Ramified Deformable Particles in Simple Shear

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Abstract

The behavior of deformable structures in fluid flows is a standard problem, but normally involves the interaction between a complex flow and a complex structure. Fibers are an example such an interaction: the curvature of a fiber in a fluid flow will correspond to the derivative of the velocity gradient tensor. In simpler flows however, where the velocity gradient tensor remains constant over time, fibers exhibit no deformation, making them no more useful than non-deformable structures. [1]

We have identified a new opportunity for deformable structures by using deformable ramified particles. These particles interactions with linear velocity fields (i.e. flows with constant velocity gradient tensors) are simple enough that we can extract the full velocity gradient from a single deformable particle. Normally, numerous non-deformable particles would be required to extract the same information.

This is of particular value when studying turbulent flows. Turbulent flows exhibit linear behavior at the kolmogorov length, but because this length is often exceedingly small, the seed density (density of tracer particles) needed to re-assemble the full velocity gradient tensor is prohibitively high. Using deformable ramified particles, which extract far more information on a per particle basis, we can make the same measurements while maintaining a low seed density.

The use of ramified deformable particles represents a novel development in the field of fluid dynamics. As a proof of concept, we set out to test them in a simple, well understood fluid flow with well known fluid structure interactions.
Chapter 1- Introduction

Simple shear is one of the most fundamental types of fluid flows, and can be easily created and controlled. To create a shear flow, two parallel plates, separated by a distance $h$, are moved at constant relative velocity parallel to the plates.

![Simple shear flow](image)

**Figure 1.1** A shear flow induced by two parallel plates, with the bottom plate being held fix while the top plate is moved with some constant velocity, $V$.

Simple shear has a linear flow profile, such that the velocity along a line perpendicular to the flow is given by some constant, $k$, multiplied by the distance from the origin. In the above example, the velocity as a function of height is given by the equation $u(y) = ky$ where $k = \frac{V}{h}$.

To create a continuous shear flow, one can run two belts, separated by some distance $h$, at equal and opposite speeds. For simplicity, our apparatus made use of a single belt with a fixed distance between the opposing sides of the belt. Figure 2.5 is a schematic of the apparatus with the corresponding dimensions.

Every linear fluid flow can be decomposed into two components: strain and vorticity. In the case of simple shear, the relationship between strain and vorticity is particularly simple.

Combining equal parts incompressible strain and vorticity yields a perfectly linear shear flow, as illustrated in figure 1.2.
Measuring strain and vorticity independently of one another is quite difficult, as researchers must track many particles in a region of space over which the velocity gradient is nearly constant. In complex flows, this requires a huge seeding density, something like ten particles in a volume \(4\eta\). Given that \(\eta\) is often quite small, a seed density this high would effectively turn a fluid into a soup.

To combat this problem, we sought to design a particle that responds instantaneously and independently to strain and vorticity. In effect, we want the particle to have a unique reaction to the strain rate of the flow, and a unique reaction to the vorticity of the flow, and for this to reaction to occur over a short time scale.

Deformable particles allow us to do that by capitalizing on a key distinction between strain and vorticity: vorticity rotates fluid elements, while strain deforms them. Consequently, a deformable particle will only deform if the flow has a strain component to it, otherwise it will simply rotate, undeformed, with the vorticity of the flow.

By placing our deformable particles in a simple shear flow, which has well understood fluid flow and fluid structure interactions, we can easily compare the measurements extracted by our particle with the actual properties of the fluid.
Chapter 2- Methods: The Experiment

Figure 2.1 is a picture of the deformable particle we designed for our experiment. The particle has three arms, each with a diameter of 3mm and a length of 45mm from the center.

Every arm is attached to the center via a “joint,” at which the particle’s diameter narrows and then widens again. Using such joints has multiple advantages, most significant of which is their ability to behave as an ideal spring. There is a linear relationship between arm deformation and torque on a given arm in the small angle limit, enabling us to easily measure the torque exerted on a given arm. The joints also localize particle deformation, such that all deformation occurs about each arm’s respective joint.
To measure the torque spring constant of each arm, the particle is held fixed with one arm extending out horizontally with a protracted placed behind it. Weights are then incrementally added and the corresponding deformation recorded. For ease of measurement, torque is measured about the center, rather than about the joints. Below are the measured values for the 1.5mm and 1.0mm particle torque spring constants

\[
\alpha_{1.5\text{mm}} = \frac{\tau}{\phi} = \frac{0.0011772\ N\cdot\text{m}}{\text{degree}} \quad \alpha_{1.0\text{mm}} = \frac{\tau}{\phi} = \frac{0.0004782\ N\cdot\text{m}}{\text{degree}}
\]

Two types of particles were used in the experiment: one with a joint that, at its narrowest, measures 1.5 mm in diameter, and another with a joint diameter of 1.0mm.

The particles are mounted on a frictionless air bearing whose axle extends into the center of the flow. The air bearing allows the particle to freely rotate while keeping it in a fixed position. If the particle is exactly centered within the flow, than fixing its position will not affect its behavior. We do this to make sure the particle remains within the field of view of the camera.

Figure 2.2 Particle mount inside of belt
To create our shear flow, we constructed a belt conveyor and submerged it in a glycerol-water solution. Figure 2.3 is a schematic of the shear flow generator.

27"

$D$

4.5"

$68.58 \text{ cm}$

Figure 2.3 The rollers are 4.5" in diameter and are separated by a distance of 27" from axle to axle.

The solution is 90% glycerol by weight, and has a kinematic viscosity of 193 mm$^2$/s. A Cannon-Fenske 350 capillary viscometer was used to measure the kinematic viscosity.

Figure 2.4 A capillary Viscometer
To measure viscosity with a capillary viscometer, one draws the fluid to be measured up to the start mark and then records the time it takes for the fluid to reach the stop mark. This time is then multiplied by a constant (.4586 mm$^2$/s$^2$ in this case) to determine the kinematic viscosity.

The belt in our belt conveyor is made of polyurethane and intended for use in wet environments; it measures 5.3' x 10”.

For the rollers, two 4.5” diameter PVC pipes were cut into 1 ft segments, capped, and mounted on axles made from ⅛” threaded rods. The axles are attached to the conveyor via four sealed, high load, corrosion resistant bearings. The bottom two bearings, which were submerged during the experiment, make use of ceramic balls to prevent rust.

The conveyor frame is made out of anodized extruded aluminum, which is lightweight, corrosion resistant, and capable of supporting large loads. The frame measures 33” x 9” x 18,” and sits in a 40 gallon glass bottom aquarium.

The conveyor is powered by an aerotech BMS60 variable speed brushless DC motor, with a maximum torque of 1.31 Nm. The motor is connected to a motor control unit that enables precise tuning of the motor. It's crucial that the belt runs at a constant, non-variable speed in order to maintain the flow profile.
Centered on the top of the conveyor frame is a half inch thick plexiglass plate, to which the air bearing is attached. The plexiglass plate allows light to pass through unobstructed to the particle. The plate has four slits, one for each bolt, allowing the plate to move horizontally to center the particle in the flow. The plate is leveled before each run to make sure it stays in alignment with the camera.

The air bearing is an OAVTB16i04 - 4mm thrust bearing. More precisely, the 16i04 is actually a “bushing,” as it remains stationary. The air bearing works by creating a thin film of high pressure air that allows the support block, to which the axle is attached, to rotate freely. The only source of friction in the system is air resistance, which at low speeds is negligible. The bearing is connected to the lab’s air supply, which provides a steady 40 psi of pressure.

![Air bearing diagram](image)

*Figure 2.6 Air bearing diagram*

A light diffuser rests atop the aquarium above the plexiglass plate in order to provide a uniformly lit surface. This provides the needed backlighting for tracking the particle.

A camera sits on a tripod underneath the aquarium, and uses a wide aperture 50 mm lens to provide a narrow depth of field. This helps ensure that any debris between the particle and the camera remains out of focus.
The belt is run over a range of speeds, from 0.5 cm/s to 15 cm/s, with increments of 0.5 cm/s. The particle is photographed at 10 Hz for 600 frames (60 seconds.) The images are then run through an image analysis program that measures the angles between each arm. Below is a sample image taken from the 1 cm/s dataset.
The program first draws two circles around the center of particle, and then looks between these two circles for shapes that are black and have more than 4000 connected pixels.

Figure 2.10 Frame from 1cm/s dataset

Once these regions are located, it draws an ellipse around them, and measures their angle relative to the x-axis. Finally it computes the difference between these angles, and plots them for each frame. The coordinate system established below is the one used throughout the experiment.
Making it Work

Particle Design

As is the case with all experiments, a host of unforeseen obstacles presented themselves at every step of the. Below is a short recounting of such obstacles and how we managed to overcome them.

The first challenge was designing a particle that would deform in a predictable and dependable manner. A particle that responded chaotically or inconsistently would be of little experimental use.

We knew that we wanted our final particle to be a deformable triad, since we make frequent use of triads in our other experiments due to their ability to mimic disc in fluid flows. Our task was to design the triads arms to be flexible and dependable.

Our first prototypes were straight cylinders made from a flexible resin. We then placed these arms in a channel flow, and measured their deformations at varying fluid velocities.

![Arm prototype in channel flow](image)
We quickly discovered that a simple, cylindrical arm would not suffice: thick cylinders were not very deformable, and thin cylinders would either break or fail to print properly from the 3D printer. To combat this, we tried a variety of particle designs, shown below.

We settled on a straight cylinder with a single joint elevated from the base. (Second from the left in figure 2.13) The original motivation for adding such a joint was to improve flexibility, but we soon discovered, quite accidentally, that these joints are nearly ideal springs.

We had originally planned to use beam theory to relate the curvature of the arm to the drag force. The joints offered a far more elegant approach as we only needed to measure the arms angle relative to its equilibrium position to uncover the drag force.
Conveyor Construction

While belt conveyors are simple machines, there are many subtitles in their operation that often go overlooked. For the sake of our experiment, the conveyor had to function with a high degree of precision: proper tracking, uniform tension, no slippage, little to no vibration, flat belt faces, and constant belt velocity.

The first issue is belt tracking. If the two rollers of a conveyor system are even slightly out of alignment, the belt will begin to shift towards one side. If left uncorrected, the belt will eventually rub up against the frame of the conveyor, causing it to either fray or break.

To solve this, conveyors often make use of “crown” rollers. These rollers have a slight taper at each edge, such that if the belt begins to slide towards either side of the roller, the taper induces a non-uniform tension in the belt that pulls the belt back towards the center.

![A crown roller](image)

Figure 2.15 Standard beam deformation vs joint deformation

Only one of the conveyor rollers has to be crowned to properly track. A slight taper is all that is needed to make a successful crown roller; we used a ⅛ “ taper across the top and bottom three inches of our roller.

However, even with proper crowning, roller alignment is still vital to proper belt tracking. Consequently, the axle of the crowned roller has to be adjustable, such that either side of the axle can be raised or lowered to adjust the alignment of the roller. Unfortunately, the only way to check belt alignment is to run the conveyor for an extended period of time. If the belt tracks towards either side, the roller alignment needs to be accordingly adjusted.
The extruded aluminum proved invaluable in this process. The connecting brackets can be easily loosened and adjusted, making it easy to tilt the axle of the roller. However, the belt is extremely sensitive to even slight axle adjustments. While the extruded aluminum made large adjustments easy, it was nearly impossible to gradually tilt the roller. Once the belt is brought up to tension, loosening the brackets that hold the axle causes the roller to snap out of alignment. As a result, adjusting the the rollers with the belt under tension required a rubber mallet to nudge the brackets without loosening them. This is not a recommended approach. A proper tensioning device would make use of screw mounts that can make small adjustments to the axle mount while under tension.

Another crucial factor in a conveyor system is the belt seam. To function properly, both ends of the belt need to be properly fastened to one another. If there is even a slight angle to the belt seam, the tension throughout the belt will not be uniform, inhibiting belt tracking and inducing a vibration in the conveyor. Additionally, the seam must be capable of withstanding the tension placed on the belt without sacrificing flexibility.

Without question, properly joining the two ends of our belt was one of the most tedious and difficult task of the experiment. Multiple methods of belt joining were considered, from sewing to mechanical belt fasteners, but ultimately a thin layer of superglue proved the most effective. One side of the belt would be placed atop the other, glued, then clamped in place until cured.

Effectively joining the belt was a matter of trial and error: a thin joint with a small overlap would snap under tension, while a thick joint with a large overlap would be too rigid, preventing it from making it around the roller or causing the conveyor to jolt violently as it went around. The need for the joint to be flexible yet strong, as well as be properly aligned, made for a long and exhausting process of goldilocks, replete with several superglued fingertips.

Roller alignment and belt joining are the fundamentals of any effective conveyor system, but to make a precision conveyor, careful attention needs to be paid to belt velocity and
vibration. Even minor aberrations in conveyor speed can cause irregularities in the shear flow; the same is true for belt vibrations.

A host of factors will influence the belts speed and vibration, (belt seam, roller alignment, belt tension, bearing type, axle rigidity, etc.) making it difficult to isolate the source of either. As a result, some level of belt vibration and speed variation is inevitable, however both should be minimized as much as possible.

Regarding speed, the motor is the most important factor. Ideally, the amount of torque required to power the belt will remain constant. In reality, irregularities in the roller profile, bearing friction, belt tension and the likes will affect the amount of torque needed to maintain constant belt speed.

The belt seam is one example of such irregularities: the seam is stiffer than the rest of the belt, making it more resistant to bending and thus harder to pull across a roller. As a result, every time the seam reaches a roller, the amount of torque needed to maintain belt speed increases slightly. The best solution for this is a motor capable of varying its power, and thus torque, to maintain a set speed.

To do this, the motor (or its corresponding motor control unit) must make continuous measurements of its velocity, and adjust its power output whenever it detects a deviation. Depending on the application, the motor is “tuned” to be more or less sensitive to deviations. Motor tuning is largely a trial and and error process. If the motor is too sensitive, it will end up overcompensating for tiny aberrations, leading to a halting stop and go motion rather than a smooth rotation. If it’s not sensitive enough, then the motor will fail to compensate for deviations in velocity, leading the conveyor to rotate at an inconsistent speed.

Belt vibration is more difficult to control than belt velocity. While successful motor tuning can compensate for most irregularities in belt speed, belt vibrations can’t be controlled, only mitigated. To make matters worse, the belt becomes more susceptible to vibration as the tension increases. Reducing tension is not an option, as tension is necessary for belt tracking
and keeping the belt profile flat. (Remember, simple strain requires two parallel plates, so any bend or fold in the belt will disrupt the flow.)

However, if we place large flat plates alongside the belt, a negative pressure region will be created between the belt and the plate as the belt moves past, in effect pulling the belt towards the plate and dampening any residual vibrations. Additionally, the plates also help maintain a flat belt profile.

While it proved impossible to eliminate all vibrations, careful roller alignment, a small flexible seam, and the addition of dampening plates kept the vibrations small enough that they had no effect on the experiment.

Belt slippage was another problem we faced, though only at higher speeds. Normally, when the belt makes contact with the roller, the fluid between the roller and the belt is "squeezed" out of the way. At higher speeds though, a thin film of fluid begins to form between the belt and the roller, causing the belt to slide down the roller. To counter this, several holes were drilled in each roller, allowing the belt to push the fluid out through the roller and prevent belt slippage.

**Particle Mounting & Centering**

Due to the uniform nature of simple shear, the vorticity and strain will be identical at any point in the flow, meaning our particle will experience the same deformation and solid body rotation regardless of location.

At the center of the flow, $x=0$, the net fluid velocity is zero. Moreover, the fluid velocity some distance from the origin, $x=d$, is equal and opposite to the fluid velocity at $x=-d$. (See figure 1.2) The symmetry of the flow means that as long as the particle remains perfectly centered, the particle will stay there indefinitely. Nevertheless, even a tiny perturbation away from the center will cause the particle to start moving, eventually carrying it out of view of the
camera. To prevent this, we hold the particle at the center of the flow while allowing it to rotate in place.

So long as the particle is centered in the flow and can rotate without resistance, fixing its position has no affect on its behavior. If the particle is not centered in the flow though, then the flow will try and carry the particle away, exerting an equal and opposite force on the particle to keep it in place. This will affect the particles behavior, as a given arm will experience a higher velocity when its on one side of the flow than the other. As a result, the arm deformation will be more pronounced when the arm is on a particular side of the flow.
Looking at the above figures, we can clearly see how the arm deformation for arm 1 will be identical in figure 2.16 and figure 2.17. The reason for this is obvious: in 2.16 and 2.17, from the particle’s reference frame, the flow field is identical when rotated 180 degrees.

When we shift the particle to the right, however, we can see that arm 1 will have a different deformation in figure 2.18 than in figure 2.19: the arm deformation will be far larger in 2.18 than 2.19. In this configuration, the particle’s reference frame sees the flow field is not identical when rotated 180 degrees.

We can easily see this effect when we look at a test run with the particle center shifted in the positive $x$ direction.

![Relative Arm Angles](image)

**Figure 2.20**

Note the asymmetry in deformation amplitude between arm 1 and arm 2. At frame 50, arm 1 is in the same position as in figure 2.18, and at frame 125 arm 1 is in the same position as in figure 2.19. The deformation of arm 1 is substantially larger than in frame 50 than 125.
Keeping the particle centered in the flow required several modifications to apparatus. Originally, the particles were mounted on a stainless steel rod with a sharpened tip, which kept the torque due to friction on the axle at a minimum. However, the axle would wander as it rotated, constantly shifting the center of the particle. This, combined with the appeal of an even lower friction system, motivated us to purchase the air bearing mentioned above.

Using the air bearing our particle remained fixed. Nevertheless, we were still measuring asymmetries in our deformation. The sensitivity of the particle to its location was so great that if the particle was off center by a few pixels, we would see asymmetric deformation. To make finer adjustments to the location of the particle mount, we modified the plexiglass plate by adding bolt slits that allowed for horizontal adjustments to its location. As we will see in our results section, this modification proved successful in producing the symmetric deformation amplitudes we expected.
Deformation Model

According to resistive force theory, the differential viscous drag forces acting on a small segment of a slender body are [2]

\[ dF = 2a \cdot C_d \cdot u_\perp(r) \, dr \]
\[ df = a \cdot C_d \cdot u_\parallel(r) \, dr \]

Where \( df \) is represents the force acting parallel to the rod and \( dF \) represents the force acting perpendicular to the rod. The constant \( a \) is the width of the rod, \( C_d \) is the drag coefficient, \( r \) is the vector along the axis of the slender body, and \( u(r) \) is the relative velocity between the segment and the fluid as a function of \( r \), the distance from the center of the flow.

The torque comes from the component of the velocity perpendicular to the arm, \( u_\perp(r) \), allowing us to eliminate the cross product. Thus the torque exerted on a small segment of the rod by the relative fluid velocity is

\[ d\tau = r \cdot dF \]

The simplest model of particle motion assumes the rotation matches the vorticity of our flow, meaning the relative velocity due to vorticity is always zero. [4] As a result, the relative velocity between the particle and the fluid is solely a result of the strain rate of our flow.

In a strain flow, \( v_x = \frac{r}{2} y \) and \( v_y = \frac{r}{2} x \), which we can rewrite in polar coordinates as \( v_x = \frac{r}{2} \cos(\theta) \) and \( v_y = \frac{r}{2} \sin(\theta) \), where \( \theta \) represents the angle between a given arm and the horizontal. We want to know \( u_\perp \), the relative fluid velocity that is perpendicular to the arm, so we decompose the velocity into parallel and perpendicular components relative to the arm.
The component of \( v_y \) that is perpendicular to \( r \) is given by \( u_{y\perp} = v_y \cos(\theta) \), which becomes \( u_{y\perp} = r \cdot \cos(\theta) \cos(\theta) \) in polar coordinates. The component of \( v_x \) that is perpendicular to \( r \) is given by \( u_{x\perp} = -v_x \cos(90 - \theta) \), since the angle between \( v_x \) and \( r \) is given by \( (90 - \theta) \), this becomes \( u_{\perp} = -r \sin(\theta) \cos(90 - \theta), \) or \( u_{\perp} = -\frac{1}{2} \cdot r \sin(\theta) \sin(\theta) \). The negative sign accounts for the fact that, relative to the arm, \( u_{x\perp} \) is negative. Combing \( u_{x\perp} \) and \( u_{y\perp} \) yields

\[
u_{\perp}(r) = \frac{1}{2} r [\cos^2(\theta) - \sin^2(\theta)]
\]

Plugging the above equation into our equation for \( dF \) we get

\[
dF = 2a \ C \ d \ \frac{1}{2} r [\cos^2(\theta) - \sin^2(\theta)] \cdot dr
\]

Which makes the torque per unit length
\[ d\tau = r \left\{ 2a C_d \frac{k}{2} r \left[ \cos^2(\theta) - \sin^2(\theta) \right] \right\} dr \]

\[ d\tau = 2a C_d \frac{k}{2} \left[ \cos^2(\theta) - \sin^2(\theta) \right] r^2 dr \]

Integrating over the length of the arm, \( R \), with respect to \( dr \)

\[ \tau = \int_0^R 2a C_d \frac{k}{2} \left[ \cos^2(\theta) - \sin^2(\theta) \right] r^2 dr \]

\[ \tau = 2a C_d \frac{k}{2} \left[ \cos^2(\theta) - \sin^2(\theta) \right] \frac{R^3}{3} \]

Our particle has three arms, each separated by \( \frac{2\pi}{3} \) radians. Therefore, the torque on arm 2 and 3 will be

\[ \tau_2 = 2a C_d \frac{k}{2} \left[ \cos^2\left(\theta + \frac{2\pi}{3}\right) - \sin^2\left(\theta + \frac{2\pi}{3}\right) \right] \frac{R^3}{3} \]

\[ \tau_3 = 2a C_d \frac{k}{2} \left[ \cos^2\left(\theta + \frac{4\pi}{3}\right) - \sin^2\left(\theta + \frac{4\pi}{3}\right) \right] \frac{R^3}{3} \]

There is, however, a problem with the above equations: they assume our arms are always offset by \( \frac{2\pi}{3} \) radians. Therefore to get an accurate measurement of the torque we have to include each arm’s deviation from equilibrium, which we’ll represent with \( \phi \). The equations for torque now become

\[ \tau_1 = 2a C_d \frac{k}{2} \left[ \cos^2\left(\theta + \phi_1\right) - \sin^2\left(\theta + \phi_1\right) \right] \frac{R^3}{3} \]

\[ \tau_2 = 2a C_d \frac{k}{2} \left[ \cos^2\left(\theta + \frac{2\pi}{3} + \phi_2\right) - \sin^2\left(\theta + \frac{2\pi}{3} + \phi_2\right) \right] \frac{R^3}{3} \]

\[ \tau_3 = 2a C_d \frac{k}{2} \left[ \cos^2\left(\theta + \frac{4\pi}{3} + \phi_3\right) - \sin^2\left(\theta + \frac{4\pi}{3} + \phi_3\right) \right] \frac{R^3}{3} \]

To solve for a given arm’s deviation from equilibrium, \( \phi \), we divide the torque by the arms torque spring constant, \( \alpha \).

\[ \phi = \frac{\tau}{\alpha} \]

Plugging the above equation into our torque equations yields

\[ \phi_1 = \alpha \left\{ 2a C_d \frac{k}{2} \left[ \cos^2(\theta + \phi_1) - \sin^2(\theta + \phi_1) \right] \right\} \frac{R^3}{3} \]
These are transcendental equations for each $\phi_i$. To solve for $\phi_i$ we use an iterative method.

We know $\phi$ is small relative to $\theta$, so we’ll approximate $\tau$ with $\phi = 0$, solve for $\phi$ using this approximation of $\tau$, then put $\phi$ back into our equation to get a better approximation of $\tau$.

It’s worth taking a moment to visualize how this approach works. Imagine we place our particle in some given strain flow, with all the arms located in their equilibrium positions, (i.e. unbent) and hold the particle there. A torque will be exerted on the arms by the strain flow, leading them to deform from equilibrium in response. Now, with the arms deformed, the amount of torque experience by the arm will change. We recalculate the torque, taking into account the previously solved for deformation, and calculate a new deformation. As we’ll see in the results section, this approach provides us with a fairly accurate model of behavior for particle deformation.
Chapter 3 - Results

In this section, we demonstrate the viability of deformable particles in extracting the velocity gradient tensor. As predicted earlier, the particle’s rotation rate is shown to be unaffected by their deformation in the non-turbulent, low deformation limit. As a result, our particles can independently extract the vorticity and strain components of the velocity gradient tensor, as each component evokes a unique response from the particle. More succinctly, the vorticity only affects the rotation rate, while the strain only affects the arm bending, allowing us to determine the vorticity without having to worry about the effects of the strain, and vice versa.

Our particles also allow for near instantaneous measurements of both components: one only need to know the rotation rate and deformation over a short time span to decompose the velocity gradient tensor. In a turbulent flow, the velocity gradient tensor changes over the same time scale that a particle rotates, so turbulent applications need to be able to extract the velocity gradient tensor from short time measurements and not from following a full rotation. In turbulent flows the time scale of change for the velocity gradient tensor is the kolmogorov time, as is the time scale associated with the magnitude of the velocity gradient tensor.

Below are the relative arm angles, along with the modeled relative arm angles, for our 1.5mm particle at belt speeds of 2.5 cm/s, 5.0 cm/s and 7.5 cm/s, respectively.
Figure 3.0 - velocity profile

Figure 3.1

Figure 3.2

Figure 3.3

Figure 3.4
Figure 3.6 is a graph of the angular velocity of our 1.5mm joint particle as a function of belt speed. As predicted, in the low speed limit, the particle has the same solid body rotation rate as a stiff armed, rigid particle. Moreover, the rotation rate scales linearly with the magnitude of vorticity.

Figure 3.7 shows the amplitude of deformation as a function of belt speed. The arm deformation scales linearly with the shear rate of our flow, though this relation begins to break down as we reach higher speeds.

Comparing figure 1 and figure 2 allows us to see the unique effects vorticity and strain exert on the particle. Up until 6 cm/s, the deformation of the particle has no effect on the solid body rotation rate, even as the deformation becomes increasingly pronounced.

It’s worth noting that this relation only holds true in the small deformation limit. To illustrate this phenomena we use a far more flexible particle, with a 1.0mm joint, which achieves higher amplitudes of deformation at lower speeds than our previous particle.
In figure 3.8, we again see the linear relationship between the solid body rotation rate and the vorticity of the shear flow. However, the relationship between the two decouples at a lower belt speed than our 1.5mm joint particle, namely 4 cm/s as opposed to 6 cm/s.

Figure 3.9 re-illustrates the linear relationship between the amplitude of deformation and shear rate, though as was the case with the solid body rotation rate, this relationship becomes nonlinear at a lower belt speed with the 1.0mm particle than with the 1.5 mm particle.

To more clearly compare and contrast the behaviors of the two particles, the above plots are repeated below, this time with both particles plotted together.
The above effects are easily understood when we consider the isotropy of the particle. As long as the particle remains largely isotropic, the solid body rotation rate will be unaffected by particle deformation. However, as we run our experiment at higher speeds, and increase the flexibility of our particles, the isotropy of our particles begins to break down, in turn causing the effects discussed above. We can clearly see the loss of isotropy when we look at the 1.00mm particle at two different speeds. Figure 3.12 & 3.13 show the particle at 1 cm/s and 15 cm/s, respectively.
When it comes to measuring the strain and vorticity components of the flow, each particle offers unique advantages. While the thicker joint particle allows us to measure the coefficients over a wider range of speeds, the thinner joint particle is significantly more responsive to slight variations in the velocity gradient.

To extract the vorticity component of our flow, all we need to know is the solid body rotation rate. Figure 5 demonstrates that the measured vorticity component, does, in fact, correspond with the measured vorticity in the low deformation, low speed limit.

To extract the strain rate component of the flow, we measure the torque exerted on each arm, as the torque on correlates to the strain rate. (Remember, in the particle's reference frame, the flow has no vorticity, because its rotating at exactly the same rate as the fluid, meaning any torque on an arm must originate from the strain rate.)

The torque exerted on a given arm due to the strain rate is given by the equation:

\[ \tau_1 = 2a C_d \tau_{strain} \left[ \cos^2(\theta) - \sin^2(\theta) \right] \cdot \frac{R^3}{3} \]

Since our joints are nearly ideal springs, a linear relationship exists between arm deformation and torque. So to determine the torque on a given arm, we divide the amount of deformation by the torque spring constant.

\[ \tau_1 = \frac{\phi_1}{\alpha} \]

In order to solve for \( \tau_{strain} \) using our measured arm deformation \( \phi_1 \), we rearrange equation 1.1 and sub in \( \frac{\phi_1}{\alpha} \) for \( \tau_1 \).

\[ \tau_{strain} = \frac{\frac{\phi_1}{\alpha}}{2a C_d \left[ \cos^2(\theta + \phi_1) - \sin^2(\theta + \phi_1) \right]} \cdot \frac{3}{R^3} \]

Once the drag coefficient \( C_d \) is known, we can use equation 1.3 to extract \( \tau_{strain} \) in any arbitrary flow.
Fortunately, in our experiment the value of $\tau_{\text{strain}}$ is already known to be $\frac{1}{2}$. To solve for $C_d$ then, we measure the arm deformation at several different values of $\theta$ and use a least squared fit to match it to the model derived in chapter three and extract the corresponding drag coefficient. Figure 9 below is a plot of the measured drag coefficient vs $\tau_{\text{strain}}$. As with our past plots, the 1.0mm & 1.5mm particles show strong agreement in the low speed, low deformation limit, but experience significant deviations outside that regime.

![Figure 3.14](image)

While simply knowing the amplitude of deformation may suffice in extracting $\tau_{\text{strain}}$, in order to arrive at the orientation of the strain flow we must also incorporate the particles orientation. Certain orientations will correspond to a maximum in the relative angle between the two arms, while other orientations will correspond to a minimum.

Figure 3.15 shows the orientation for which arms 1 and 3 will have the largest angle between them, while figure 3.16 shows the orientation for which this angle will be at a minimum.
Looking at figures 3.15 & 3.16, we can clearly see that the minimum angle between arms 1 & 3 occurs when the angle bisector lies along the extensional eigenvector of the strain, while the maximum angle between 2 & 3 occurs when the angle bisector lies along the compressional eigenvector of the strain. Therefore we can determine the orientation of the strain rate by calculating the direction of the angle bisector between arms 1 & 3 when their relative angle is at a minimum.

From figure 3.15 we can tell that the compressional eigenvectors of the strain flow should be at angles of 135 and 315 degrees, respectively. The angle bisector of arms 1 and 3 when their relative angle is at a maximum always lies within a few degrees of these orientations.

From figure 3.16 we can tell that the extensional eigenvectors of the strain flow should be at angles of 45 and 225 degrees, respectively. The angle bisector of arms 1 and 3 when their relative angle is at a maximum always lies within a few degrees of these orientations as well.
### Orientation of Arm 1 and 3 with Maximum Relative Angle

<table>
<thead>
<tr>
<th>Frame</th>
<th>Arm 1 orientation</th>
<th>Arm 3 orientation</th>
<th>Angle bisector</th>
</tr>
</thead>
<tbody>
<tr>
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<td>72.04</td>
<td>134.52</td>
</tr>
<tr>
<td>88</td>
<td>15.78</td>
<td>251.75</td>
<td>313.77</td>
</tr>
<tr>
<td>159</td>
<td>195.69</td>
<td>71.22</td>
<td>133.455</td>
</tr>
<tr>
<td>230</td>
<td>14.69</td>
<td>250.10</td>
<td>312.4</td>
</tr>
</tbody>
</table>

### Orientation of Arm 1 and 3 with Minimum Relative Angle

<table>
<thead>
<tr>
<th>Frame</th>
<th>Arm 1 orientation</th>
<th>Arm 3 orientation</th>
<th>Angle bisector</th>
</tr>
</thead>
<tbody>
<tr>
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<td>174.25</td>
<td>231.97</td>
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<tr>
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<td>104.02</td>
<td>351.068</td>
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<tr>
<td>197</td>
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</tr>
<tr>
<td>268</td>
<td>105.35</td>
<td>349.65</td>
<td>47.5</td>
</tr>
</tbody>
</table>

Unfortunately, this approach requires particle measurements over a full rotation to determine the eigenvectors of the strain. Due to time constraints, the model needed to calculate the eigenvectors of the strain flow has not been fully developed, though early results are promising. We see a unique set of relative arm angles for every orientation, which implies that the particles particular deformation is specific to a given orientation in the strain flow.

**Conclusion**

As a proof of concept, our experiment was highly successful. Our ramified deformable particles were shown to have unique, independent and immediate reactions to the strain and vorticity of our shear flow. The particles also experienced pronounced deformations despite being placed in a linear flow profile. Moreover, we were able to extract not only the magnitude of the vorticity and strain, but also the eigenvectors of the strain as well.

The flow structure interaction of our particles also proved to be far simpler than other deformable particles, allowing for easier and more accurate measurements. The near ideal spring behavior of our arms was integral to this.
The belt conveyor succeeded in generating a continuous, laminar shear flow. Our air bearing allowed the particle to rotate in place while experiencing little to no friction, and our careful alignment of the particle with the center of the flow essentially negated the effects of being held in place while maintaining symmetric deformation amplitudes.

The model developed for our particle showed strong agreement with our data. Over a wide range of speeds, the solid body rotation rate was well within in a few percentage points of the expected rotation rate. Additionally, the deformation amplitude of our rarefied arms scaled linearly with belt speed, and showed a similarly strong agreement with predicted deformations.

Rarefied deformable particles have significant potential to enhance the information that can be extracted from future experiments. Moving forwards, we intend to develop three dimensional rarefied particles capable of reconstructing the velocity gradient tensor in three dimensional flows.

References

[3]