, each private agent has its own private estimate which it has not immediate motivation to share. This section applies Morris and Shin's technique to the model at hand, where the important economic parameter to be measured is the natural rate of unemployment. The step of showing that natural rate misestimates fit into the highly abstracted Morris-Shin model is unique.

Despite the natural rate of unemployment's importance in economic theory, Staiger, Stock, and Watson (1997) found sophisticated econometric estimates (via the Kalman filter) have error bands of up to three percentage points; concurrently at the time of their publication, the 95% confidence interval for the level of the NAIRU was 5.1-7.7%. This magnitude of error has led some to suggest, such as Galbraith (1996), that the parameter should be attributed relatively less importance in economic theory.

In practice, the Federal Reserve seems to avoid major catastrophe out of natural rate mismeasurement by attributing relatively more weight to inflation, and inflation expectations.⁵¹ However, in current theory and this particular New-Keynesian model the natural rate plays a vital role, which has been thus far abstracted from. The current section will illuminate the importance in the estimation of the natural rate of unemployment, specifically, its link with inflation expectations, and the effect of monetary authority communication on this link.

Suppose that in each period, the monetary authority develops its own estimate of the natural rate of unemployment. This estimate is equal to the actual value of the natural rate plus some normal white-noise error having a potentially large degree of variance.

$$u_{t,ma}^n = u_t^n + e_{t,ma} \quad (7.1)$$

$$\sigma_{e,ma}^2 \quad (7.2)$$

Thus, the estimate of the monetary authority's estimate has a mean of the actual natural rate, u_t^n , with an error band quantifiable by the relative variance of the error, $\sigma_{e,ma}^2$. For the time-being, it is assumed that there is perfect information available to the public, so that the monetary authority announces its estimate, $u_{t,ma}^n$, and the relative uncertainty of its estimate, $\sigma_{e,ma}^2$. Thus in Geraats' (2002) terminology, we begin by assuming the monetary authority is perfectly transparent in the "economic" sense.

⁵¹ See, for instance, the recent *Wall Street Journal* article: Ip, 2007b.

Each agent from a continuum of private agents has its own estimate of the natural rate as well. Like the monetary authority, each agent i estimates the actual natural rate u_t^n correctly on-average, but its estimates have potentially great uncertainty as measurable by the variance of the error:

$$u_{t,i}^n = u_t^n + e_{t,i} \quad (7.3)$$

$$\sigma_{e,i}^2 \quad (7.4)$$

Thus, the accuracy of the signals of both the monetary authority and private agent i are measurable by the inverse of their variances:

$$A_{ma} = \frac{1}{\sigma_{e,ma}^2} \quad (7.5)$$

$$A_i = \frac{1}{\sigma_{e,i}^2} \quad (7.6)$$

A smaller variance in the error of either signal therefore makes that signal relatively more accurate as A_x increases. Let it be given that each individual agent has information on the value of its own private estimate and that of the monetary authority, and their relative accuracies. Then each agent forms its estimate of the natural rate of unemployment in any period is a weighted average of the two signals, where the weights are the accuracies themselves:

$$E_i(u_t^n | u_{t,ma}^n, u_{t,i}^n) = \frac{A_{ma}u_{t,ma}^n + A_i u_{t,i}^n}{A_{ma} + A_i} \quad (7.7)$$

However, rational private agents have additional information that could inform their estimate. They understand there is inherent imprecision in the estimate of any one agent, but that all agents are potentially right on average. In fact, given

that each agent faces the same uncertainty in its individual estimate, the natural rate estimates of the continuum of agents should form a normal distribution around the natural rate's true value. Thus, the average estimate of the natural rate from all agents would influence each agent's estimate were this cumulative private information available. However, any single agent must settle for only the *expected value* of the global (i.e., all-agent) mean of private estimates when these private estimates are not made public.

At the same time, each public agent faces a “second-guessing” motive; were its own estimate of the natural rate superior to the expected value of the mean of all others, it may be able to capitalize at the expense of others.⁵² Given this second-guessing motive is quantifiable by a term $0 < r < 1$, agent i 's natural rate estimate given the weighted average of the monetary authority's estimate and its own estimate, along with the expected value of the global mean of natural rate estimates, \bar{u}_t^n , is:

$$E_i(u_t^n | u_{t,ma}^n, u_{t,i}^n, E_i(\bar{u}_t^n)) = (1-r) \frac{A_{ma}u_{t,ma}^n + A_i u_{t,i}^n}{A_{ma} + A_i} + r E_i(\bar{u}_t^n) \quad (7.8)$$

This represents the “Keynesian beauty contest” phenomenon from *The General Theory* (1936) referred to in the *Introduction*, in which estimates of other agents matter to each individual agent. The expected value of \bar{u}_t^n is calculated individually by each agent with imperfect information on the other agents. Consistent with the formula of (7.7), each agent i believes the expected value of the natural rate of each other agent j is a linear combination of its own individual estimate and the

⁵² At the very least, in reputation for accurate estimation might improve its business.

monetary authority's publicly available estimate. That is, where K is a constant,

$$0 < K < 1:$$

$$E_j(u_t^n | u_{t,ma}^n, u_{t,j}^n) = Ku_{t,j}^n + (1-K)u_{t,ma}^n \quad (7.9)$$

Then, given the error of each agent's estimate is mean zero, agent i 's best estimate of the average of all other agents' individual private estimates is its own private estimate:

$$E_i(\bar{u}_t^n | u_{t,ma}^n, u_{t,i}^n) = K \frac{A_{ma}u_{t,ma}^n + A_i u_{t,i}^n}{A_{ma} + A_i} + (1-K)u_{t,ma}^n \quad (7.10)$$

Then, by multiplying by identities, equation (7.10) may be rewritten:

$$E_i(\bar{u}_t^n | u_{t,ma}^n, u_{t,i}^n) = \left(\frac{KA_i}{A_{ma} + A_i} \right) u_{t,i}^n + \left(1 - \frac{KA_i}{A_{ma} + A_i} \right) u_{t,ma}^n \quad (7.11)$$

Thus, plugging (7.11) and (7.9) into (7.8) yields:

$$E_i(u_t^n | u_{t,ma}^n, u_{t,i}^n, E_i(\bar{u}_t^n)) = (1-r) \frac{A_{ma}u_{t,ma}^n + A_i u_{t,i}^n}{A_{ma} + A_i} + r \left[\left(\frac{KA_i}{A_{ma} + A_i} \right) u_{t,i}^n + \left(1 - \frac{KA_i}{A_{ma} + A_i} \right) u_{t,ma}^n \right] \quad (7.12)$$

$$= \left[(1-r) \frac{A_{ma}}{A_{ma} + A_i} + r \left(1 - \frac{KA_i}{A_{ma} + A_i} \right) \right] u_{t,ma}^n + (1-r) \left[\frac{A_i}{A_{ma} + A_i} + r \frac{KA_i}{A_{ma} + A_i} \right] u_{t,i}^n \quad (7.13)$$

$$= \left[\frac{A_{ma}(1-r+rK)}{A_{ma} + A_i} \right] u_{t,ma}^n + \left[1 - \frac{A_{ma}(1-r+rK)}{A_{ma} + A_i} \right] u_{t,i}^n \quad (7.14)$$

Since (7.14) and (7.9) both represent estimates of the natural rate, the similarity of

their form allows us to write:

$$K = \frac{A_{ma}(1-r+rK)}{A_{ma} + A_i} \quad (7.15)$$

from which it may be solved:

$$K = \frac{A_{ma}(1-r)}{A_{ma}(1-r) + A_i} \quad (7.16)$$

So, expectations of each private agent i may be written:

$$E_i(u_t^n | u_{t,ma}^n, u_{t,i}^n) = \frac{A_{ma}(1-r)}{A_{ma}(1-r) + A_i} u_{t,i}^n + \left(1 - \frac{A_{ma}(1-r)}{A_{ma}(1-r) + A_i}\right) u_{t,ma}^n \quad (7.17)$$

$$= \frac{A_{ma}(1-r)u_{t,i}^n + A_i u_{t,ma}^n}{A_{ma}(1-r) + A_i} \quad (7.18)$$

Recall from the optimal policy condition equation (2.15) that inflation policy is linearly related to unemployment gap policy.⁵³ Then, where η_t^s is the value of control error communicated to private agents and (7.18) represents the value of the natural rate of unemployment a representative (or average) agent estimates, the average agent will have the following identity:

$$\pi_t - \eta_t^s = \frac{\alpha_t^e}{\lambda} \left(u_t - \frac{A_{ma}(1-r)u_{t,i}^n + A_i u_{t,ma}^n}{A_{ma}(1-r) + A_i} \right) \quad (7.19)$$

The variable α_t^e is the value of monetary authority preferences the representative private agent perceives given its estimate of the natural rate of unemployment, and π_t and u_t are the realized (actual) values of inflation and unemployment, respectively. Given this, the average private agent's inference of the monetary authority's preferences is:

$$\alpha_t^e = \frac{\lambda(\pi_t - \eta_t^s)}{\left(u_t - \frac{A_{ma}(1-r)u_{t,i}^n + A_i u_{t,ma}^n}{A_{ma}(1-r) + A_i} \right)} \quad (7.20)$$

⁵³ Where the monetary authority is only functionally concerned with inflation policy, unemployment gap policy is implicit via the Phillips curve (2.7).

b. Expectation formation

Given public and private signals of the natural rate of unemployment, the public is able to draw an inference of the value of monetary authority preferences, (7.20). Given this estimate of the monetary authority's preferences, using the rational form of expectations, equation (2.22), expectations are:

$$E_t \pi_{t+1} = \tau \rho r \{a_t^e\} (\pi_t - \eta_t^s) \quad (7.21)$$

With this specification of expectations, the comparative static of the accuracy of the monetary authority's signal on expectations may be found. In the last subsection it had been assumed that the monetary authority revealed its exact estimate to the public, and that any noise represented only the accuracy of this estimate, not the opacity of communication. However, if it is found that the partial affect of the accuracy of the monetary authority's signal on expectations is positive, then this would suggest that if the monetary authority were to add additional noise to its estimate in its announcement, it could moderate expectations.

Such an approach of masking the true value of the natural rate is potentially only *not* detrimental to the public if the estimates of private agents are as accurate as those of the monetary authority. However, it will be shown that whether the accuracy of the private estimate of each individual agent is exactly equal to, or less than that of the monetary authority, the same comparative static effects hold. The exact partial effect of the accuracy of the monetary authority's signal on expectations is⁵⁴:

⁵⁴ See Mathematica notebook 9 in the Mathematica appendix.

$$\frac{\partial E_t \pi_{t+1}}{\partial A_{ma}} = \frac{-(x_0 x_1 (x_2 + x_3))}{x_4^2 (x_5 + x_6)^2} \quad (7.22)$$

$$x_0 = -2A_i (u_{t,ma}^n - u_{t,i}^n) (\eta_t^s - \pi_t) (r-1) \chi (\lambda \rho \tau)^2 \quad (7.23)$$

$$x_1 = A_i u_{t,ma}^n \lambda + A_i (\eta_t^s - \pi_t - u_t \lambda) + (1-r) (\pi_t - \eta_t^s + (u_t - u_{t,i}^n) \lambda) A_{ma} \quad (7.24)$$

$$x_2 = A_i u_{t,ma}^n (\lambda^2 + 2\chi\rho) - A_i (u_t (\lambda^2 + 2\chi\rho) + \lambda (\pi_t - \eta_t^s)) \quad (7.25)$$

$$x_3 = (r-1) ((u_t - u_{t,i}^n) (\lambda^2 + 2\chi\rho) + \lambda (\pi_t - \eta_t^s)) A_{ma} \quad (7.26)$$

$$x_4 = (A_i (u_t - u_{t,ma}^n) + (r-1) (u_{t,i}^n - u_t) A_{ma})^2 \quad (7.27)$$

$$x_5 = x_2 - A_i u_{t,ma}^n \chi \rho \quad (7.28)$$

$$x_6 = x_3 - A_{ma} (r-1) (u_t - u_{t,i}^n) \chi \rho \quad (7.29)$$

The sign of this partial effect is immediately not obvious, though its denominator is unambiguously positive. However, what is apparent by experimenting with different values of the parameters of the model is that there are ranges of values in which added accuracy decreases expectations, and other ranges in which expectations actually increase. This seems to suggest there are ranges of parameters in the model in which the monetary authority could reduce inflation expectations by announcing its estimate of the natural rate – which already has a fair amount of noise – with additional noise in communication.

Consider first the “realistic values” of the parameters used previously, $\lambda = .1$ and $\rho = \tau = \chi = 1$, along with a small control error signal, $\eta_t^s = .1$, unemployment and inflation at the same intermediate level, $u_t = \pi_t = 5\%$, and the “second-guessing” parameter $r=1/2$. Then the accuracy of the monetary authority’s signal is set equal to the representative agent’s accuracy plus a nonnegative constant k ($A_{ma} = A_i + k$), so that the monetary authority’s signal is better than or equal to the representative agent’s signal. Even when k is positive and the monetary authority’s signal are more

precise, there are ranges of the values of the monetary authority's signal $u_{t,ma}^n$ in comparison with the representative agent's signal $u_{t,i}^n$ in which $\partial E_t \pi_{t+1} / \partial A_{ma} > 0$; these ranges form a saddle point which is depicted by *Figure 5*, and change only slightly (still retaining saddle-point shape) for varying values of k . However, it is apparent from *Figure 5* that the more accurate than the public the monetary authority is, the less of range of value for which added noise to their natural rate signal will moderate expectations. Despite this, there are always ranges in which added noise will anchor expectations, even in the most extreme case, $k=1$.

The potential for the partial effect of the accuracy of the monetary authority's signal on expectations to be positive suggests that the monetary authority could moderate expectations by announcing its own estimate of the natural rate with some additional noise. Doing so would attach additional uncertainty to their announcement in the eyes of private agents, and cause the value of A_{ma} to fall. Furthermore, this holds in the case that the monetary authority and the representative agent have equivalently accurate estimates, so withholding information in this case should be expected to have no detrimental effects. The only case in which the partial effect of the accuracy of the monetary authority's signal on expectations is in the case that the monetary authority and the representative have the same estimate, $u_{t,ma}^n = u_{t,i}^n = 0$.

The main point is that given private agents estimate the natural rate as accurately as the monetary authority, it can decrease expectations by decreasing the transparency of its estimate under certain conditions while not affecting the accuracy of the private agents' estimates.

Proposition 7.1: Given the monetary authority's period-by-period estimate of the natural rate of unemployment is at least as accurate as the representative private agent's, there are ranges of the parameters of the model in which the monetary authority may anchor inflation expectations by announcing its estimate of the natural rate with noise beyond the inherent uncertainty in the estimate.

Corollary 7.1: Increased opacity in the signal decreases expectations even when private agents' expectations are as good as the monetary authority's, so that expectations may be moderated but there are no long-term losses to the public's estimating abilities with less public information.

8. Opacity averts expectation traps

a. Markov perfect equilibria

Chari, Christiano, and Eichenbaum (1998) (henceforth CCE) famously investigate a model in which an economy experiences a transitory real shock, causing private agents' expectations to rise. The optimizing monetary authority is then left with the problem of whether to raise inflation along with expectations, or not and suffer lower employment and output. This dilemma is referred to by Blinder (1982, p.264) as the "accommodation issue," and was potentially a reason for the high inflation experienced in the late 1960's and throughout the 1970's. (Ball, 1995) CCE find that when the private sector is modeled in detail, there are indeed dynamic situations in which a transitory real shock can lead an accommodative monetary authority to a new higher-inflation level equilibria than before the shock; CCE call such an experience an "expectations trap."

Prior to CCE, Ball (1995) studies a similar model of the persistent effect of transitory shocks on inflation. Ball begins his investigation by citing that a well-known problem of infinite-horizon monetary policy games is that there exist multiple Nash equilibria. He argues that the choice of one of these equilibria, such as the inflationary bias equilibrium with time-inconsistent discretionary policy making in the Barro-Gordon (1983) model, is ambiguous. Furthermore, says Ball, models of finite monetary policy regimes such as Backus and Driffill (1985) address the problems associated with an infinite horizon but introduce a new problem; in the

finite-horizon models, exiting regimes may have the incentive to play the endgame of sacrificing all reputation for economic stimulus, which is not seen in practice.⁵⁵

In response to these problems, Ball uses Markov perfect equilibria (henceforth, MPE) which are Nash equilibria in which the actions of monetary authority depend only on Markovian, or “state-contingent” variables. (Maskin and Tirole, 2001) MPE depend not on past histories of variables but only their current state, thereby greatly simplifying the problem. Ball adopts this approach from Maskin and Tirole (1988) who studied a model of dynamic oligopoly. The variation conducted by Ball is relatively straightforward, as instead of oligopolists, he is concerned with two types of monetary authorities; a relatively “strong” monetary authority (more conservative; hawkish), and a relatively “weak” one (less conservative; dovish). Ball’s additional extension off of Maskin and Tirole’s perfect information model is to assume imperfect information available to private agents. The public therefore must estimate the relevant parameters of the model it uses in expectations formation.

Using such a model, Ball shows that a transitory shock can lead to persistent inflation as the product of an accommodative monetary authority. The appeal of Ball’s use of Markov perfect equilibria (MPE) is that the “expectations trap” phenomenon of persistent inflation arising out of a transitory shock made famous by CCE is demonstrated in a much simpler manner. Whereas CCE model the incentives and loss functions of the private sector in detail, Ball abstracts from overcomplicating roles of agents and the particular parameters of their actions and loss functions. As Maskin and Tirole (p. 553) say, the benefit of MPE is that “actions depend on as little

⁵⁵ Dr. Greenspan is currently surmounting a formidable pension off of his maintained reputation with a book deal and up to \$100,000 per hour-long speech. Availability: *CNNMoney.com*: http://money.cnn.com/2007/03/20/news/newsmakers/greenspan/index.htm?eref=rss_topstories

as possible while still being consistent with rationality.” By adapting the MPE framework of Ball (1995) to the current New-Keynesian framework, it will be shown that some lack of transparency can help a monetary authority avoid an expectations trap under certain conditions. I believe the specificity of this result to be unique to the literature.

b. Hawks, doves, and expectation traps

First, let the preferences of the monetary authority be simplified so that they take one of two forms; one preference type is that of a “hawk” or zero weight on the unemployment gap in (2.1), and that of a “dove” with some weight on the unemployment gap. In other words, preferences s_t from (2.4) may be written as one of two states:

$$s_t = B(*) \tag{8.1}$$

$$B(1) = 0 \quad (\text{Hawk}) \tag{8.2}$$

$$B(2) \gg 0 \quad (\text{Dove}) \tag{8.3}$$

The dove’s preferences $B(2)$ need not be very greatly positive, but necessarily distinguishable from zero; the precise value of $B(2)$ is not known before such a policy-maker takes power, and is only potentially visible to the public through enacted policy. The public does not know the exact identity of the monetary authority in period $t+1$ because neither does the monetary authority in time t itself. The probability that $B(1)$ is in power in any given period is $b(1)$ and the probability $B(2)$ is in power is $b(2) = 1 - b(1)$. These probabilities adhere to the Markov

criterion because they do not depend on past histories. Additionally, once either $B(1)$ or $B(2)$ is in power, there is some persistence to its tenure. Once $B(1)$ is in power, let the (relatively high) probability that $B(1)$ is in power next period be h (hawk). Once $B(2)$ is in power, let the probability it is in power next period be d (dove). The probabilities h and d likewise adhere to the Markov criterion because they depend only on the relatively small set of information on the present state of the monetary authority, not the entire history of play.

Each period, there is also potentially a cost-push shock. However, to simplify, let us assume the cost-push shock takes one of two values, zero, or $\hat{\varepsilon} > 0$. Ball associated non-zero shocks with major announcements by OPEC, which is an allusion to the oil price shocks of the 70's that may have been responsible for what may have been an ensuing expectations trap. Either of the values of the shock are realized with the following associated probabilities:

$$\varepsilon_t = 0 \quad \text{with probability } q \quad (8.4)$$

$$\varepsilon_t = \hat{\varepsilon} > 0 \quad \text{with probability } (1-q) \quad (8.5)$$

Without information about who is in power or what the cost-push shock will be next period, the public forms expectations of inflation next period, $E_t \pi_{t+1}$, using only payoff-relevant variables. In this case whether or not $B(1)$ will be in power is the only payoff-relevant variable which is at all predictable. Thus without knowledge of who is in power the current period, agents form expectations as a function of the

probability that $B(1)$ is in charge in any period, $b(1)$, which again is Markovian in that it does not depend on past values.

$$E_t \pi_{t+1} = E_t \pi_{t+1} (b(1)) \quad (8.6)$$

The monetary authority on the other hand forms policy as a function of expectations and the current cost-push shock which is stochastic and thereby a Markov variable. It is assumed that the “hawk” and “dove” are separate entities, so policy formation is not a function of the highly persistent $B(\bullet)$, thereby potentially conflicting with the Markov criterion. Thus for each the hawk and dove individually, policy is a function of expectations and the cost-push shock:

$$\pi_t^p = \pi_t^p (b(1), \varepsilon_t) \quad (8.7)$$

Explicitly,

$$\pi_t^p = \frac{E_t \pi_{t+1} + B(\bullet) \varepsilon_t}{\lambda - B(\bullet)} \quad (8.8)$$

Policy (8.8) is the optimized value using the loss function implicit of the objective function (2.1) with the New-Keynesian Phillips curve (2.7) used as a restriction. Where the cost-push shock is random, an MPE is fully defined by the form of (8.7) since it incorporates all payoff-relevant variables and the variables do not depend on past histories.

Similarly but not equivalent to Ball, given (8.8), I posit the following set of $b(1)$ for which $\pi_t^p = 0$ when $\varepsilon_t = 0$ and when $\varepsilon_t = \hat{\varepsilon}$ constitute an equilibrium.

Preliminarily the sets are written in terms of expectations, $E_t \pi_{t+1}$, which are themselves dependent on $b(1)$. Then the exact connection between $E_t \pi_{t+1}$ and $b(1)$

will be made, and it will be shown that this equilibrium constitutes a Nash perfect equilibrium (Nash equilibrium in every subgame). With the assumption of dependence on Markov variables, the following set constitutes a MPE:

<i>Monetary Authority Type</i>	<i>Shock</i>	<i>Set of $b(1)$ (via $E_t\pi_{t+1}$)</i>
$B(1)$	$\varepsilon_t = 0$	$\pi_t^p = 0$ iff $E_t\pi_{t+1} = 0$
	$\varepsilon_t = \hat{\varepsilon}$	$\pi_t^p = 0$ iff $E_t\pi_{t+1} = 0$
$B(2)$	$\varepsilon_t = 0$	$\pi_t^p = 0$ iff $E_t\pi_{t+1} = 0$
	$\varepsilon_t = \hat{\varepsilon}$	$\pi_t^p \neq 0$

From the set, it is clear inflation policy will equal zero if $B(1)$ is in power, or if $B(2)$ is in power and the cost-push shock is non-existent, $\varepsilon_t = 0$. Thus, expectations of inflation next period are rationally zero if $B(1)$ is expected to be in power, or if the cost-push shock is expected to be zero. Since the cost-push shock is unpredictable, the public predicts inflation equal to zero next period if and only if they expect $B(1)$ to be in power. Where the public has no information on the monetary authority's identity in the current period, I posit that they rationally predict $B(1)$ will be in power next period if $b(1) > 1/2$. Under this assumption the table representing the MPE may truly be written as a set of $b(1)$:

<i>Monetary Authority Type</i>	<i>Shock</i>	<i>Set of $b(1)$</i>
$B(1)$	$\varepsilon_t = 0$	$\pi_t^p = 0$ iff $b(1) > 1/2$
	$\varepsilon_t = \hat{\varepsilon}$	$\pi_t^p = 0$ iff $b(1) > 1/2$
$B(2)$	$\varepsilon_t = 0$	$\pi_t^p = 0$ iff $b(1) > 1/2$
	$\varepsilon_t = \hat{\varepsilon}$	$\pi_t^p \neq 0$

The proof that this set represents a subgame-perfect equilibrium is straightforward. The monetary authority type, shock value, and values of $b(1)$ (standing in for expectations) correspond to the optimal inflation choice as in (8.8). Thus if $B(1)$ is in power or $B(2)$ is in power and $\varepsilon_t = 0$, a choice of zero inflation is optimal to any other choice. Likewise, when $B(2)$ is in power and $\varepsilon_t = \hat{\varepsilon}$, a choice of inflation as specified by (8.8) is optimal to any other choice. Additionally, the values of $b(1)$ which constitute the set correspond to the public's rational expectations of zero inflation given the probability $b(1)$ is greater than one half. Thus, the actions of the public and the monetary authority within the set are Pareto-optimal and thereby the set constitutes a Nash perfect equilibrium. Since behavior depends on fundamentals (i.e., the Markov variable $b(1)$ does not depend on history), the set is a Markov perfect equilibrium as well.

Say the economy starts at zero-inflation with expectations of inflation equal to zero, and no cost-push shocks. Then, according to the MPE set, whether $B(1)$ or $B(2)$ is in power, there will be zero-inflation. This is consistent in the model of Clarida, Gali, and Gertler (1999) outline in *Section 2* in that no cost-push shocks

implies immediate zero-inflation targeting is optimal.⁵⁶ Thus, expectations are at least to some degree self-fulfilling since even under the dove regime, there is zero-inflation consistently. Furthermore, it does not seem particularly important from the public's perspective as to who is in power.

However in some period, the shock $\varepsilon_t = \hat{\varepsilon}$ is realized. If $B(1)$ is in power, inflation will remain at zero; the precommitted zero-inflation hawk regime avoids accommodative inflation just as the zero-inflation targeting central banker does in the traditional Barro-Gordon model. If $B(2)$ is in power, the accommodative dove regime will immediately inflate, thereby revealing its identity as the dove by the value of inflation policy, where (8.8) may be reformulated:⁵⁷

$$B(\bullet) = \frac{\chi E_t \pi_{t+1} + \lambda \pi_t^p}{\pi_t^p - \varepsilon_t} \quad (8.9)$$

Since regime-type is highly persistent, the public should rationally expect that the period after the shock-induced inflation, the dove regime will still be in power. If the probability that the dove regime will continue to be in power, d , is greater than or equal to 1/2, then expectations will be greater than zero as explained previously.

Given expectations are greater than zero, according to the equilibrium set, inflation will be greater than zero. Then, as long as $B(2)$ is in power, inflation will continue to be positive, unambiguously. Thus the expectations trap: a transitory shock caused an accommodative dove regime to inflate, sparking expectations and resulting in a self-fulfilling prophecy of prolonged inflation.

⁵⁶ See the analysis immediately following equation (2.34).

⁵⁷ Note, if $\pi_t^p = \varepsilon_t = 0$, the value of $B(\bullet)$ is intractable, so in the original equilibrium of zero-inflation the public does not know which $B(\bullet)$ is in power.

If the shock hits when $B(2)$ is in power, the regime faces a stark reality; the expectations trap is permanent in their tenure and may only be reduced once a hawkish regime comes into power. Such a scenario is reminiscent of the environment of the 1970's when the relatively hawkish Volcker regime stepped in to ease the inflation incurred under Burns, at the same time causing a major recession. The times series of inflation under the different Chairmen is depicted in *Figure 6*, clearly showing upward-trending inflation through the Burns regime and downward-sloping through Volcker's appointment.

c. Ambiguity's cure

It seems that any dovish regime is inevitably doomed to an inflation trap when a transitory shock does hit. However, this is not necessarily the case if the dovish regime is able to make ambiguous the exact value of its policy decision. If policy is not directly visible to the public, then the true identity of the monetary authority is not necessarily viewable through (8.9).

Consider the fact that when inflation policy is enacted, there may be some degree of control error, as used in previous sections:

$$\pi_t = \pi_t^p + \eta_t \tag{8.10}$$

The parameter η_t is a white-noise shock with variance σ_η^2 . The variance of the control error shock is taken as a technological parameter which may be purposefully increased, but not decreased below the concurrently technologically viable level.

Consider the scenario again in which the economy begins with consistent expectations, and realizations, of zero-inflation. When the dove is in power, the shock $\hat{\varepsilon}$ hits. The dove has to inflate at least minutely, since this is simply its identity as an accommodative regime. However, whether the public can tell if it is in fact inflating may be unclear if there is control error. The reason for this is that even when the hawk is in power, with control error its given policy is not always precisely zero.

Thus say the minute the shock hits, the public begins monitoring inflation to see if they can spot the dove. In particular, using (8.9), the public will rationally collect inflation data and test for the null hypothesis:

$$H_0 : B(\bullet) = 0 \tag{8.11}$$

$$H_A : B(\bullet) > 0 \tag{8.12}$$

The t-statistic for such a test would be:

$$t = \frac{\bar{B}(\bullet)_n}{\left(\frac{s_n}{\sqrt{n}} \right)} \tag{8.13}$$

where n is the number of sample points, $\bar{B}(\bullet)_n$ is the sample mean according to (8.9), and s_n is the sample variance of $\bar{B}(\bullet)_n$.

Since the public views only π_t and not π_t^p , from (8.9) the sample variance they calculate, $\bar{B}(\bullet)_n$, uses π_t without a signal of control error. The monetary authority has at least partial control over the variance of realized inflation since they have control over the variance of control error. In particular, by increasing the variance in control error they could potentially manipulate the sample variance of

$\bar{B}(\bullet)_n$, s_n , which the public calculates from its inflation observations. By increasing control error purposefully, in Geraats' (2002) terminology, the monetary authority is becoming less transparent "policy"-wise. Recall from equation (6.20) that the variance of a function of a random variable x may be estimated by the delta variance estimation technique:

$$Var[f(x)] \approx (f'(E[x]))^2 Var[x] \quad (8.14)$$

Using this approximation of the variance of a function of x , the variance of $B(\bullet)$ (equation (8.9)) as a function of η_t (control error) may be estimated. Notice, this is possible because $B(\bullet)$ meets the necessary condition that it is a twice-differentiable function of η_t , where η_t is a random variable⁵⁸:

$$\frac{\partial^2 B(\bullet)}{\partial (\eta_t)^2} = \frac{2(\chi E_t \pi_{t+1} + \lambda(\pi_t - \eta_t))}{(\pi_t - \varepsilon_t - \eta_t)^3} - \frac{2\lambda}{(\pi_t - \varepsilon_t - \eta_t)^2} \quad (8.15)$$

Thus according to (8.14), the estimated variance of $B(\bullet)$ as a function of the variance of η_t is, where $E[\eta_t] = 0$:

$$Var(B(\bullet)) = \left(\frac{\chi E_t \pi_{t+1} + \lambda \pi_t}{(\pi_t - \varepsilon_t)^2} - \frac{\lambda}{\pi_t - \varepsilon_t} \right)^2 Var(\eta_t) \quad (8.16)$$

Additionally, trivially:

$$Var(\pi_t) = Var(\pi_t^p) + Var(\eta_t) + 2Cov(\pi_t^p, \eta_t) \quad (8.17)$$

Control error η_t is independent of the level of inflation, so $Cov(\pi_t^p, \eta_t) = 0$.

⁵⁸ See Mathematica notebook 10 in the Mathematica appendix for this and (8.16).

Thus, an increase in the variance of control error results in an increase in realized inflation variance and consequently an unambiguous increase in the variance of $B(\bullet)$. So, the monetary authority has the power to increase the variance of $B(\bullet)$, and through this the sample variance $\bar{B}(\bullet)_n$, s_n , by increasing the variance of control error which is always possible (control error can only be *decreased* to the technologically viable level).

The ability to increase the sample variance of $B(\bullet)$ to high levels in fact enables a dove regime to “hide” its identity. Consider, for example, if $n=20$ (a small sample consistent with ever-changing regimes), the 95% critical value for such a t-statistic is 2.08. If $\bar{B}(\bullet)_n = .2$,⁵⁹ then by (8.13), a relatively meager sample variance $s_n \approx .43$ achieves the critical value; as the sample variance increases, which is possible solely by increasing control error variance, the t-statistic retreats into the “fail-to-reject” region. When the null hypothesis can not be rejected, a dove has successfully convinced the public it is a hawk. This will result in the public expecting inflation to be zero next period, and by the equilibrium table presented earlier, absent cost-push shocks the actual inflation level will return to zero. Thus, an expectations trap has been avoided by limited transparency in the “political” sense, in Geraats’ (2002) terminology.

Thus, by increasing the variance of control error, a dovish monetary authority may be able to avoid an expectations trap. This increased noise in inflation, itself being the signal of inflation policy, is an example of how limited transparency may be able to curb increasing inflation. An increase in the variance of control error was the

⁵⁹ Even if this sample mean is exactly correct, it will not affect the outcome of this exercise.

original means by which Cukierman and Meltzer (1986) showed the potential benefits of decreased transparency, and which spawned much of the literature I have referred to throughout this thesis.

One important caveat of this model is that the dove is not perceivable simply because control error is higher. Whereas it is assumed there is some control error endogenous to the system so that both the hawk and the dove experience it, if the dove must induce very high levels of control error to hide its identity, which the hawk need not do, this paradoxically may in itself reveal the dove's identity. Thus, in the case that the difference in the variance of control error is otherwise perceivable between the tenures of a hawk or dove, the dove may only use ambiguity to hide its identity if the hawk colludes and always uses the same variance of control error. Although the hawk has no specific incentive to do so, if it is assumed that the hawk and dove are simply different characters of the same monetary authority entity, the hawk is protecting its future self by colluding.

Proposition 8.1: Given the monetary authority may control the degree of control error in realized inflation, a “dove” regime may hide its identity and avoid an expectations trap as the consequence of a transitory cost-push shock.

Corollary 8.1: In the case sufficiently high control error is needed to hide the dove's identity such that an increase in control error is detectable, the hawk necessarily must collude and maintain the same degree of control error necessary to hide the dove's identity.

9. Conclusion

"When you get this far away from a recession, invariably forces build up for the next recession, and indeed we are beginning to see that sign. ... For example in the U.S., profit margins ... have begun to stabilize, which is an early sign we are in the later stages of a cycle."

Quotation of Alan Greenspan
The Wall Street Journal
(Mon, February 26, 2007)

Although there is undeniable rationale for transparency in academic and policy-making discussion, there are some things a central banker may want to communicate only with some ambiguity. Even though no longer in office, when Greenspan made this statement last February, along with a proclamation that the risk of a recession this year is "one third," stocks and bonds plummeted and the dollar weakened. (Browning, *The Wall Street Journal*, Feb. 27, 2007). This thesis has ventured to point out that there are yet some potential benefits to monetary policy secrecy despite the recent move towards transparency.

First, it was shown that the New-Keynesian model of Clarida, Gali, and Gertler (1999) implicitly imbeds incentives for the monetary authority to "lie," but that outright deception is plagued by credibility problems. Ambiguity avoids these credibility issues while still accomplishing similar benefits as lying.

Despite the fact that the Characterization Theorem for second-order stochastic dominance implies a risk-averse monetary authority (it is risk-averse for "feasible" values of the parameters of the model) will prefer full transparency to any ambiguity in signals, further evidence was presented in support of ambiguity. First, when the public has its own collective signal with given relative accuracy, opacity may be

preferable as in Geraats (2007). Second, even when the monetary authority is equally accurate as a representative public agent in estimating the natural rate of unemployment, some ambiguity in the signal from the monetary authority can be beneficial for anchoring expectations due to a Keynesian beauty contest phenomenon as in Morris and Shin (2001). Thirdly, when an accommodative “dove” regime is in power, ambiguity in the public signal can help prevent an expectations trap in a Markov perfect equilibrium model similar to that of Ball (1995).

This thesis has presented many original ideas that are critical of the prevalent “cure-all” attitude toward transparency. First, the “backward-bending” nature of the curve representing the relationship between inflation and unemployment gap variation for different values of time-varying preferences (see *Figure 2a* and *Figure 4*) is at odds with Clarida et al’s distinction of an unambiguous trade-off (see *Figure 1*) with time-homogenous preferences. This provided the original intuition for the possible benefits from “lying.”

The “credibility for honesty” criterion assessing correlations in signals represents a unique contribution to this thesis, although the “credibility for accuracy” criterion was adapted from Geraats (2007). The “baseline case” for opacity is likewise completely original, as was showing that a risk-averse monetary authority may prefer transparency in the sense of second-order stochastic dominance. Additionally, the demonstration that Morris and Shin’s (2001) highly-regarded theoretic framework may be applied to the practical case of mis-estimating the natural rate of unemployment was the first instance I know of making their case in a formal policy-making framework. Finally, although the Markov perfect equilibrium

framework showing the expectations trap was adapted from Ball (1995), the intuition in showing how opacity can avert such an expectations trap is new to the literature.

One benefit of such an extended study of monetary authority opacity is that many possible definitions of transparency and related concepts may be investigated. Here, two definitions of credibility consistent with the literature, and four of the five types of transparency defined by Geraats (2002) are explicitly addressed. Prior studies in academic journals tend to concentrate on one particular example that is seemingly disconnected from the rest and highly model-specific. The analysis here is additionally powerful in that it collects many strong theories of why transparency is not unambiguously good for policy-making and presents them in a single coherent framework. In all, four complete and consistent reasons are given for why transparency is not limitlessly beneficial.

Future direction of study of the optimal degree of transparency would be toward surveying the degree of transparency in central banks worldwide, and their respective successes and failures, as the *Preface* suggested Bernanke would like. Such a study, although intensive, could provide valuable insights into the actual benefits of transparency worldwide. Although it would be additionally be difficult to draw out the specific effects of transparency, any inferences at this time would be valuable, as current technology leaves most study at the theory level. An investigation of this kind might, it is the theory of this thesis, find that transparency is not unambiguously beneficial for monetary policy.

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11. Figures

Figure 1 (a) *Efficient Policy Frontier with Time-Consistent Preferences:*

Approximate reproduction from Clarida, Gali, and Gertler (p.1673).

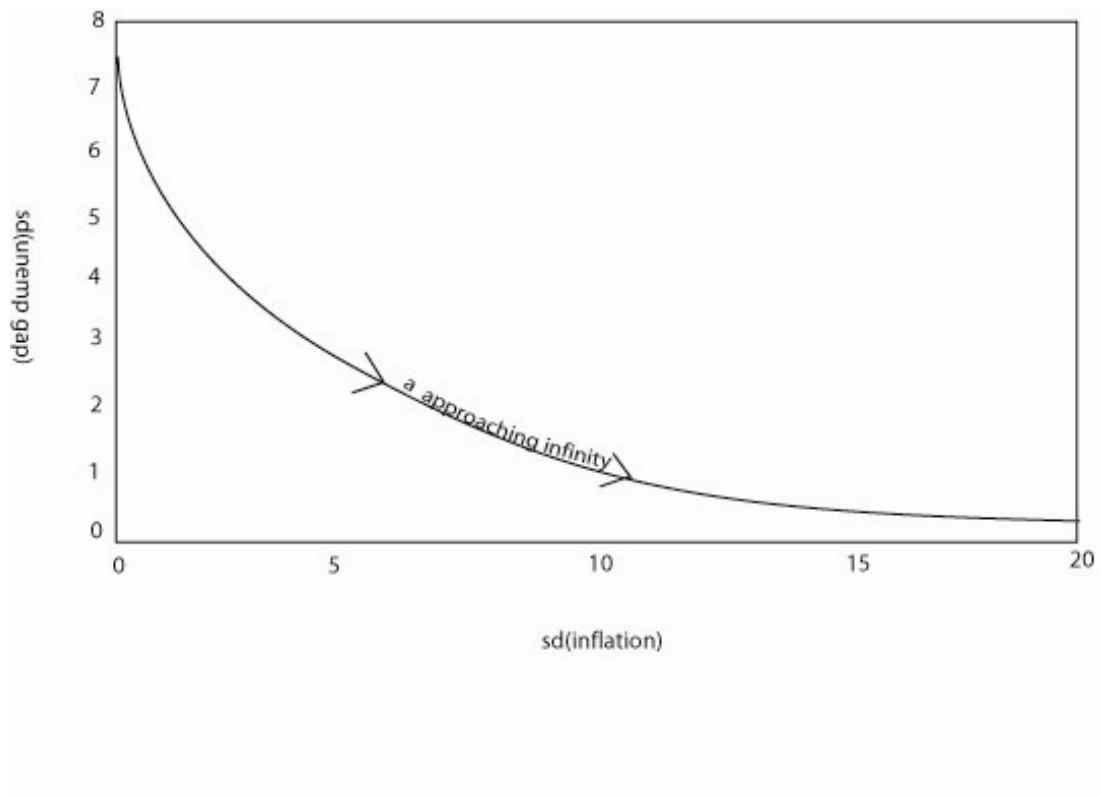


Figure 1 (b) Approximate effect of a on $sd(\text{inflation})$ in CGG: nondecreasing:

(strictly concave)

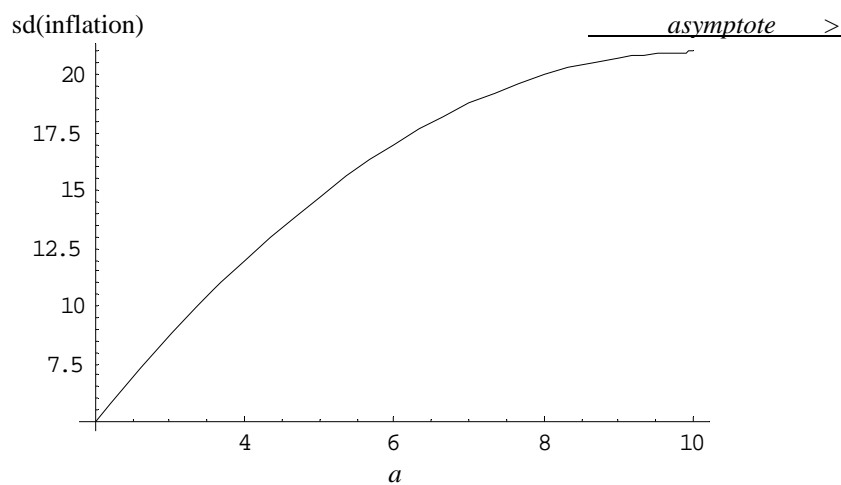


Figure 1 (c) Approximate effect of a on $sd(unemp\ gap)$ in CGG.
(strictly convex)

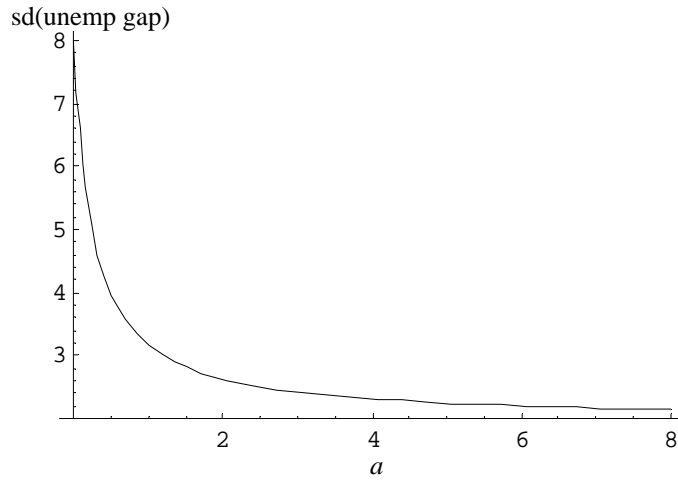


Figure 2 (a) *Efficient Policy Frontier with Time-Varying Preferences*: Inflation standard deviation against output gap standard deviation for varying values of a_t when a_t is time-varying instead of constant as in *Figure 1 (a)*. Parameter values: $\lambda = 1/2$, $\chi = 1/2$, $\rho = 1/2$, $\tau = 1/2$, $\sigma_\varepsilon = 3$; this would appear helical in three-dimensional space. (Graph created by the author using point-by-point calculation and Photoshop. The label s corresponds to a_t with A included; see equation (2.4) for clarification.)

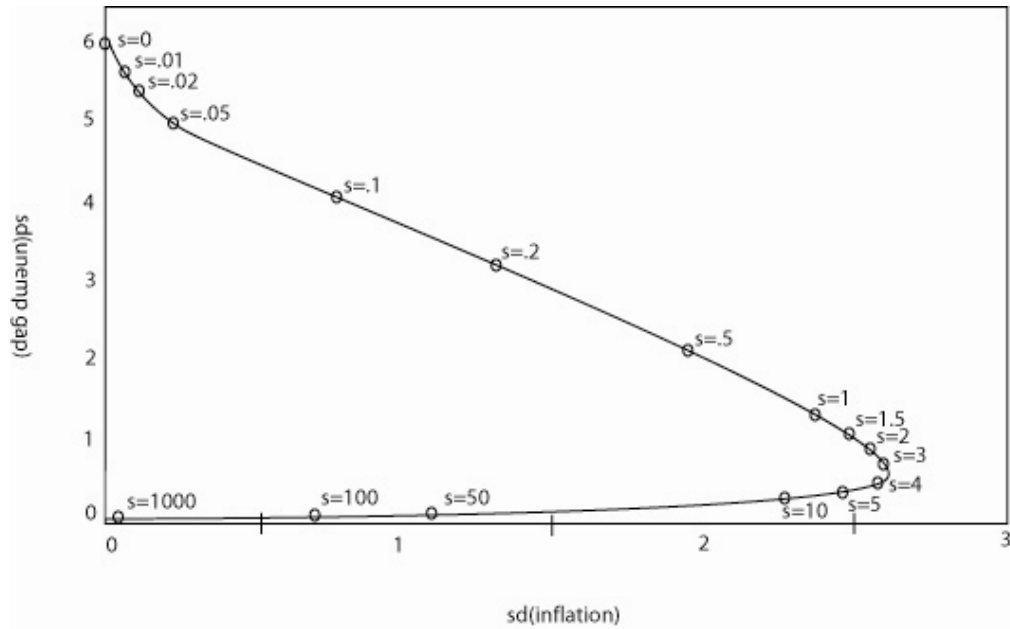


Figure 2 (b) Approximate effect of a_t on $sd(inflation)$ when preferences are time-varying for parameter values: $\lambda = 1/2$, $\chi = 1/2$, $\rho = 1/2$, $\tau = 1/2$, $\sigma_\varepsilon = 3$.

Ensuing graphs through figure 3 (e) created in Mathematica. Code in Mathematica notebook (3) in the Mathematica appendix.

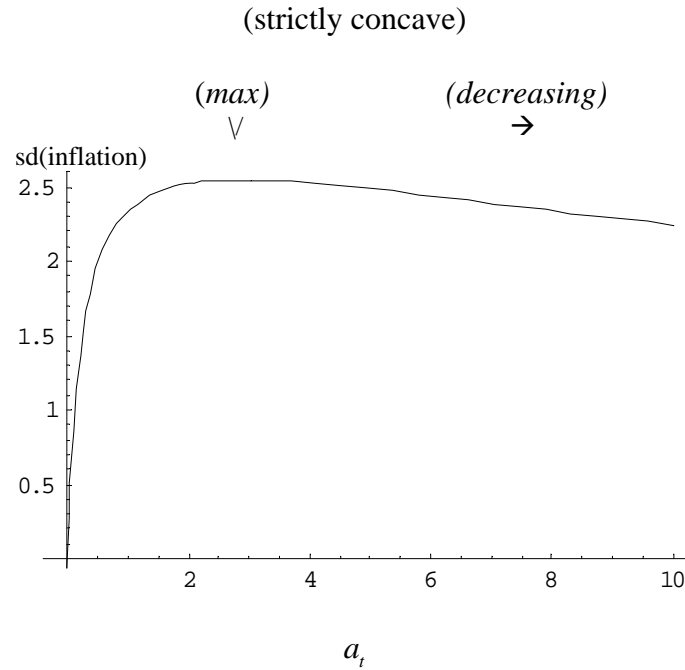


Figure 2 (c) Approximate effect of a_t on $sd(unemp\ gap)$ when preferences are time-varying for parameter values: $\lambda = \frac{1}{2}$, $\chi = \frac{1}{2}$, $\rho = \frac{1}{2}$, $\tau = \frac{1}{2}$, $\sigma_\varepsilon = 3$ (same as 2(a)).

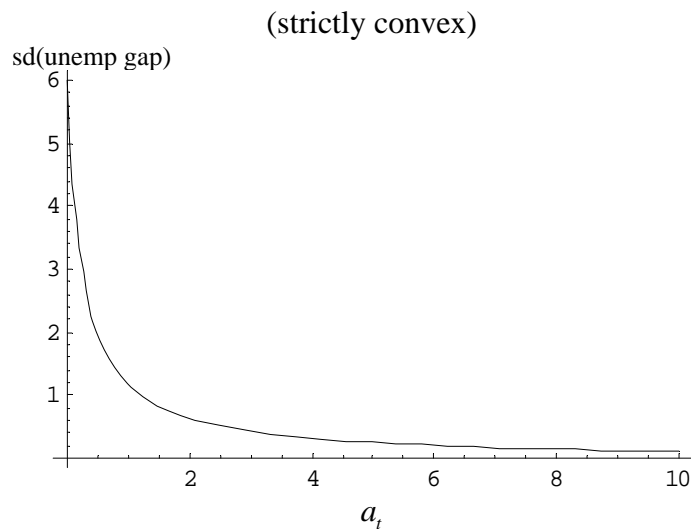


Figure 3 (a) Approximate effect of a_t on $sd(\text{inflation})$ and $sd(\text{unemp gap})$ when

$\lambda = \frac{1}{2}$, $\chi = \frac{1}{2}$, $\rho = \frac{1}{2}$, $\tau = \frac{1}{2}$, and σ_ε varies:

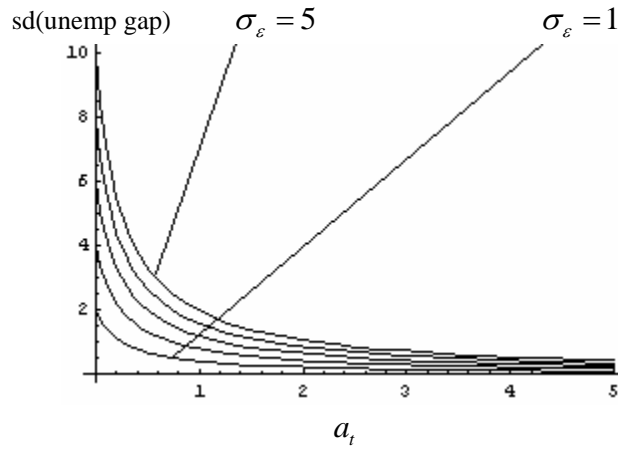
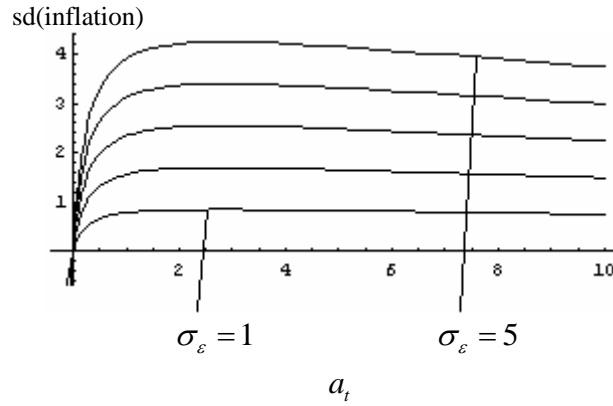
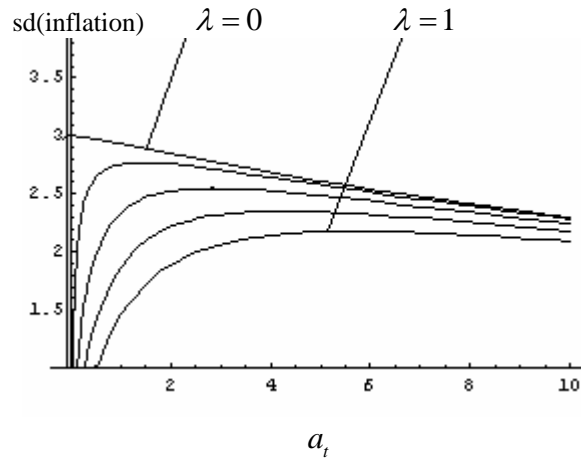


Figure 3 (b) Approximate effect of a_t on $sd(\text{inflation})$ and $sd(\text{unemp gap})$ when

$\sigma_\varepsilon = 3$, $\chi = \frac{1}{2}$, $\rho = \frac{1}{2}$, $\tau = \frac{1}{2}$, and λ varies:



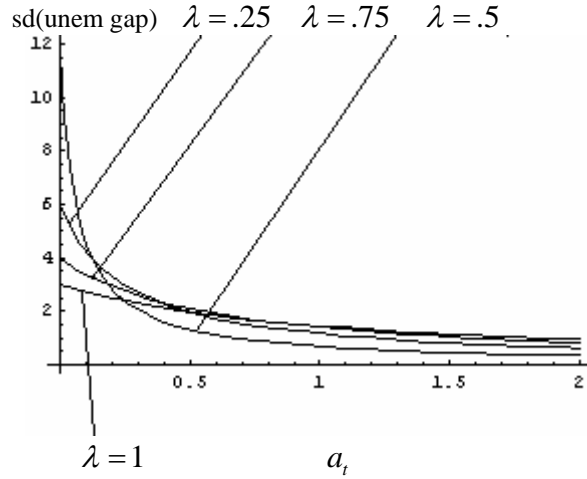


Figure 3 (c) Approximate effect of a_t on $sd(inflation)$ and $sd(unemp\ gap)$ when

$\sigma_\varepsilon = 3$, $\lambda = \frac{1}{2}$, $\rho = \frac{1}{2}$, $\tau = \frac{1}{2}$, and χ varies:

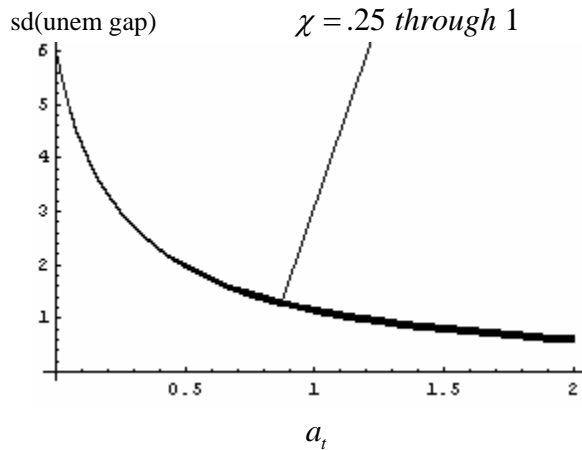
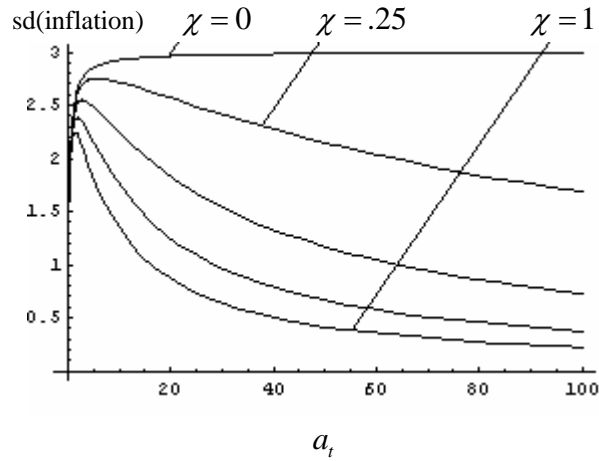


Figure 3 (d) Approximate effect of a_t on $sd(\text{inflation})$ and $sd(\text{unemp gap})$ when

$\sigma_\varepsilon = 3$, $\lambda = \frac{1}{2}$, $\chi = \frac{1}{2}$, $\tau = \frac{1}{2}$, and ρ varies:

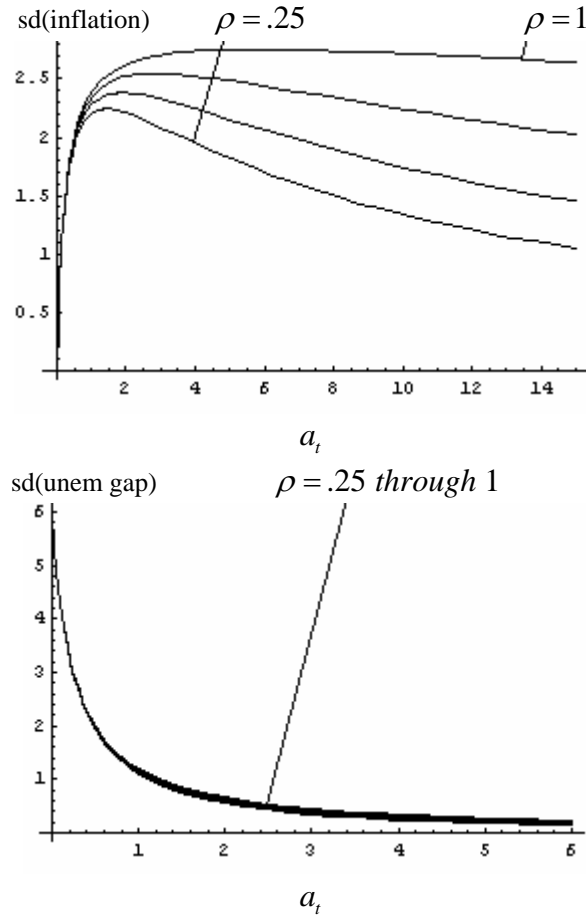
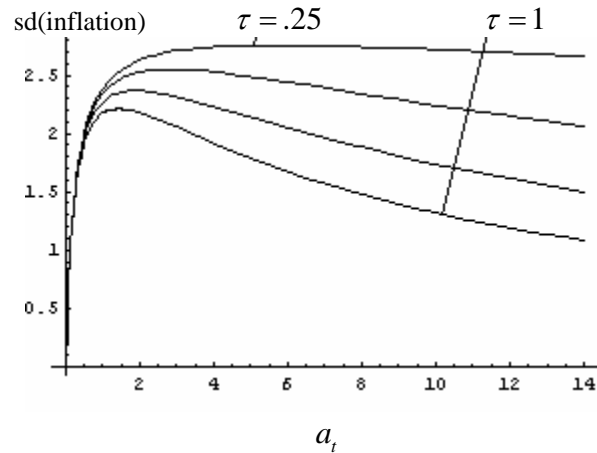


Figure 3 (e) Approximate effect of a_t on $sd(\text{inflation})$ and $sd(\text{unemp gap})$ when

$\sigma_\varepsilon = 3$, $\lambda = \frac{1}{2}$, $\chi = \frac{1}{2}$, $\rho = \frac{1}{2}$, and τ varies:



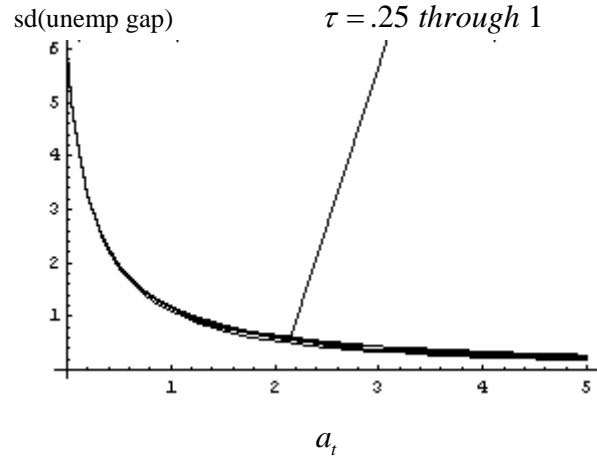
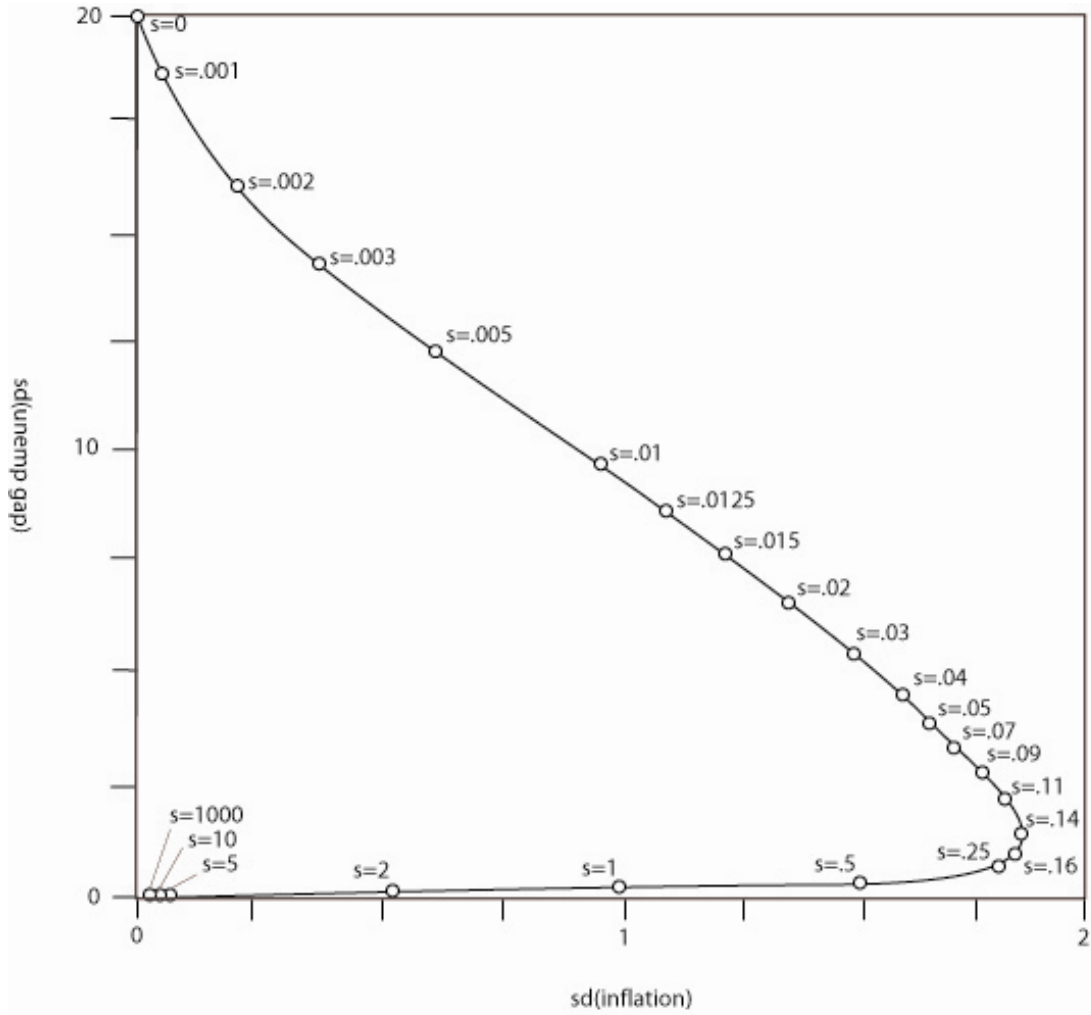


Figure 4

Efficient Policy Frontier with Time-Varying Preferences: $\lambda = .1$, $\rho = \tau = \chi = 1$, $\sigma_\varepsilon = 2$. (s corresponds to a_t with A included; see equation (2.4) for clarification.)



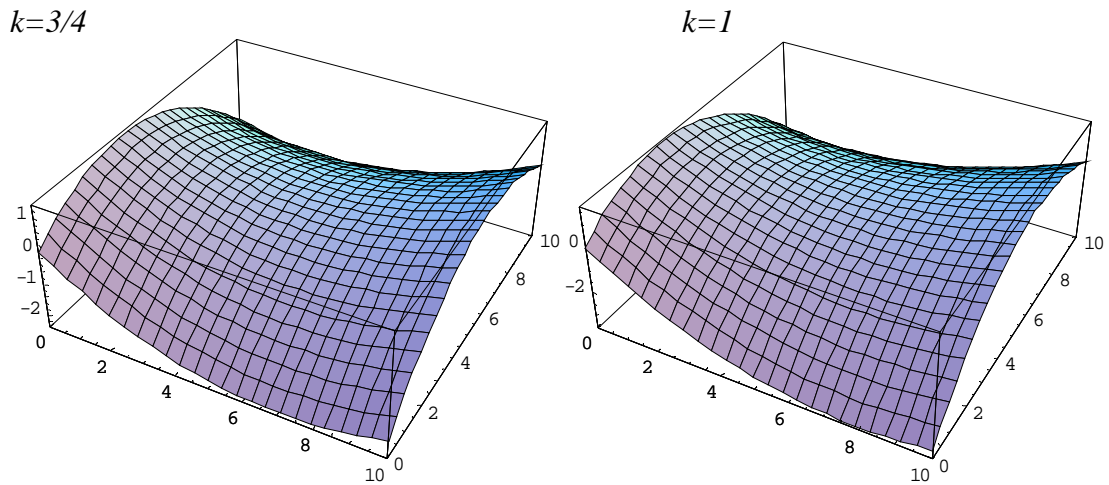
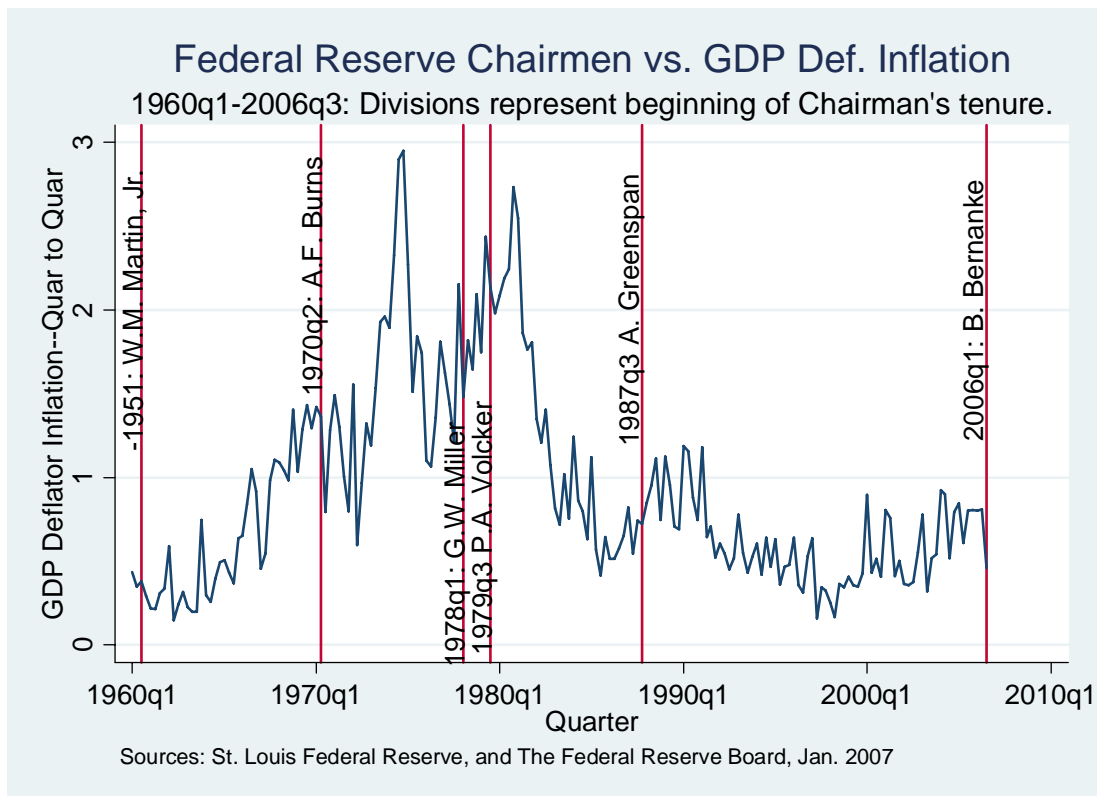


Figure 6.



12. Tables

Table 1.

History of Fed Communication

(all information from the Federal Reserve's website: <http://www.federalreserve.gov/>)

1913 Federal Reserve created by act of Congress.

1935 "Record of Policy Actions," or, "Policy Record" published once a year; these include a paragraph or two of motivation behind each policy action. Over time these records grew in detail. Extensive minutes kept at each FOMC meeting but not made available to the public.

1964 Record of Policy Actions reach an average length of about five pages. Semi-annual meeting minutes from 1936-60 made public.

June 1967 "Record of Policy Actions" and "Minutes of Actions" (list of all actions at FOMC meetings) begin to be released 90 days after each meeting. The Fed continues taking internal minutes, and starts compiling a "Memorandum of Discussion" detailing the discussion at each meeting; these are released at a delay of 5 years following each meeting.

1975 Records of Policy Actions and Minutes of Actions begin to be released 45 days after each meeting; this is a result of the Freedom of Information Act (July 4, 1966).

1976 Fed announces it will release Records of Policy Actions and Minutes of Actions the Friday after the subsequent meeting (meaning about 30 days later).

1978 Humphrey-Hawkins Testimony begins in which the Chairman must give biannual testimony to Congress; Fed fights against this decision in court, and loses.

1987 Greenspan: "Since I have become a central banker I have learned to mumble with great incoherence."

1993 Records of Policy Actions and Minutes of Actions combined into "Minutes of the FOMC Meeting," which continues to be released the Friday after the subsequent meeting. Also, edited transcripts of meeting begin to be released to the public at a lag of five years. These include verbatim conversation and the graphs and charts used by Governors at the meeting.

February 1994 The FOMC begins to release immediate statements to note changes in its target of the federal funds rate, but continues to remain silent after meetings with no policy changes.

December 1999 Statements released immediately after every meeting, along with “balance of risks” assessment which expands to time horizon beyond the next FOMC meeting.

2000 Humphrey-Hawkins Testimony no longer required by law, but Greenspan and later Bernanke continue to talk to Congress twice a year.

2003 Greenspan: “Openness is an obligation of a central bank in a free and democratic society.”

August 2003 Statements begin to be notably more explicit about future path of interest rates. For example, the August 2003 statement of the FOMC indicated that "policy accommodation can be maintained for a considerable period," a formulation replaced a few meetings later with the comment that the Committee could be "patient" in removing policy accommodation. These statements conveyed information to markets about the Committee's economic outlook as well as its policy approach. (Bernanke 2004)

January 2005 FOMC decides to expedite release of minutes to three weeks after each meeting.

February 2006 Benjamin S. Bernanke, an open advocate of inflation targeting, sworn in as Chairman of the Federal Reserve System.

13. Appendix

A(2.15)

The associate first-order condition is:

$$\frac{-a_t}{\lambda} \left(-\frac{1}{\lambda} \pi_t^p + G_t \right) + \pi_t^p = 0$$

Rewritten,

$$\pi_t^p = \frac{a_t}{\lambda} \left(-\frac{1}{\lambda} \pi_t^p + G_t \right)$$

plugging in (1.8), the optimality condition for inflation and the unemp. gap is:

$$\pi_t^p = \frac{a_t}{\lambda} (\tilde{u}_t)$$

QED

A(2.17)

The Phillips curve is written:

$$\pi_t = \chi E_t \pi_{t+1} - \lambda \tilde{u}_t + \varepsilon_t$$

plugging in expectations as given, and the optimality condition for π_t and \tilde{u}_t ,

$$\pi_t^p = \chi c r_t E_t \pi_t^p - \lambda \left(\frac{\lambda}{a_t} \right) \pi_t^p + \varepsilon_t$$

where the public observes π_t , not π_t^p directly. When policy is visible,

$$\pi_t^p = \chi c r_t \pi_t^p - \lambda \left(\frac{\lambda}{a_t} \right) \pi_t^p + \varepsilon_t$$

$$\left[1 - \chi c r_t + \frac{\lambda^2}{a_t} \right] \pi_t^p = \varepsilon_t$$

$$\pi_t^p = \frac{\varepsilon_t}{1 - \chi c r_t + \frac{\lambda^2}{a_t}}$$

$$\pi_t^p = \frac{a_t \varepsilon_t}{a_t (1 - \chi c r_t) + \lambda^2}$$

QED

A(2.22)

In order to find the appropriate values of c and r_t for rational expectations, we first set:

$$E_t \pi_{t+1}^p = cr_t \pi_t^p = \frac{\tau a_t \rho \varepsilon_t}{\tau a_t (1 - \chi cr_t) + \lambda^2}$$

Since $\pi_t^p = a_t \varepsilon_t q_t$, can say that the expectations of the public are:

$$E_t \pi_{t+1}^p = cr_t \frac{a_t \varepsilon_t}{a_t (1 - \chi cr_t) + \lambda^2}$$

Since c is a constant,

$$c = \tau \rho$$

and so, it must be the case that:

$$r_t = \frac{a_t (1 - \chi \tau \rho r_t) + \lambda^2}{\tau a_t (1 - \chi \tau \rho r_t) + \lambda^2}$$

given this specification of c and r_t , expectations are rational. One can solve for r_t :

$$r_t [\tau a_t (1 - \chi \tau \rho r_t) + \lambda^2] = a_t (1 - \chi \tau \rho r_t) + \lambda^2$$

$$(r_t \tau - 1)(a_t [1 - \chi \tau \rho r_t] + \lambda^2) = 0$$

$$r_t \tau a_t - r_t^2 \tau^2 \rho \chi - a_t + \chi \tau \rho r_t + r_t \tau \lambda^2 - \lambda^2 = 0$$

$$r_t (\tau a_t + \chi \tau \rho + \tau \lambda^2) - r_t^2 \chi \tau^2 \rho = a_t + \lambda^2$$

$$r_t = \frac{a_t + \lambda^2 + r_t^2 \chi \tau^2 \rho}{\tau a_t + \chi \tau \rho + \tau \lambda^2}$$

$$r_t = \left(\frac{\chi \tau^2 \rho}{\tau a_t + \chi \tau \rho + \tau \lambda^2} \right) r_t^2 + \left(\frac{a_t + \lambda^2}{\tau a_t + \chi \tau \rho + \tau \lambda^2} \right)$$

$$\left(\frac{\chi \tau^2 \rho}{\tau a_t + \chi \tau \rho + \tau \lambda^2} \right) r_t^2 + (-1)r_t + \left(\frac{a_t + \lambda^2}{\tau a_t + \chi \tau \rho + \tau \lambda^2} \right) = 0$$

Using the quadratic formula,

$$r_t = \frac{1 \pm \sqrt{1 - 4 \left(\frac{\chi \tau^2 \rho (a_t + \lambda^2)}{(\tau a_t + \chi \tau \rho + \tau \lambda^2)^2} \right)}}{\left(\frac{2 \tau^2 \chi \rho}{\tau a_t + \chi \tau \rho + \tau \lambda^2} \right)}$$

$$r_t = \left(1 \pm \sqrt{\frac{(\tau a_t + \chi \tau \rho + \tau \lambda^2)^2 - 4 \chi \tau^2 \rho (a_t + \lambda^2)}{(\tau a_t + \chi \tau \rho + \tau \lambda^2)^2}} \right) \left(\frac{\tau a_t + \chi \tau \rho + \tau \lambda^2}{2 \tau^2 \chi \rho} \right)$$

$$r_t = \left(\frac{\tau a_t + \tau \chi \rho + \tau \lambda^2}{2\tau^2 \chi \rho} \right) \pm \frac{\sqrt{(\tau a_t + \tau \chi \rho + \tau \lambda^2)^2 - 4\chi \tau^2 \rho (a_t + \lambda^2)}}{2\tau^2 \chi \rho (\tau a_t + \tau \chi \rho + \tau \lambda^2)}$$

$$r_t = \left(\frac{\tau a_t + \tau \chi \rho + \tau \lambda^2}{2\tau^2 \chi \rho} \right) \pm \frac{((\tau a_t + \tau \chi \rho + \tau \lambda^2) - 2\chi \tau^2 \rho (a_t + \lambda^2))T}{2\tau^2 \chi \rho (\tau a_t + \tau \chi \rho + \tau \lambda^2)}$$

$$T = ((\tau a_t + \tau \chi \rho + \tau \lambda^2) + 2\chi \tau^2 \rho (a_t + \lambda^2))$$

$$\text{let } f(a_t) = \tau(a_t + \chi \rho + \lambda^2)$$

$$\text{and let } g(a_t) = 2\chi \tau^2 \rho (a_t + \lambda^2)$$

then,

$$r_t = \left(\frac{f(a_t)}{\left(\frac{g(a_t)}{(a_t + \lambda^2)} \right)} \right) \pm \frac{(f(a_t) - g(a_t))(f(a_t) + g(a_t))}{\left[\frac{g(a_t)}{(a_t + \lambda^2)} f(a_t) \right]}$$

$$r_t = (a_t + \lambda^2) \left[\left(\frac{f(a_t)}{g(a_t)} \right) \pm \frac{(f(a_t)^2 - g(a_t)^2)}{f(a_t)g(a_t)} \right]$$

QED

A(2.38)

When *plus* is used in the quadratic specification of r_t ,

$$\text{As } a_t \rightarrow \alpha: \sigma_{\tilde{u}} = d\lambda\sigma_\varepsilon; \sigma_\pi = d\alpha\sigma_\varepsilon$$

where

$$d = \frac{-(\alpha + \lambda^2 + \chi \rho)}{d_1 + d_2}$$

where

$$d_1 = (-\lambda^2(\lambda^2 + \chi \rho) + (\alpha^3 + \alpha\lambda^4)(1 + 2\chi^2\rho^2\tau^2))$$

and

$$d_2 = \alpha((2\lambda^2 + \chi \rho)(-1 + \chi \rho) + \alpha^2(-1 + 2\chi \rho + \lambda^2(2 + 4\chi^2\rho^2\tau^2)))$$

which implies d may be negative.

QED

A (4.4)

The Phillips curve is written:

$$\pi_t = \chi E_t \pi_{t+1} - \lambda \tilde{u}_t + \varepsilon_t$$

plugging in expectations as given, and the optimality condition for π_t and \tilde{u}_t ,

$$\pi_t^p = \chi cr_t E_t \pi_t^p - \lambda \left(\frac{\lambda}{a_t} \right) \pi_t^p + \varepsilon_t$$

Which equals:

$$\pi_t^p = \chi cr \{a_t^s\} (\pi_t - \eta_t^s) - \lambda \left(\frac{\lambda}{a_t} \right) \pi_t^p + \varepsilon_t$$

But since actual inflation and control error signal are unknown at the time of policy formation,

$$\pi_t^p = E_t \left[\chi cr \{a_t^s\} (\pi_t - \eta_t^s) \right] - \lambda \left(\frac{\lambda}{a_t} \right) \pi_t^p + \varepsilon_t$$

$$E_t \pi_t = \pi_t^p, \quad E_t \eta_t^s = 0$$

$$\pi_t^p = \chi cr \{a_t^s\} (\pi_t^p) - \lambda \left(\frac{\lambda}{a_t} \right) \pi_t^p + \varepsilon_t$$

$$\left[1 - \chi cr \{a_t^s\} + \frac{\lambda^2}{a_t} \right] \pi_t^p = \varepsilon_t$$

$$\pi_t^p = \frac{\varepsilon_t}{1 - \chi cr \{a_t^s\} + \frac{\lambda^2}{a_t}}$$

$$\pi_t^p = \frac{a_t \varepsilon_t}{a_t (1 - \chi cr \{a_t^s\}) + \lambda^2}$$

QED

A (4.6)

$$\kappa_t = a_t^s - a_t$$

$$\begin{aligned} \text{Var}(\kappa_t) &= E_t \left[\kappa_t^2 \right] - \left[E_t (\kappa_t) \right]^2 \\ &= E \left[(a_t^s - a_t)^2 \right] - E \left[\kappa_t \right]^2 \\ &= E \left[(a_t^s - a_t)^2 \right] \\ &= \sigma_\kappa^2 \end{aligned}$$

QED

A (6.22)

Given by the formulation of expectations,

$$E_t \pi_{t+1}^p = \rho \tau r \left\{ E \left[a_t \mid a_t^s \right] \right\} \left(\pi_t - E \left[\eta_t \mid \eta_t^s \right] \right)$$

Where $\tilde{A}_{\eta^s} = A_{\eta^s}$ and $\tilde{A}_{a^s} = A_{a^s}$,

$$E \left[a_t \mid a_t^s \right] = (1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s \quad \text{and,}$$

$$E \left[\eta_t \mid \eta_t^s \right] = (1 - A_{\eta^s}) \tilde{\eta}_t + A_{\eta^s} \eta_t^s$$

Thus,

$$\frac{\partial E_t \pi_{t+1}}{\partial \eta_t^s} = -A_{\eta^s} \rho \tau \left[(1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s \right]$$

So,

$$\text{Var} \left[E_t \pi_{t+1} \{ \eta_t^s \} \right] \approx \left(\frac{\partial E_t \pi_{t+1}}{\partial \eta_t^s} \{ E \left[\eta_t^s \right] \} \right)^2 \left(\frac{\sigma_{\eta^s}^2}{A_{\eta^s}} \right)$$

$$\text{Var} \left[E_t \pi_{t+1} \{ \eta_t^s \} \right] \approx \left(-A_{\eta^s} \rho \tau \left[(1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s \right] \right)^2 \left(\frac{\sigma_{\eta^s}^2}{A_{\eta^s}} \right)$$

$$= A_{\eta^s} \sigma_{\eta^s}^2 (\rho \tau)^2 \left[(1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s \right]^2$$

$$\text{Var} \left[E_t \pi_{t+1} \{ \eta_t^s \} \right] \approx A_{\eta^s} \sigma_{\eta^s}^2 (\rho \tau)^2 \left[(1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s \right]^2$$

QED

A (6.23)

$$\sigma_{E_t \pi_{t+1}}^2 = A_{\eta^s} \sigma_{\eta^s}^2 (\rho \tau)^2 \left[(1 - A_{a^s}) \tilde{a}_t + A_{a^s} a_t^s \right]^2$$

$$= \left(\left[(1 - A_{a^s}) \tilde{a}_t \right]^2 + 2(1 - A_{a^s}) \tilde{a}_t A_{a^s} a_t^s + \left[A_{a^s} a_t^s \right]^2 \right) A_{\eta^s} \sigma_{\eta^s}^2 (\rho \tau)^2$$

$$= \left[(1 - 2A_{a^s} + A_{a^s}^2) \tilde{a}_t^2 + 2\tilde{a}_t A_{a^s} a_t^s - 2\tilde{a}_t A_{a^s}^2 a_t^s + A_{a^s}^2 a_t^{s2} \right] A_{\eta^s} \sigma_{\eta^s}^2 (\rho \tau)^2$$

$$\frac{\partial \sigma_{E_t \pi_{t+1}}^2}{\partial A_{a^s}} = \left((2A_{a^s} - 2) \tilde{a}_t^2 + 2\tilde{a}_t a_t^s - 4A_{a^s} \tilde{a}_t a_t^s + 2A_{a^s} a_t^{s2} \right) \tilde{a}_t^2 A_{\eta^s} \sigma_{\eta^s}^2 (\rho \tau)^2$$

$$= 2 \left[(A_{a^s} - 1) \tilde{a}_t^2 + \tilde{a}_t a_t^s (1 - 2A_{a^s}) + A_{a^s} a_t^{s2} \right] \tilde{a}_t^2 A_{\eta^s} \sigma_{\eta^s}^2 (\rho \tau)^2$$

$$> 0 \text{ iff } (A_{a^s} - 1) \tilde{a}_t^2 + \tilde{a}_t a_t^s (1 - 2A_{a^s}) + A_{a^s} a_t^{s2} > 0$$

QED

14. Mathematica appendix

Original Mathematica notebooks available upon request.

Notebook 1

```

p[a_, e_, r_, x_, rho_, lambda_, tau_] :=
(a * e) /
(lambda^2 +
a
(1 - x * tau * rho *
((a + lambda^2) *
(((tau * (a + rho * x + lambda^2)) / (2 * x * tau^2 * rho * (a + lambda^2))) +
(((tau * (a + rho * x + lambda^2))^2 + (2 * x * tau^2 * rho * (a + lambda^2))^2) /
((tau * (a + rho * x + lambda^2)) *
(2 * x * tau^2 * rho * (a + lambda^2))))))))))
s[a_, e_, r_, x_, rho_, lambda_, tau_] :=
(a * e) /
(lambda^2 +
a
(1 - x * tau * rho *
((a + lambda^2) *
(((tau * (a + rho * x + lambda^2)) / (2 * x * tau^2 * rho * (a + lambda^2))) -
(((tau * (a + rho * x + lambda^2))^2 + (2 * x * tau^2 * rho * (a + lambda^2))^2) /
((tau * (a + rho * x + lambda^2)) *
(2 * x * tau^2 * rho * (a + lambda^2))))))))))
q[a_, e_, r_, x_, rho_, lambda_, tau_] :=
(lambda * e) /
(lambda^2 +
a
(1 - x * tau * rho *
((a + lambda^2) *
(((tau * (a + rho * x + lambda^2)) / (2 * x * tau^2 * rho * (a + lambda^2))) +
(((tau * (a + rho * x + lambda^2))^2 + (2 * x * tau^2 * rho * (a + lambda^2))^2) /
((tau * (a + rho * x + lambda^2)) *
(2 * x * tau^2 * rho * (a + lambda^2))))))))))
t[a_, e_, r_, x_, rho_, lambda_, tau_] :=
(lambda * e) /
(lambda^2 +
a
(1 - x * tau * rho *
((a + lambda^2) *
(((tau * (a + rho * x + lambda^2)) / (2 * x * tau^2 * rho * (a + lambda^2))) -
(((tau * (a + rho * x + lambda^2))^2 + (2 * x * tau^2 * rho * (a + lambda^2))^2) /
((tau * (a + rho * x + lambda^2)) *
(2 * x * tau^2 * rho * (a + lambda^2))))))))))

```

Limit[p[a,ε, r,x,ρ,λ,τ], a→∞]
 Limit[p[a,ε, r,x,ρ,λ,τ], a→0]
 Limit[s[a,ε, r,x,ρ,λ,τ], a→∞]
 Limit[s[a,ε, r,x,ρ,λ,τ], a→0]
 Limit[q[a,ε, r,x,ρ,λ,τ], a→∞]
 Limit[q[a,ε, r,x,ρ,λ,τ], a→0]
 Limit[t[a,ε, r,x,ρ,λ,τ], a→∞]
 Limit[t[a,ε, r,x,ρ,λ,τ], a→0]

0
 0
 0
 0
 0
 $\frac{\epsilon}{\lambda}$
 0
 $\frac{\epsilon}{\lambda}$
 0

Notebook 2

```

p[a_, ε_, r_, x_, ρ_, λ_, τ_] :=
(a*ε) /
(λ² +
a
(1-x*τ*ρ*
((a+λ²) *
(((τ*(a+ρ*x+λ²)) / (2x*τ²*ρ*(a+λ²))) +
(((τ*(a+ρ*x+λ²))² + (2x*τ²*ρ*(a+λ²))²) /
((τ*(a+ρ*x+λ²)) *
(2x*τ²*ρ*(a+λ²))))))))))
s[a_, ε_, r_, x_, ρ_, λ_, τ_] :=
(a*ε) /
(λ² +
a
(1-x*τ*ρ*
((a+λ²) *
(((τ*(a+ρ*x+λ²)) / (2x*τ²*ρ*(a+λ²))) -
(((τ*(a+ρ*x+λ²))² + (2x*τ²*ρ*(a+λ²))²) /
((τ*(a+ρ*x+λ²)) *
(2x*τ²*ρ*(a+λ²))))))))))
  
```

$$\begin{aligned}
& \mathbf{q}[\mathbf{a}, \epsilon, \mathbf{r}, \mathbf{x}, \rho, \lambda, \tau] := \\
& (\lambda * \epsilon) / \\
& (\lambda^2 + \\
& \mathbf{a} \\
& (1 - \mathbf{x} * \tau * \rho * \\
& ((\mathbf{a} + \lambda^2) * \\
& (((\tau * (\mathbf{a} + \rho * \mathbf{x} + \lambda^2)) / (2 \mathbf{x} * \tau^2 * \rho * (\mathbf{a} + \lambda^2))) + \\
& (((\tau * (\mathbf{a} + \rho * \mathbf{x} + \lambda^2))^2 + (2 \mathbf{x} * \tau^2 * \rho * (\mathbf{a} + \lambda^2))^2) / \\
& ((\tau * (\mathbf{a} + \rho * \mathbf{x} + \lambda^2)) * \\
& (2 \mathbf{x} * \tau^2 * \rho * (\mathbf{a} + \lambda^2)))))))))
\end{aligned}$$

$$\begin{aligned}
& \mathbf{t}[\mathbf{a}, \epsilon, \mathbf{r}, \mathbf{x}, \rho, \lambda, \tau] := \\
& (\lambda * \epsilon) / \\
& (\lambda^2 + \\
& \mathbf{a} \\
& (1 - \mathbf{x} * \tau * \rho * \\
& ((\mathbf{a} + \lambda^2) * \\
& (((\tau * (\mathbf{a} + \rho * \mathbf{x} + \lambda^2)) / (2 \mathbf{x} * \tau^2 * \rho * (\mathbf{a} + \lambda^2))) - \\
& (((\tau * (\mathbf{a} + \rho * \mathbf{x} + \lambda^2))^2 + (2 \mathbf{x} * \tau^2 * \rho * (\mathbf{a} + \lambda^2))^2) / \\
& ((\tau * (\mathbf{a} + \rho * \mathbf{x} + \lambda^2)) * \\
& (2 \mathbf{x} * \tau^2 * \rho * (\mathbf{a} + \lambda^2)))))))))
\end{aligned}$$

Limit[p[a, ε, r, x, ρ, λ, τ], a → u]

Limit[s[a, ε, r, x, ρ, λ, τ], a → u]

Limit[q[a, ε, r, x, ρ, λ, τ], a → u]

Limit[t[a, ε, r, x, ρ, λ, τ], a → u]

$$\begin{aligned}
& -(\mathbf{u} \epsilon (\mathbf{u} + \lambda^2 + \mathbf{x} \rho)) / (-\lambda^2 (\lambda^2 + \mathbf{x} \rho) + \mathbf{u}^3 (1 + 2 \mathbf{x}^2 \rho^2 \tau^2) + \\
& \mathbf{u} (2 \lambda^2 (-1 + \mathbf{x} \rho) + \mathbf{x} \rho (-1 + \mathbf{x} \rho) + \lambda^4 (1 + 2 \mathbf{x}^2 \rho^2 \tau^2)) + \\
& \mathbf{u}^2 (-1 + 2 \mathbf{x} \rho + \lambda^2 (2 + 4 \mathbf{x}^2 \rho^2 \tau^2)))
\end{aligned}$$

$$\frac{\mathbf{u} \epsilon (\mathbf{u} + \lambda^2 + \mathbf{x} \rho)}{(\mathbf{u} + \lambda^2) (\mathbf{u} + \lambda^2 + \mathbf{x} \rho + 2 \mathbf{u}^2 \mathbf{x}^2 \rho^2 \tau^2 + 2 \mathbf{u} \mathbf{x}^2 \lambda^2 \rho^2 \tau^2)}$$

$$\begin{aligned}
& -(\epsilon \lambda (\mathbf{u} + \lambda^2 + \mathbf{x} \rho)) / (-\lambda^2 (\lambda^2 + \mathbf{x} \rho) + \mathbf{u}^3 (1 + 2 \mathbf{x}^2 \rho^2 \tau^2) + \\
& \mathbf{u} (2 \lambda^2 (-1 + \mathbf{x} \rho) + \mathbf{x} \rho (-1 + \mathbf{x} \rho) + \lambda^4 (1 + 2 \mathbf{x}^2 \rho^2 \tau^2)) + \\
& \mathbf{u}^2 (-1 + 2 \mathbf{x} \rho + \lambda^2 (2 + 4 \mathbf{x}^2 \rho^2 \tau^2)))
\end{aligned}$$

$$\frac{\epsilon \lambda (\mathbf{u} + \lambda^2 + \mathbf{x} \rho)}{(\mathbf{u} + \lambda^2) (\mathbf{u} + \lambda^2 + \mathbf{x} \rho + 2 \mathbf{u}^2 \mathbf{x}^2 \rho^2 \tau^2 + 2 \mathbf{u} \mathbf{x}^2 \lambda^2 \rho^2 \tau^2)}$$

Notebook 3

$\lambda := .5$
 $\rho := .5$
 $x := .5$
 $\tau := .5$
 $\epsilon := 3$

$$\text{Plot}\left[\frac{a(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2\tau^2 + 2ax^2\lambda^2\rho^2\tau^2)}, \{a, 0, 10\}\right]$$

$$\text{Plot}\left[\frac{\lambda(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2\tau^2 + 2ax^2\lambda^2\rho^2\tau^2)}, \{a, 0, 10\}\right]$$

$\lambda := .5$
 $\rho := .5$
 $x := .5$
 $\tau := .5$
 $\epsilon := 3$

$$\text{Plot}\left[\left\{\frac{a(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 0^2 + 2ax^2\lambda^2\rho^2 0^2)}, \frac{a(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 .25^2 + 2ax^2\lambda^2\rho^2 .25^2)}, \frac{a(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 .5^2 + 2ax^2\lambda^2\rho^2 .5^2)}, \frac{a(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 .75^2 + 2ax^2\lambda^2\rho^2 .75^2)}, \frac{a(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 1^2 + 2ax^2\lambda^2\rho^2 1^2)}\right\}, \{a, 0, 14\}\right]$$

$$\text{Plot}\left[\left\{\frac{\lambda(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 0^2 + 2ax^2\lambda^2\rho^2 0^2)}, \frac{\lambda(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 .25^2 + 2ax^2\lambda^2\rho^2 .25^2)}, \frac{\lambda(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 .5^2 + 2ax^2\lambda^2\rho^2 .5^2)}, \frac{\lambda(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 .75^2 + 2ax^2\lambda^2\rho^2 .75^2)}, \frac{\lambda(a + \lambda^2 + x\rho)\epsilon}{(a + \lambda^2)(a + \lambda^2 + x\rho + 2a^2x^2\rho^2 1^2 + 2ax^2\lambda^2\rho^2 1^2)}\right\}, \{a, 0, 5\}\right]$$

Notebook 4

$$\text{Simplify[}$$

$$\frac{(a \cdot \epsilon) / (\lambda^2 + a (1 - x \tau \rho * ((b + \lambda^2) * (((\tau * (b + \rho * x + \lambda^2)) / (2x \tau^2 \rho * (b + \lambda^2))) - ((\tau * (b + \rho * x + \lambda^2))^2 + (2x \tau^2 \rho * (b + \lambda^2))^2) / ((\tau * (b + \rho * x + \lambda^2)) * (2x \tau^2 \rho * (b + \lambda^2))))))}{\lambda^2 (b + \lambda^2 + x \rho) + a (b + \lambda^2 + x \rho + 2b^2 x^2 \rho^2 \tau^2 + 4bx^2 \lambda^2 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2)}$$

$$\frac{a \in (b + \lambda^2 + x \rho)}{\lambda^2 (b + \lambda^2 + x \rho) + a (b + \lambda^2 + x \rho + 2b^2 x^2 \rho^2 \tau^2 + 4bx^2 \lambda^2 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2)}$$

$$\text{Simplify[}$$

$$\frac{(\lambda \cdot \epsilon) / (\lambda^2 + a (1 - x \tau \rho * ((b + \lambda^2) * (((\tau * (b + \rho * x + \lambda^2)) / (2x \tau^2 \rho * (b + \lambda^2))) - ((\tau * (b + \rho * x + \lambda^2))^2 + (2x \tau^2 \rho * (b + \lambda^2))^2) / ((\tau * (b + \rho * x + \lambda^2)) * (2x \tau^2 \rho * (b + \lambda^2))))))}{\lambda^2 (b + \lambda^2 + x \rho) + a (b + \lambda^2 + x \rho + 2b^2 x^2 \rho^2 \tau^2 + 4bx^2 \lambda^2 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2)}$$

$$\frac{\epsilon \lambda (b + \lambda^2 + x \rho)}{\lambda^2 (b + \lambda^2 + x \rho) + a (b + \lambda^2 + x \rho + 2b^2 x^2 \rho^2 \tau^2 + 4bx^2 \lambda^2 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2)}$$

$$g[b] := \frac{a \in (b + \lambda^2 + x \rho) / (\lambda^2 (b + \lambda^2 + x \rho) + a (b + \lambda^2 + x \rho + 2b^2 x^2 \rho^2 \tau^2 + 4bx^2 \lambda^2 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2))}{\lambda^2 (b + \lambda^2 + x \rho) + a (b + \lambda^2 + x \rho + 2b^2 x^2 \rho^2 \tau^2 + 4bx^2 \lambda^2 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2)}$$

$$h[b] := \frac{\epsilon \lambda (b + \lambda^2 + x \rho) / (\lambda^2 (b + \lambda^2 + x \rho) + a (b + \lambda^2 + x \rho + 2b^2 x^2 \rho^2 \tau^2 + 4bx^2 \lambda^2 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2))}{\lambda^2 (b + \lambda^2 + x \rho) + a (b + \lambda^2 + x \rho + 2b^2 x^2 \rho^2 \tau^2 + 4bx^2 \lambda^2 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2)}$$

Derivative[1] [g]

Derivative[1] [h]

$$-(a \in (\lambda^2 + x \rho + \#1) (\lambda^2 + a (1 + 4x^2 \lambda^2 \rho^2 \tau^2 + 4x^2 \rho^2 \tau^2 \#1))) / (\lambda^2 (\lambda^2 + x \rho + \#1) + a (\lambda^2 + x \rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))^2 + (a \in) / (\lambda^2 (\lambda^2 + x \rho + \#1) + a (\lambda^2 + x \rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2)) \&$$

$$-(\epsilon \lambda (\lambda^2 + x \rho + \#1) (\lambda^2 + a (1 + 4x^2 \lambda^2 \rho^2 \tau^2 + 4x^2 \rho^2 \tau^2 \#1))) / (\lambda^2 (\lambda^2 + x \rho + \#1) + a (\lambda^2 + x \rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))^2 + (\epsilon \lambda) / (\lambda^2 (\lambda^2 + x \rho + \#1) + a (\lambda^2 + x \rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2)) \&$$

$$\begin{aligned} & \text{Simplify[} \\ & \quad -(a\epsilon (\lambda^2 + x\rho + \#1) (\lambda^2 + a(1 + 4x^2 \lambda^2 \rho^2 \tau^2 + 4x^2 \rho^2 \tau^2 \#1))) / \\ & \quad (\lambda^2 (\lambda^2 + x\rho + \#1) + a(\lambda^2 + x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))^2 + \\ & \quad (a\epsilon) / (\lambda^2 (\lambda^2 + x\rho + \#1) + a(\lambda^2 + x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))] \\ & \text{Simplify[} \\ & \quad -(\epsilon \lambda (\lambda^2 + x\rho + \#1) (\lambda^2 + a(1 + 4x^2 \lambda^2 \rho^2 \tau^2 + 4x^2 \rho^2 \tau^2 \#1))) / \\ & \quad (\lambda^2 (\lambda^2 + x\rho + \#1) + a(\lambda^2 + x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))^2 + \\ & \quad (\epsilon \lambda) / (\lambda^2 (\lambda^2 + x\rho + \#1) + a(\lambda^2 + x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))] \\ & \text{Simplify[} \\ & \quad -(a\epsilon (\lambda^2 + x\rho + \#1) (\lambda^2 + a(1 + 4x^2 \lambda^2 \rho^2 \tau^2 + 4x^2 \rho^2 \tau^2 \#1))) / \\ & \quad (\lambda^2 (\lambda^2 + x\rho + \#1) + a(\lambda^2 + x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))^2 + \\ & \quad (a\epsilon) / (\lambda^2 (\lambda^2 + x\rho + \#1) + a(\lambda^2 + x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))] \\ & \text{Simplify[} \\ & \quad -(\epsilon \lambda (\lambda^2 + x\rho + \#1) (\lambda^2 + a(1 + 4x^2 \lambda^2 \rho^2 \tau^2 + 4x^2 \rho^2 \tau^2 \#1))) / \\ & \quad (\lambda^2 (\lambda^2 + x\rho + \#1) + a(\lambda^2 + x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))^2 + \\ & \quad (\epsilon \lambda) / (\lambda^2 (\lambda^2 + x\rho + \#1) + a(\lambda^2 + x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 4x^2 \lambda^2 \rho^2 \tau^2 \#1 + 2x^2 \rho^2 \tau^2 \#1^2))] \\ & \quad -(2a^2 x^2 \epsilon \rho^2 \tau^2 (\lambda^2 + \#1) (\lambda^2 + 2x\rho + \#1)) / \\ & \quad (\lambda^2 (\lambda^2 + x\rho + \#1) + a(x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 2x^2 \rho^2 \tau^2 \#1^2 + \lambda^2 (1 + 4x^2 \rho^2 \tau^2 \#1)))^2 \\ & \quad -(2ax^2 \epsilon \lambda \rho^2 \tau^2 (\lambda^2 + \#1) (\lambda^2 + 2x\rho + \#1)) / \\ & \quad (\lambda^2 (\lambda^2 + x\rho + \#1) + a(x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 + 2x^2 \rho^2 \tau^2 \#1^2 + \lambda^2 (1 + 4x^2 \rho^2 \tau^2 \#1)))^2 \end{aligned}$$

Notebook 5

$$\begin{aligned} & \text{Simplify[} \\ & \quad ((a * \epsilon) / \\ & \quad (\lambda^2 + \\ & \quad a \\ & \quad (1 - x * \tau * \rho * \\ & \quad ((b + \lambda^2) * \\ & \quad (((\tau * (b + \rho * x + \lambda^2)) / \\ & \quad (2x * \tau^2 \rho * (b + \lambda^2))) - \\ & \quad (((\tau * (b + \rho * x + \lambda^2))^2 + \\ & \quad (2x * \tau^2 \rho * (b + \lambda^2))^2) / \\ & \quad ((\tau * (b + \rho * x + \lambda^2)) * \\ & \quad (2x * \tau^2 \rho * (b + \lambda^2)))))))))))] \end{aligned}$$

$$\frac{a \in (b + \lambda^2 + x \rho)}{(\lambda^2 (b + \lambda^2 + x \rho) + a (b + \lambda^2 + x \rho + 2 b^2 x^2 \rho^2 \tau^2 + 4 b x^2 \lambda^2 \rho^2 \tau^2 + 2 x^2 \lambda^4 \rho^2 \tau^2))}$$

r :=

$$\begin{aligned} & ((b + \lambda^2) * \\ & \quad (((\tau * (b + \rho * x + \lambda^2)) / (2 x * \tau^2 \rho * (b + \lambda^2))) - \\ & \quad \quad ((\tau * (b + \rho * x + \lambda^2))^2 + \\ & \quad \quad \quad (2 x * \tau^2 \rho * (b + \lambda^2))^2) / \\ & \quad \quad \quad ((\tau * (b + \rho * x + \lambda^2)) * \\ & \quad \quad \quad (2 x * \tau^2 \rho * (b + \lambda^2)))))) \end{aligned}$$

g[b] :=

$$\begin{aligned} & \rho \tau (a \in (b + \lambda^2 + x \rho)) / \\ & \quad (\lambda^2 (b + \lambda^2 + x \rho) + \\ & \quad \quad a (b + \lambda^2 + x \rho + 2 b^2 x^2 \rho^2 \tau^2 + 4 b x^2 \lambda^2 \rho^2 \tau^2 + \\ & \quad \quad \quad 2 x^2 \lambda^4 \rho^2 \tau^2)) * \\ & \quad ((b + \lambda^2) * \\ & \quad \quad (((\tau * (b + \rho * x + \lambda^2)) / \\ & \quad \quad \quad (2 x * \tau^2 \rho * (b + \lambda^2))) - \\ & \quad \quad \quad ((\tau * (b + \rho * x + \lambda^2))^2 + \\ & \quad \quad \quad \quad (2 x * \tau^2 \rho * (b + \lambda^2))^2) / \\ & \quad \quad \quad \quad ((\tau * (b + \rho * x + \lambda^2)) * \\ & \quad \quad \quad \quad (2 x * \tau^2 \rho * (b + \lambda^2)))))) \end{aligned}$$

Derivative[1][g]

Output of derivative is omitted, but its simplification shown (derivative is within *Simplify[.]* upcoming)

$$-(2ax\epsilon (a+\lambda^2) \rho \rho \tau \tau (\lambda^2 + \#1) (\lambda^2 + 2x\rho + \#1)) /$$

$$(\lambda^2 (\lambda^2 + x\rho + \#1) + a (x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + \#1 +$$

$$2x^2 \rho^2 \tau^2 \#1^2 + \lambda^2 (1 + 4x^2 \rho^2 \tau^2 \#1)))^2$$

Notebook 6

Simplify[

$$(a\epsilon) /$$

$$(\lambda^2 +$$

$$a$$

$$(1 -$$

$$x\tau\rho$$

$$(((a+z) + \lambda^2) * (((\tau * (a+z) + \rho * x + \lambda^2)) / (2x\tau\rho * (a+z) + \lambda^2))) -$$

$$(((\tau * (a+z) + \rho * x + \lambda^2))^2 + (2x\tau\rho * (a+z) + \lambda^2)^2) /$$

$$((\tau * (a+z) + \rho * x + \lambda^2) * (2x\tau\rho * (a+z) + \lambda^2)))))]$$

$$(a\epsilon (a+z+\lambda^2+x\rho)) / (\lambda^2 (z+\lambda^2+x\rho) + 2a^3 x^2 \rho^2 \tau^2 +$$

$$a(z+2\lambda^2+x\rho+2x^2 z \rho^2 \tau^2 + 4x^2 z \lambda^2 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2) + a^2 (1+4x^2 (z+\lambda^2) \rho^2 \tau^2))$$

Simplify[

$$(a\epsilon (a + \lambda^2 + x\rho)) /$$

$$(\lambda^2 (\lambda^2 + x\rho) + 2a^3 x^2 \rho^2 \tau^2 + a (2\lambda^2 + x\rho + 2x^2 \lambda^4 \rho^2 \tau^2 + 2x^2 \lambda^4 \rho^2 \tau^2) +$$

$$a^2 (1 + 4x^2 (\lambda^2 \rho^2 \tau^2)))]$$

$$(a\epsilon (a + \lambda^2 + x\rho)) / (\lambda^4 + x\lambda^2 \rho + 2a^3 x^2 \rho^2 \tau^2 +$$

$$a^2 (1 + 4x^2 \lambda^2 \rho^2 \tau^2) + a (2\lambda^2 + 2x^2 \lambda^4 \rho^2 \tau^2 + x\rho (1 + 2Ax\rho\tau^2)))$$

g[A_] :=

$$(a\epsilon (a + \lambda^2 + x\rho)) /$$

$$(\lambda^4 + x\lambda^2 \rho + 2a^3 x^2 \rho^2 \tau^2 + a^2 (1 + 4x^2 \lambda^2 \rho^2 \tau^2) +$$

$$a (2\lambda^2 + 2x^2 \lambda^4 \rho^2 \tau^2 + x\rho (1 + 2Ax\rho\tau^2)))$$

Derivative[1] [g]

$$-(2a^2 x^2 \epsilon \rho^2 (a + \lambda^2 + x\rho) \tau^2) / (\lambda^4 + x\lambda^2 \rho + 2a^3 x^2 \rho^2 \tau^2 +$$

$$a^2 (1 + 4x^2 \lambda^2 \rho^2 \tau^2) + a (2\lambda^2 + 2x^2 \lambda^4 \rho^2 \tau^2 + x\rho (1 + 2x\rho\tau^2 \#1)))^2 \&$$

Simplify[
 $(\lambda * \epsilon) /$
 $(\lambda^2 +$
 a
 $(1 -$
 $x * \tau * \rho *$
 $((a + z) + \lambda^2) * (((\tau * (a + z) + \rho * x + \lambda^2)) / (2 * x * \tau^2 * \rho * ((a + z) + \lambda^2))) -$
 $((\tau * (a + z) + \rho * x + \lambda^2)^2 + (2 * x * \tau^2 * \rho * ((a + z) + \lambda^2))^2) /$
 $((\tau * (a + z) + \rho * x + \lambda^2) * (2 * x * \tau^2 * \rho * ((a + z) + \lambda^2)))))))]$

$$(\epsilon \lambda (a + z + \lambda^2 + x \rho)) / (\lambda^2 (z + \lambda^2 + x \rho) + 2 a^3 x^2 \rho^2 \tau^2 +$$

$$a (z + 2 \lambda^2 + x \rho + 2 x^2 z \rho^2 \tau^2 + 4 x^2 z \lambda^2 \rho^2 \tau^2 + 2 x^2 \lambda^4 \rho^2 \tau^2) + a^2 (1 + 4 x^2 (z + \lambda^2) \rho^2 \tau^2))$$

Simplify[
 $(\epsilon \lambda (a + \lambda^2 + x \rho)) /$
 $(\lambda^2 (\lambda^2 + x \rho) + 2 a^3 x^2 \rho^2 \tau^2 + a (2 \lambda^2 + x \rho + 2 x^2 \lambda^2 \rho^2 \tau^2 + 2 x^2 \lambda^4 \rho^2 \tau^2) +$
 $a^2 (1 + 4 x^2 (\lambda^2) \rho^2 \tau^2))]$

$$\frac{\epsilon \lambda (a + \lambda^2 + x \rho)}{\lambda^4 + x \lambda^2 \rho + 2 a^3 x^2 \rho^2 \tau^2 + a^2 (1 + 4 x^2 \lambda^2 \rho^2 \tau^2) + a (2 \lambda^2 + 2 x^2 \lambda^4 \rho^2 \tau^2 + x \rho (1 + 2 A x \rho \tau^2))}$$

g[A_] :=
 $(\epsilon \lambda (a + \lambda^2 + x \rho)) /$
 $(\lambda^4 + x \lambda^2 \rho + 2 a^3 x^2 \rho^2 \tau^2 + a^2 (1 + 4 x^2 \lambda^2 \rho^2 \tau^2) +$
 $a (2 \lambda^2 + 2 x^2 \lambda^4 \rho^2 \tau^2 + x \rho (1 + 2 A x \rho \tau^2)))$

Derivative[1][g]
 $-(2 a x^2 \epsilon \lambda \rho^2 (a + \lambda^2 + x \rho) \tau^2) / (\lambda^4 + x \lambda^2 \rho + 2 a^3 x^2 \rho^2 \tau^2 +$
 $a^2 (1 + 4 x^2 \lambda^2 \rho^2 \tau^2) + a (2 \lambda^2 + 2 x^2 \lambda^4 \rho^2 \tau^2 + x \rho (1 + 2 x \rho \tau^2 \#1)))^2 \&$

Simplify[$-(2 a x^2 \epsilon \lambda \rho^2 (a + \lambda^2 + x \rho) \tau^2) /$
 $(\lambda^4 + x \lambda^2 \rho + 2 a^3 x^2 \rho^2 \tau^2 + a^2 (1 + 4 x^2 \lambda^2 \rho^2 \tau^2) +$
 $a (2 \lambda^2 + 2 x^2 \lambda^4 \rho^2 \tau^2 + x \rho (1 + 2 x \rho \tau^2 \#1)))^2]$

$-(2 a x^2 \epsilon \lambda \rho^2 (a + \lambda^2 + x \rho) \tau^2) / (\lambda^4 + x \lambda^2 \rho + 2 a^3 x^2 \rho^2 \tau^2 +$
 $a^2 (1 + 4 x^2 \lambda^2 \rho^2 \tau^2) + a (2 \lambda^2 + 2 x^2 \lambda^4 \rho^2 \tau^2 + x \rho (1 + 2 x \rho \tau^2 \#1)))^2$

Notebook 7

$$g[b] := (2*a*x^2*\rho^2*(b+\lambda^2)*(b+\lambda^2+2*x*\rho)*\tau^2) /$$

$$(\lambda^2*(b+\lambda^2+x*\rho) + a*(b+\lambda^2+x*\rho) +$$

$$(1+2*\lambda^2+\lambda^4)*2*b*x^2*\rho^2*\tau^2)^2$$

Derivative[1][g]

$$-(4ax^2\rho^2\tau^2(a+\lambda^2+2x^2(1+2\lambda^2+\lambda^4)\rho^2\tau^2)(\lambda^2+\#1)$$

$$(\lambda^2+2x\rho+\#1)) / (2x^2(1+2\lambda^2+\lambda^4)\rho^2\tau^2\#1+$$

$$a(\lambda^2+x\rho+\#1)+\lambda^2(\lambda^2+x\rho+\#1))^3 +$$

$$(2ax^2\rho^2\tau^2(\lambda^2+\#1)) / (2x^2(1+2\lambda^2+\lambda^4)\rho^2\tau^2\#1+$$

$$a(\lambda^2+x\rho+\#1)+\lambda^2(\lambda^2+x\rho+\#1))^2 +$$

$$(2ax^2\rho^2\tau^2(\lambda^2+2x\rho+\#1)) / (2x^2(1+2\lambda^2+\lambda^4)\rho^2\tau^2\#1+$$

$$a(\lambda^2+x\rho+\#1)+\lambda^2(\lambda^2+x\rho+\#1))^2 \&$$

Simplify[

$$-(4ax^2\rho^2\tau^2(a+\lambda^2+2x^2(1+2\lambda^2+\lambda^4)\rho^2\tau^2)(\lambda^2+\#1)$$

$$(\lambda^2+2x\rho+\#1)) /$$

$$(2x^2(1+2\lambda^2+\lambda^4)\rho^2\tau^2\#1+a(\lambda^2+x\rho+\#1)+$$

$$\lambda^2(\lambda^2+x\rho+\#1))^3 +$$

$$(2ax^2\rho^2\tau^2(\lambda^2+\#1)) /$$

$$(2x^2(1+2\lambda^2+\lambda^4)\rho^2\tau^2\#1+a(\lambda^2+x\rho+\#1)+$$

$$\lambda^2(\lambda^2+x\rho+\#1))^2 +$$

$$(2ax^2\rho^2\tau^2(\lambda^2+2x\rho+\#1)) /$$

$$(2x^2(1+2\lambda^2+\lambda^4)\rho^2\tau^2\#1+a(\lambda^2+x\rho+\#1)+$$

$$\lambda^2(\lambda^2+x\rho+\#1))^2]$$

$$(4ax^4\rho^4\tau^2(a-2\lambda^8\tau^2-2x\rho\tau^2\#1-2\lambda^6\tau^2(2+2x\rho+\#1)-$$

$$\lambda^2(-1+2\tau^2\#1+4x\rho\tau^2(1+\#1))-$$

$$2\lambda^4\tau^2(1+2\#1+x\rho(4+\#1)))) /$$

$$(2x^2\rho^2\tau^2\#1+a(\lambda^2+x\rho+\#1)+\lambda^4(1+2x^2\rho^2\tau^2\#1)+$$

$$\lambda^2(x\rho+\#1+4x^2\rho^2\tau^2\#1))^3$$

$$(4ax^4\rho^4\tau^2$$

$$(a-2\lambda^8\tau^2-2x\rho\tau^2b-2\lambda^6\tau^2(2+2x\rho+b)-$$

$$\lambda^2(-1+2\tau^2b+4x\rho\tau^2(1+b))-$$

$$2\lambda^4\tau^2(1+2b+x\rho(4+b)))) /$$

$$(2x^2\rho^2\tau^2b+a(\lambda^2+x\rho+b)+\lambda^4(1+2x^2\rho^2\tau^2b)+$$

$$\lambda^2(x\rho+b+4x^2\rho^2\tau^2b))^3$$

Notebook 8

```

x:=1
ρ:=1
τ:=1
λ:=.1
(4ax4ρ4τ2
(a-2λ8τ2-2xρτ2b-2λ6τ2(2+2xρ+b) -
λ2(-1+2τ2b+4xρτ2(1+b)) -
2λ4τ2(1+2b+xρ(4+b))) /
(2x2ρ2τ2b+a(λ2+xρ+b) + λ4(1+2x2ρ2τ2b) +
λ2(xρ+b+4x2ρ2τ2b))3

(4a(-2.×10-8+a-2b-2.×10-6(4+b) -
0.0002(5+3b) - 0.01(-1+2b+4(1+b))) /
(2b+a(1.01+b) + 0.0001(1+2b) + 0.01(1+5b))3

Simplify[
(4a(-2.0000000000000001-8+a-2b-
2.0000000000000008-6(4+b) -
0.00020000000000000004-5(5+3b) -
0.010000000000000002-4(-1+2b+4(1+b))) /
(2b+a(1.01-1+b) + 0.00010000000000000002-2(1+2b) +
0.010000000000000002-3(1+5b))3]

4. a(-0.031008+a-2.0606b)
(0.0101+2.0502b+a(1.01+b))3

-0.031008020000000008-8+a-2.060602-2b<0

True

```

Notebook 9

```

ρτ(π-n)
((a+λ2)*
(((τ*(a+ρ*x+λ2))/(2x*τ2ρ*(a+λ2))) -
(((τ*(a+ρ*x+λ2))2+(2x*τ2ρ*(a+λ2))2)/((τ*(a+ρ*x+λ2))*(2x*τ2ρ*(a+λ2))))))
(λ(π-n)/(u-(M(1-r)j+An)/(M(1-r)+A)))

```

$$(-n+\pi) (a+\lambda^2) \rho \tau \left(\frac{a+\lambda^2+x\rho}{2x (a+\lambda^2) \rho \tau} - \frac{(a+\lambda^2+x\rho)^2 \tau^2 + 4x^2 (a+\lambda^2)^2 \rho^2 \tau^4}{2x (a+\lambda^2) \rho (a+\lambda^2+x\rho) \tau^3} \right)$$

$$\frac{(-n+\pi) \lambda}{-\frac{Am+jM(1-r)}{A+M(1-r)} + u} + u$$

$\rho \tau (\pi - n)$

$$\begin{aligned} & \left((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \lambda^2 \right) * \\ & \left(((\tau * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \rho * x + \lambda^2)) / \right. \\ & \quad \left. (2x * \tau^2 \rho * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \lambda^2))) - \right. \\ & \quad \left. (((\tau * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \rho * x + \lambda^2))^2 + \right. \\ & \quad \left. (2x * \tau^2 \rho * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \lambda^2))^2) / \right. \\ & \quad \left. ((\tau * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \rho * x + \lambda^2)) * \right. \\ & \quad \left. (2x * \tau^2 \rho * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \lambda^2)))) \right) \end{aligned}$$

$$(-n+\pi) \left(\frac{(-n+\pi) \lambda}{-\frac{Am+jM(1-r)}{A+M(1-r)} + u} + \lambda^2 \right) \rho \tau$$

$$\left(\frac{\frac{(-n+\pi) \lambda}{-\frac{Am+jM(1-r)}{A+M(1-r)} + u} + \lambda^2 + x\rho}{2x \left(\frac{(-n+\pi) \lambda}{-\frac{Am+jM(1-r)}{A+M(1-r)} + u} + \lambda^2 \right) \rho \tau} - \frac{\left(\frac{(-n+\pi) \lambda}{-\frac{Am+jM(1-r)}{A+M(1-r)} + u} + \lambda^2 + x\rho \right)^2 \tau^2 + 4x^2 \left(\frac{(-n+\pi) \lambda}{-\frac{Am+jM(1-r)}{A+M(1-r)} + u} + \lambda^2 \right)^2 \rho^2 \tau^4}{2x \left(\frac{(-n+\pi) \lambda}{-\frac{Am+jM(1-r)}{A+M(1-r)} + u} + \lambda^2 \right) \rho \left(\frac{(-n+\pi) \lambda}{-\frac{Am+jM(1-r)}{A+M(1-r)} + u} + \lambda^2 + x\rho \right) \tau^3} \right)$$

$g[M] :=$

$\rho \tau (\pi - n)$

$$\begin{aligned} & \left((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \lambda^2 \right) * \\ & \left(((\tau * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \rho * x + \lambda^2)) / \right. \\ & \quad \left. (2x * \tau^2 \rho * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \lambda^2))) - \right. \\ & \quad \left. (((\tau * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \rho * x + \lambda^2))^2 + \right. \\ & \quad \left. (2x * \tau^2 \rho * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \lambda^2))^2) / \right. \\ & \quad \left. ((\tau * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \rho * x + \lambda^2)) * \right. \\ & \quad \left. (2x * \tau^2 \rho * ((\lambda (\pi - n) / (u - (M(1-r)j + Am) / (M(1-r) + A))) + \lambda^2)))) \right) \end{aligned}$$

Derivative[1][g]

Resulting derivative omitted due to length, but available upon request.

Simplified version follows:

$$\begin{aligned} & -(2(Am - Aj)(n - \pi)^2(-1 + r)x\lambda^2\rho\rho\tau\tau(Am\lambda + A(n - \pi - u\lambda) + (-1 + r)(-n + \pi - j\lambda + u\lambda)\#1) \\ & \quad (Am(\lambda^2 + 2x\rho) - A(-n\lambda + \pi\lambda + u(\lambda^2 + 2x\rho)) + (-1 + r)(-n\lambda + \pi\lambda - (j - u)(\lambda^2 + 2x\rho))\#1)) / \\ & \quad ((-Am + Au + (-1 + r)(j - u)\#1)^2 \\ & \quad (Am(\lambda^2 + x\rho) - A(-n\lambda + \pi\lambda + u(\lambda^2 + x\rho)) + (-1 + r)(-n\lambda + \pi\lambda - (j - u)(\lambda^2 + x\rho))\#1)^2) \end{aligned}$$

$$\begin{aligned}
& - (2 (A^*m - A j) (n - \pi)^2 (-1 + r) x \lambda^2 \rho \tau \tau (A^*m \lambda + A (n - \pi - u \lambda) + (-1 + r) (-n + \pi - j \lambda + u \lambda) M) \\
& \quad (A^*m (\lambda^2 + 2x\rho) - A (-n \lambda + \pi \lambda + u (\lambda^2 + 2x\rho)) + (-1 + r) (-n \lambda + \pi \lambda - (j - u) (\lambda^2 + 2x\rho)) M) / \\
& \quad ((-A^*m + Au + (-1 + r) (j - u) M)^2 \\
& \quad (A^*m (\lambda^2 + x\rho) - A (-n \lambda + \pi \lambda + u (\lambda^2 + x\rho)) + (-1 + r) (-n \lambda + \pi \lambda - (j - u) (\lambda^2 + x\rho)) M)^2)
\end{aligned}$$

$$\begin{aligned}
& - (2 (-A j + Am) (n - \pi)^2 (-1 + r) x \lambda^2 (Am \lambda + A (n - \pi - u \lambda) + A (-1 + r) (-n + \pi - j \lambda + u \lambda)) \rho \\
& \quad (Am (\lambda^2 + 2x\rho) + A (-1 + r) (-n \lambda + \pi \lambda - (j - u) (\lambda^2 + 2x\rho)) - A (-n \lambda + \pi \lambda + u (\lambda^2 + 2x\rho))) \\
& \quad \rho \tau \tau / ((-Am + A (-1 + r) (j - u) + Au)^2 \\
& \quad (Am (\lambda^2 + x\rho) + A (-1 + r) (-n \lambda + \pi \lambda - (j - u) (\lambda^2 + x\rho)) - A (-n \lambda + \pi \lambda + u (\lambda^2 + x\rho)))^2)
\end{aligned}$$

$$\begin{aligned}
& (2 (A^*m - A j) (n - \pi)^2 (-1 + r) x \lambda^2 \rho \tau \tau (A^*m \lambda + A (n - \pi - u \lambda) + (-1 + r) (-n + \pi - j \lambda + u \lambda) M) \\
& \quad (A^*m (\lambda^2 + 2x\rho) - A (-n \lambda + \pi \lambda + u (\lambda^2 + 2x\rho)) + (-1 + r) (-n \lambda + \pi \lambda - (j - u) (\lambda^2 + 2x\rho)) M)
\end{aligned}$$

M := A + k

k := k

$$\begin{aligned}
& -0.0925129 (-1.06248 - 0.5 (3.54159 - 0.1 j) (0.3 + k) + 0.03 m) (-0.3 j + 0.3 m) \\
& (-3.10625 - 0.5 (0.304159 - 2.01 (-5 + j)) (0.3 + k) + 0.603 m)
\end{aligned}$$

\rho := 1

\tau := 1

\lambda := .1

x := 1

n := 0.1

u := 5

P := 5

r := .5

$$\begin{aligned}
& 2 (-A j + Am) (n - \pi)^2 (-1 + r) x \lambda^2 (Am \lambda + A (n - \pi - u \lambda) + A (-1 + r) (-n + \pi - j \lambda + u \lambda)) \rho \\
& \quad (Am (\lambda^2 + 2x\rho) + A (-1 + r) (-n \lambda + \pi \lambda - (j - u) (\lambda^2 + 2x\rho)) - A (-n \lambda + \pi \lambda + u (\lambda^2 + 2x\rho))) \\
& \quad \rho \tau \tau
\end{aligned}$$

$$\begin{aligned}
& -0.0925129 (-3.54159 A - 0.5 A (3.54159 - 0.1 j) + 0.1 A m) (-A j + A m) (-10.3542 \\
& A - 0.5 A (0.304159 - 2.01 (-5 + j)) + 2.01 A m)
\end{aligned}$$

$$\begin{aligned}
& \text{Simplify}[-0.09251285870371401^{\sim} (-1.0624777960769378^{\sim} - 0.5^{\sim} \\
& (3.541592653589793^{\sim} - 0.1^{\sim} j) (0.3^{\sim} + k) + 0.03^{\sim} m) (-0.3^{\sim} j + 0.3^{\sim} m) (- \\
& 3.1062477796076933^{\sim} - 0.5^{\sim} (0.3041592653589793^{\sim} - 2.01^{\sim} (-5 + j)) (0.3^{\sim} \\
& + k) + 0.6029999999999999^{\sim} m)]
\end{aligned}$$

$$\begin{aligned}
& -0.0925129 (-1.59372 + j (0.015 + 0.05 k) - 1.7708 k + 0.03 m) (-0.3 j + 0.3 m) (- \\
& 4.65937 - 5.17708 k + j (0.3015 + 1.005 k) + 0.603 m)
\end{aligned}$$

j := j

m := m

k := 1

$$\begin{aligned}
& -0.09251285870371401^{\sim} (-1.5937166941154066^{\sim} + j (0.015^{\sim} + 0.05^{\sim} k) - \\
& 1.7707963267948965^{\sim} k + 0.03^{\sim} m) (-0.3^{\sim} j + 0.3^{\sim} m) (-4.65937166941154^{\sim} - \\
& 5.177079632679489^{\sim} k + j (0.30149999999999993^{\sim} + 1.005^{\sim} k) + 0.6029999999999999^{\sim} \\
& m)
\end{aligned}$$

$$\begin{aligned}
& -0.0925129 (-3.36451 + 0.065 j + 0.03 m) (-0.3 j + 0.3 m) (-9.83645 + 1.3065 \\
& j + 0.603 m)
\end{aligned}$$


```

j := 5
m := 4
0.2401000000000001`A^3 (j - m) (-8.100000000000001` + 0.05` j + 0.1` m)
(-15.809999999999999` + 1.005` j + 2.01` m)

4.91011A^3

j := 4.9
m := 5.1
0.2401000000000001`A^3 (j - m) (-8.100000000000001` + 0.05` j + 0.1` m)
(-15.809999999999999` + 1.005` j + 2.01` m)

-0.223793 (k + M)^3

j := j
m := m
A := .3
0.09251285870371401`A^3 (j - 1.` m) (-5.312388980384689` + 0.05` j + 0.1` m)
(-15.531238898038467` + 1.005` j + 2.01` m)

0.00249785 (j - 1. m) (-5.31239 + 0.05 j + 0.1 m) (-15.5312 + 1.005 j + 2.01 m)

Plot3D[-0.09251285870371401` (-1.5937166941154066` + 0.015` j + 0.03` m) (-0.3`
j + 0.3` m) (-4.65937166941154` + 0.30149999999999993` j + 0.6029999999999999` m),
{m, 0, 10}, {j, 0, 10}]

Plot3D[-0.09251285870371401` (-2.0364157758141306` + 0.0275` j + 0.03` m) (-
0.3` j + 0.3` m) (-5.953641577581412` + 0.5527499999999999`
j + 0.6029999999999999` m), {m, 0, 10}, {j, 0, 10}]

Plot3D[-0.09251285870371401` (-2.479114857512855` + 0.04` j + 0.03` m) (-0.3`
j + 0.3` m) (-7.247911485751285` + 0.8039999999999998` j + 0.6029999999999999` m),
{m, 0, 10}, {j, 0, 10}]

Plot3D[-0.09251285870371401` (-2.921813939211579` + 0.052500000000000005`
j + 0.03` m) (-0.3` j + 0.3` m) (-8.542181393921156` + 1.0552499999999998`
j + 0.6029999999999999` m), {m, 0, 10}, {j, 0, 10}]

Plot3D[-0.09251285870371401` (-3.3645130209103034` + 0.065` j + 0.03` m) (-0.3`
j + 0.3` m) (-9.836451302091028` + 1.3064999999999998` j + 0.6029999999999999` m),
{m, 0, 10}, {j, 0, 10}]

```

Notebook 10

```
g[n_] := (x*e+λ*(p-n))/(p-n)-ε
```

```
Derivative[1][g]
```

$$\frac{ex + \lambda(p - \#1)}{(p - \epsilon - \#1)^2} - \frac{\lambda}{p - \epsilon - \#1} \&$$

```
g[n_] := (x*e+λ*(p-n))/(p-n)-ε
```

```
Derivative[2][g]
```

$$\frac{2(ex + \lambda(p - \#1))}{(p - \epsilon - \#1)^3} - \frac{2\lambda}{(p - \epsilon - \#1)^2} \&$$

15. Glossary of propositions and corollaries

Proposition 2.1 (p. 35): When preferences a_t are time-varying (specifically, $AR(1)$), and additionally cost-push inflation is present ($\sigma_\varepsilon > 0$) and expectations matter in the Phillips curve ($\chi > 0$), the variability of the unemployment gap may, in the short-run, be unambiguously decreased when a_t increases at no expense of variability of inflation.

Corollary 2.1 (p. 37): Given time-varying preferences, a relatively less-conservative (i.e., high a_t) monetary authority may, in the short-run, achieve lower variability and level of the unemployment gap than a more conservative (low a_t) monetary authority, while also delivering equivalent or less inflation variability and level, for realistic values of the parameters of the model.

Proposition 3.1 (p. 42): Given $\lambda \tilde{u}_t^p + \pi_t^p \neq 0$, a monetary authority may achieve unambiguously lower inflation and unemployment gap variability and level, thereby increasing utility, for *any* values of the parameters of the model, if they can convince private agents that their preferences are not what they actually are.

Corollary 3.1 (p. 42): Given a formal inflation target of zero, the monetary authority has no incentive to “lie.”

Proposition 3.2 (p. 48): Given even one preference “lie” is detectable to the public (that is, reveals some correlation between announced signals of independent variables), expectations will be biased upwards or downwards according to the monetary authority’s deceitful incentives for the current level of inflation and the unemployment gap.

Proposition 3.3 (p. 54): Under the assumption of normally distributed signals for control error and preferences, losses to credibility from spurious *or* man-made correlations in v_t^* and η_t^* may be avoided the more accurate a_t^* and η_t^* are. However, gains similar to “lying” at the same time diminish with added accuracy.

Proposition 4.1 (p. 63): If the monetary authority must choose a permanent value of σ_κ^2 to avoid credibility problems, and this choice is made according to a weighted average of expected future $\lambda\tilde{u}_i^p + \pi_i^p$, if $\lambda\tilde{u}_i^p + \pi_i^p$ is expected to be positive *but one time* in the future, a choice of $\sigma_\kappa^2 > 0$ is made, even if only marginally greater than zero.

Proposition 4.2 (p. 63): Releasing a noisy public signal of preferences (and control error) may result in the same minimizing effects to inflation as “lying,” but without the associated “credibility for honesty” problems.

Proposition 4.3 (p. 64): Opacity achieves the utility-increasing benefits of lying without suffering the credibility problems from dishonesty. However, there exists a trade-off between the benefits from opacity and its detractions from credibility for accuracy *if* the public has an idea (accurate or otherwise) of the actual variances of control error and preferences.

Proposition 5.1 (p. 71): When the monetary authority is risk-averse and $\lambda\tilde{u}_i^p + \pi_i^p$ is positive, the expected value of utility is always greater with lower variance in the signal for preferences due to a second-order stochastically dominating shift.

Proposition 6.1 (p. 79): Given the public has its own estimate of preferences and control error, and knows the accuracy of public signals, a decrease in the accuracy of the control error signal always decreases the variability of expectations. A decrease in the accuracy of the preference signal does the same for certain values of the parameters of the model.

Corollary 6.1 (p. 79): Under the assumptions of *Proposition 7.1*, a decrease in the accuracy of the control error signal decreases the variability of inflation, while a decrease in the decrease in the accuracy of the preference signal does the same for certain values of the parameters of the model.

Proposition 6.2 (p. 83): Given private agents have no-communication estimates of the variances and values of the parameters of the model which are inaccurate, the monetary authority may moderate the volatility of inflation expectations by increasing the variance of the control error signal for any values of the parameters of the model, and/or by

increasing the variance of the preference signal for certain values of the parameters of the model.

Proposition 6.3 (p. 85): If the monetary authority is thought to be less conservative than it actually is ($a_t^s < \tilde{a}_t$), it will encounter unambiguously lower expectation (and hence inflation) variability than with perfect information. Thus, there is potentially the motivation to withhold actual preferences.

Corollary 6.3 (p. 85): The assumption of an inaccurately high no-communication estimate of a_t^s makes possible further moderation of expectations (and hence inflation) variability with increases in the variability of the preference signal, for certain values of the parameters of the model: $-(a_t^s - \tilde{a}_t)/(s_{a^s}^2)^2 > \tilde{\sigma}_a^2 (a_t^s - \tilde{a}_t)^2 / (s_{a^s}^2)^3$.

Proposition 7.1 (p. 95): Given the monetary authority's period-by-period estimate of the natural rate of unemployment is at least as accurate as the representative private agent's, there are ranges of the parameters of the model in which the monetary authority may anchor inflation expectations by announcing its estimate of the natural rate with noise beyond the inherent uncertainty in the estimate.

Corollary 7.1 (p. 95): Increased opacity in the signal decreases expectations even when private agents' expectations are as good as the monetary authority's, so that expectations may be moderated but there are no long-term losses to the public's estimating abilities with less public information.

Proposition 8.1 (p. 108): Given the monetary authority may control the degree of control error in realized inflation, a "dove" regime may hide its identity and avoid an expectations trap as the consequence of a transitory cost-push shock.

Corollary 8.1: (p. 108): In the case sufficiently high control error is needed to hide the dove's identity such that an increase in control error is detectable, the hawk necessarily must collude and maintain the same degree of control error necessary to hide the dove's identity.