ASYMMETRIC TRANSMISSION IN PHOTONIC STRUCTURES WITH PHASE-CHANGE COMPONENTS

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This thesis is dedicated to all of the awesome teachers I have had in my life.

Thank you so much for helping me get to where I am today.
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ABSTRACT

Phase-change materials (PCM) undergo a phase change transition from an insulating to a metallic phase via a physical mechanism like light-induced heating, external electric or magnetic field etc. Over the years, they have been proven a useful element in many photonic applications ranging from re-writable DVDs and all-optical modulators, to power limiters and switches. In most of these applications PCMs have been considered as stand-alone structures and not as an element of a photonic circuit like a photonic crystal. In this thesis we consider how the transport characteristics of such a PCM element are affected when it is a part of a photonic crystal. Using a numerical example, we show that the transmittance can become highly asymmetric within a broad range of light intensities. This effect can be utilized for directional light transmission (isolation), asymmetric optical limiting, or power switching.
INTRODUCTION

Controlling the transportation of energy, matter and especially information has been of utmost interest to humans for thousands of years. We started with letters sent by horseback from city to city and progressed to carrier pigeons. As science matured we began sending information via electrons through wires. When scientific discoveries transcended wires and began to send information through free space via electromagnetic waves of various frequencies, the transport of information was changed forever. Ongoing research aims to further enhance our grasp and control of the amplitude, phase and direction of light transport; this is the current field of photonics research. In this study, we aim to provide a dynamic way of controlling the direction and frequency of transported light.

This thesis begins with a review of important background knowledge (Chapter 1.1-1.3) that is intended to motivate the whole study. Then, we take a close look at the physics of non-reciprocal transport in electrodynamics (Lorenz reciprocity Theorem) and its applications to photonic limiters(Chapter 1.4). To this end, we derive the reciprocity theorem and identify the basic assumptions behind it. Next, we analyze the important aspects of the theory of photonic band gaps. We explain their presence in the case that the translational invariance in space is destroyed and spatial periodicity is imposed. For educational purposes,
we present the band-gap formation using a toy model consisting of delta-like scatterers placed in an equal distance from one-another (Chapter 1.5.1). This motivates us to generalize the development of band-gap theory for more realistic scenarios which are frequently found in the optics framework (Chapter 1.5.2).

Finally, we review the different types of phase change materials (associated to photonic applications) along with a detailed explanation of the simulation methods used in this research effort (Chapter 2). Chapter 3 contains the results of our study in the format of a journal publication.
1.1 Introduction to Limiters

To fully understand this study, it is important to understand the mechanism and applications for both optical limiters as well as for nonreciprocal electromagnetic transport. This chapter consists of an overview of these two concepts and the important applications for each. We also explore and analyze the photonic band gap theory and resonant defect transport within periodic dielectric media.

A limiter is a power filter that protects sensitive equipment or sensors (like the eye, antennas, radar systems etc) from incoming high-power signals. Obviously, the first requirement is that the transmitted power that is allowed to transmit through
the limiter in all occasions must be less in power than the sensor damage threshold; this is how a limiter serves its protective purpose. Specifically, in the low power regime, a limiter must act as a linear system i.e. the transmitted electromagnetic power must be proportional to the incident one. At a certain point the input power reaches the limiting threshold. For input powers greater than this limiting threshold, the amount of power transmitted has to be suppressed. In this regime, the limiter acts as a non-linear suppressive filter. Eventually as the incident power is increased even further, the limiter damage threshold is reached; this is the point where the limiter itself is irreversibly damaged. The ratio between the limiter’s damage threshold to the limiting threshold gives the figure of merit for the limiter. It is typically referred to as the limiter’s dynamic range and is presented in Figure 1. When designing a limiter it is essential to expand upon the dynamic range so that the limiter can be reused. Often times high absorption in a limiter results in the heating and melting of the device; such limiters are called sacrificial limiters because when they are damaged they are ‘sacrificed’ and must subsequently be replaced. In order to expand upon the dynamic range it is very beneficial to make the limiter highly reflective at high powers[4]. This has been experimentally realized recently in Ref. 4.
1.2 Applications of Limiters

Limiters have numerous uses both in defense and commercially. There are many instances where scientists and military assets are exposed to potentially harmful lasers. These lasers can both harm human eyes as well as damage sensors for communication. Lasers can also jam optical systems by an influx of too much energy or light, thus rendering the device temporarily unusable [2]. All of these situations require protection of either a sensor or of the human eye. In the case of protecting the human eye, there are many factors to consider. The human eye can be damaged by a large range of electromagnetic frequencies, while also needing to see outwardly in the visible range. The task of protecting the human eye necessitates that at low powers the eye must be able to see outwards but when a laser is incident on the eye the excess power must be limited. Obviously, allowing high energies into the eye can result in blindness and damage. To be specific, a limiter cannot allow irradiances of greater than 0.5 µJ/cm² or this will result in irreversible damage to the eye. Another consideration when implementing a limiter to protect eyes is that the field of vision must not be decreased significantly. The limiter mechanism used to protect the human eye is extremely important for maintaining a soldier’s ability to see and operate while still protecting their eyes. For example, in the case of two-photon absorption/carryer defocusing, part of the limiting mechanism necessitates that the incident beam is defocused and spread out.
thus decreasing the energy density and preventing damage to the eye. However, in this case the defocusing of the incident light can ‘jam’ the vision of the person because the incident light is no longer an accurate representation of the external surroundings [2]. This must be avoided in any context where even momentary jamming of vision can lead to a dangerous situation.

In protecting an optical sensor such as a receiver, the task is similar because transmittance must still be high enough at low powers to allow for full functionality of the sensor, however when high powers are directed at the sensor it must be able to be protected. The limiter must not inhibit functionality of the sensor which is often difficult in practice because of the transmission loss often associated with limiters even at low powers[2]. Another main application is in the protection of satellite communication devices. The antenna on a satellite must be protected from high power incidence but must be able to communicate at lower powers.

1.3 Limiting Mechanisms

Any material or liquid crystal with irradiance dependent material properties can potentially be used as an optical limiter. Because passive limiters are of the most relevance to this study, we will begin by reviewing the basic mechanisms for passive limiting. Passive limiting is when the incident electromagnetic radiation itself causes the limiting mechanism to occur. One of the most common mechanisms for irradiance dependent optical property change is two-photon
absorption (2PA); with 2PA, a material becomes highly absorbing as the incident irradiance increases. There are many semiconductors that exhibit two photon absorption, such as GaAs, CdS and CdSe; the basic idea is that for low power radiation they are highly transmissive and demonstrate minimal loss and as the intensity of incident waves is increased 2PA-carriers are generated and they absorb power thus limiting the amount of energy that can transmit through the material. The 2PA carriers also result in self-refractive properties thus defocusing incident light and providing another mechanism for limiting[3]; the defocusing of light decreases the overall energy density the sensor is exposed to by spreading it over a larger spatial area. A problem with the 2PA mechanism is that it requires relatively high intensity light to generate the 2PA carriers. This makes limiters made with this mechanism restricted in application because they cannot protect sensors that have a very low damage threshold, such as on-chip devices. By adding alternating layers of dielectrics on either side of a material with 2PA, the structure can be made more sensitive to incident power by increasing the q-factor of a localized resonant mode within a cavity. This results in greater absorption of light for lower incident power values, making the structure a more effective limiter for low power applications[4]. By adding these alternating layers of dielectrics to either side of the 2PA material Bragg mirrors are created on either side of the defect cavity. When the cavity itself becomes highly absorbing, this destroys the resonant mode associated with this cavity and results in a highly reflective behavior thus expanding upon the dynamic
range by orders of magnitude[4]. Using Bragg mirrors is an effective way to tune the limiting threshold depending on the necessary application because as you add more and more layers to a Bragg mirror you can concentrate more of the light in between the mirrors in the cavity, thus sensitizing the whole structure.

Phase change materials also offer many possibilities for limiting mechanisms. For example, vanadium dioxide goes through a thermally induced structural phase change from a dielectric to a metal which results in a massive change in optical properties[12]. These changes allow for transmission of light in it’s dielectric phase and then complete absorption or reflection in it’s metallic phase. This is the primary mechanism we will be exploring in this study. Using one simple layer of PCM will suffice for basic passive limiting properties. This is passive limiting because the incident power itself will lead to the heating of the PCM and thus the eventual phase change. The problem is that the PCM will continue to heat up and eventually reach it’s damage threshold and need to be replaced. This can be avoided by surrounding the material with Bragg mirrors and thus creating a cavity mode that can be destroyed in a reflective way similar to the way described previously for the 2PA case. In this study we examine a limiter which utilizes a defect cavity and two Bragg mirrors on either side of it, however we seek to combine this concept with a nonreciprocal transport mechanism.
1.4 Reciprocal and Non-reciprocal Transport

There are realistic situations where a limiter needs to have limiting threshold activation levels which are different in each propagation direction. A typical example to consider is in Radar systems, where the outgoing signal needs to be strong but at the same time we need to protect the system from strong incoming signals. It is therefore useful and often necessary to realize non-reciprocal limiter structures. As a first step, we explain the Lorentz reciprocity principle and how this reciprocity can be broken.

The Lorentz reciprocity theorem states that regardless of whether a structure has spatial asymmetries or is perfectly symmetric, transmission through the structure will be identical when sending incident waves from one side versus the other. This means that if you have two-layer structure of two different materials with different optical properties and a port to enter from material A and a port to enter from material B, the transmission will be the same regardless of which side the light enters the structure [5].

Lorentz reciprocity can be analytically derived from Maxwell’s curl equations[23]. Let’s consider two sources $J_1, J_2$ and also the electric and magnetic fields produced by those sources $E_1, H_1$ and $E_2, H_2$. These fields can be considered to be measured at the opposite sources, so $E_1, H_1 (E_2, H_2)$ is measured at the location
of $J_2(J_1)$. Starting from the curl equations where $j$ is current density, $\omega$ is angular frequency, $\mu$ is magnetic permeability, $\varepsilon$ is relative permittivity:

1. $\nabla \times E_1 = -j\omega\mu H_1$  
2. $\nabla \times H_1 = j\omega\varepsilon E_1 + J_1$  
3. $\nabla \times E_2 = -j\omega\mu H_2$  
4. $\nabla \times H_2 = j\omega\varepsilon E_2 + J_2$

Now we can use the left hand side quantity and it’s vector identity expansion in Equation 5 to get a useful relationship between sources and fields.

\[ \nabla \cdot (E_1 \times H_2 - E_2 \times H_1) \]  

(Eq 5) $= (\nabla \times E_1) \cdot H_2 - (\nabla \times H_2) \cdot E_1 - (\nabla \times E_2) \cdot H_1 + (\nabla \times H_1) \cdot E_2$  

(Eq 5) $= j\omega\mu H_1 \cdot H_2 - j\omega\mu H_1 \cdot H_2 + j\omega\varepsilon E_1 \cdot E_2 - j\omega\varepsilon E_2 \cdot E_1 + J_1 \cdot E_2 - J_2 \cdot E_1$  

\[ \text{(Eq 5)} \]
In Equation 7, we can see that the magnetic field and electric field dot product terms will cancel because the fields are commutable and so the order in which we dot them does not matter. This leaves us with just the source terms at the end of Equation 7[22].

\[ \nabla \cdot (E_1 \times H_2 - E_2 \times H_1) = J_1 \cdot E_2 - J_2 \cdot E_1 \]  \hspace{1cm} (8)

Now, if we can get the left hand side of the equation to be equal to zero, we can show Lorentz reciprocity in a very simple form. Assuming we are far away from a source, such as a sphere of infinite radius surrounding an antenna. We know that \( E \times H \) will point in the radial direction of the sphere \( \hat{r} \) and that in this case \( H = (\hat{r} \times E) \).

\[ (E_1 \times H_2 - E_2 \times H_1) \cdot \hat{r} = \eta H_1 \cdot H_2 - \eta H_2 \cdot H_1 = 0 \]  \hspace{1cm} (9)

Equation 9 shows us that we can set the left hand side of Equation 8 to zero and we get a relationship between the sources and the electric field at those sources.

\[ J_1 \cdot E_2 = J_2 \cdot E_1 \]  \hspace{1cm} (10)
We can think of $J_1$ as an input source and it’s corresponding E field $E_1$ as the output or transmission. We see that this relationship necessitates reciprocal transport if we send electromagnetic sources from two opposite directions into a structure.

It is possible to break such reciprocity and various researchers have come up with ways to do it. The breaking of this reciprocity has allowed for important applications such as optical isolators, valves and circulators which are essential for on-chip photonics and optical circuits. These applications are essential when trying to make completely light-based circuits as well as for many experimental research set ups[23].

The main way reciprocity has been broken for electromagnetic waves is by using the Faraday magneto-optical effect. From one direction the light is rotated by the magnetic field so that the polarizer blocks transmission, and from the other direction the light is rotated so that the polarizer allows transmission, due to the directional dependence of rotation in a magnetic field. The polarizer blocks the transmission in this case and is potentially subject to high levels of heating. This is a common mechanism used for optical isolation applications[6].

Another mechanism for breaking reciprocity is by using a time-modulated device, which is an active scheme. In one example, the resonant frequency of three coupled resonators is temporally modulated with a phase difference[7] for each resonator, thus creating conditions for unidirectional transport. Yet another
mechanism for asymmetric transport of light is a structure that is spatially asymmetric while incorporating materials with nonlinear irradiance dependent optical properties. The spatial asymmetry within the structure gives rise to electric field profiles that are directionally dependent. This directional dependence of field profiles makes it so the structure has directionally dependent material properties and thus can exhibit asymmetric transport, optical isolation etc. This is the primary mechanism we explore and utilize in this study.

1.5 Periodic Photonic Crystals: Bandgaps and Defect Modes

In order to grasp the resonant transmission, limiting, and unidirectional transport properties that we explore in this study, it is essential to understand how periodic photonic crystal structures work. By stacking multiple layers of two alternating dielectric materials (with different real permittivities), a periodic photonic crystal is formed (Figure 2). This leads to alternating stop bands, in which waves are reflected (as in a Bragg mirror), and transmission bands, in which waves are transmitted through. This simple structure leads to robust control of light propagation including important applications such as optical filtering. But how exactly does the photonic band gap come to be, and how can defect resonant transmission be accomplished in such a structure?

1.5.1 Quantitative Derivation and Description of Photonic Band Gaps
The immergence of allowed frequency bands in semiconductor electronics, periodic photonic crystals and all other periodic systems arises from the periodic potentials. In the case of periodic photonic crystals the alternating layers of dielectrics with different real permittivities give rise to a periodic potential. In the case of semiconductor electronics, the periodic potential comes from the periodic spacing of atoms within a semiconductor which have periodic nuclear attractive forces. The mathematics to describe the emergence of bands in each case is the same[14]. In this section we seek to first describe the emergence of bands quantitatively using a simple mathematical model. After this, we will give a more qualitative description that is more specific to periodic photonic crystals, as well as elaborating on the case for defect resonant transmission. We first start with the definition of a periodic potential where $a$ is some distance.

**Figure 2.** A 1-D stack of alternating dielectrics forming a periodic photonic crystal structure. The two layers repeat themselves at intervals of distance $= a$. [14]
\[ V(x + a) = V(x) \]  \hspace{1cm} (11)

According to Bloch’s theorem, the solutions to Schrödinger’s equation (Equation 12) must satisfy Equation 13 for a periodic potential [17].

\[ -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V(x)\psi = E\psi \]  \hspace{1cm} (12)

\[ \psi(x + a) = e^{ikx}\psi(x) \]  \hspace{1cm} (13)

The most simple way to solve a periodic potential system is by using a long sequence of evenly spaced delta-function potentials often referred to as a Dirac comb (Figure 3). \( K \) is a real constant that emerges from the boundary conditions of the problem. These delta potentials are analogous to alternating dielectric permittivities in a periodic photonic crystal.
Figure 3. A string of delta potentials form a diraq comb, a simple periodic potential system we can solve analytically.

Since it is impossible for a solid to go on forever and the edges of such a system would ruin the periodicity that we define above, we can wrap the x axis into a circle and connect the edges after a very large amount of periods \((N=10^{21})\) which gives us the following boundary condition:

\[
\psi(x + Na) = \psi(x)
\]

(14)

And considering Equation 13 we see that

\[
\psi(x) = e^{iNKa}\psi(x)
\]

(15)

Solving for the constant \(K\) we see that \(K\) must be real and it equals \(2\pi n/Na\) where \(n\) ranges from \(-N\) to \(N\). This potential can be defined mathematically when we consider the axis to wrap around in a circle, making it so the \(N\)th spike below
appears at $x=-a$ [17]. $\alpha$ is the delta function strength parameter.

$$V(x) = \alpha \sum_{j=0}^{N-1} \delta(x - ja) \quad (16)$$

Between delta functions (from $x=0$ to $x=a$ for example) the potential is zero and we can reduce Schrodinger’s equation to the differential equation in Equation 18, by getting rid of the potential term.

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = E\psi \quad (17)$$

$$\frac{d^2 \psi}{dx^2} = -k^2 \psi, \quad k = \frac{\sqrt{2mE}}{\hbar} \quad (18)$$

We know that the general solution to such a differential equation is as follows:

$$\psi(x) = Asin(kx) + Bcos(kx), (0 < x < a) \quad (19)$$

And we can apply Equation 13 (Bloch’s theorem) to Equation 19

$$\psi(x) = e^{-iKa} [Asin(k(x + a)) + Bcos(k(x + a))], (-a < x < 0) \quad (20)$$
We know that at $x=0$ we must have a continuous $\psi(x)$ and so we can plug in $x=0$ and set the above Equations 19 and 20 equal to each other $\psi(0) = e^{-iKa}[\psi(a)]$
and thus:

$$B = e^{-iKa}[A\sin(ka) + B\cos(ka)]$$  \hspace{1cm} (21)

We currently have one equation with two unknowns, in order to get another equation we can integrate the function from just before the delta potential at $x=0$ (which we will call $0_-$) to just after the delta potential (which is $0_+$) [17].

$$\int_{0_-}^{0_+} d^2\psi dx - \int_{0_-}^{0_+} \frac{2m}{\hbar^2} V(x)\psi dx = -\int_{0_-}^{0_+} E\psi dx$$  \hspace{1cm} (22)

This above equation simplifies to

$$\frac{d\psi}{dx} \bigg|_{0_+} - \frac{d\psi}{dx} \bigg|_{0_-} = \int_{0_-}^{0_+} \frac{2m\alpha}{\hbar^2} \delta(0) \psi(x) dx$$  \hspace{1cm} (23)

$$\frac{d\psi}{dx} \bigg|_{0_+} - \frac{d\psi}{dx} \bigg|_{0_-} = \frac{2m\alpha}{\hbar^2} \psi(0)$$  \hspace{1cm} (24)

$$kA - e^{-iKa}k[A\cos(ka) - B\sin(ka)] = \frac{2m\alpha}{\hbar^2} B$$  \hspace{1cm} (25)
Now using Equation 25 and 21 we have two equations with two unknowns, A and B. Solving for these two unknowns and eliminating them from the equation leaves us with an expression which we can solve.

\[ \cos(Ka) = \cos(ka) + \frac{ma}{\hbar^2 k} \sin(ka) \]  

(26)

As we can see the left side of the equation can only vary from -1 to +1 if it is going to be real, thus limiting the right hand side to those same constraints. Clearly, the right hand side can have real values far above +1 and below -1 and so for some \( k \) values (energies) there is no real solution and thus these energies or frequencies are not allowed to propagate or exist within this system. Isolating the right hand side of the equation we get the following function, for which we know the real solutions must be between -1 and +1 [17].

\[ f(x) = \cos(x) + \frac{C\sin(x)}{x} \]  

(27)

We plot the function from Equation 27 and shade in the real solutions. This separates our allowed frequencies from those that lie in the band. The band gap
structure in Figure 4 emerges beautifully from this simple Dirac comb periodic system of delta functions. In the context of periodic photonics we see ‘bands’ of frequencies in Figure 4 which are allowed to propagate and transmit through such a periodic potential. In between these bands we see the emergence of a photonic band for which there is no real wave vector $k$ solution. However, there is an imaginary $k$ solution. Physically this imaginary solution is an evanescent mode that decays exponentially upon incidence with the photonic structure with no real solutions. In this case the electromagnetic energy is reflected back into space.

Figure 4. This figure shows two plots overlayed. The x axis is energy or frequency and the y axis is $f(x)$. The sinusoidal line is a plot of Equation 27, $f(x)$. The shaded regions are a plot of the regions for which $f(x)$ is between -1 and +1. These are the only regions where there are energy solutions that are allowed to exist, thus defining the band and gap structure that emerges from all periodic potentials, including periodic dielectric media.
1.5.2 Qualitative Derivation and Description of Photonic Band Gaps

Another way of considering the origination of the photonic band gap begins with considering a homogenous media with no alternating layers of material and calculating the dispersion relation. By folding $k$ modes when $|k| > \pi/a$ we see that for the mode $k = \pi/a$ there is a degeneracy of two frequencies (Figure 5). At the $\pi/a$ mode the wavelength of light within the structure is equal to twice the periodicity.

Figure 5. a.) The dispersion relation is shown for a photonic system of homogenous media (i.e. one solid refractive index that doesn't have any spatial dependence). In this figure we have folded the bands for modes of which $|k| > \pi/a$ back into the first Brillouin Zone [19]. b.) the system has been slightly perturbed in this case by incorporating a square wave permittivity that varies with periodicity “a”[14].
(k=2π/λ, so when λ=2a, k=π/a). When the perturbation is introduced there becomes only two ways for the electric field profile to respect the periodic potential as required by the Bloch theorem in Equation 13:

\[ E(x + a) = e^{ikx}E(x) \]  

(28)

Figure 6. Two different modes (top of band 1 and bottom of band 2) and energy densities are shown for the two different frequencies after the degeneracy has been split due to perturbation of the system, which is the introduction of a periodically varying permittivity[14].
One way to respect such periodicity is by having the energy density and maximums of the field profile in the high index layers, and the other way is by having the energy density and maximums of the field profile in the low index layers as seen in Figure 6. The light prefers to be in the high index layer and so the low energy mode resides in the lowest energy configuration which has the maximums of the electric field in the high index layers. Due to orthogonality of modes, the other mode has

Figure 7. The typical transmittance and reflectance spectra for periodic dielectric media is presented. The areas where transmittance drops to zero are the photonic bands where all incident light is reflected because there are no modes for that band of frequencies.
its electric field maximums in the low index layers, thus representing a higher energy configuration.

\[ \omega_H \neq \omega_L \]  

(29)

The fundamental result from this analysis is the splitting of the degenerate \( k = \pi/a \) modes into two different allowed energies as shown in Figure 4b. The two split modes are now separated by the photonic band for which the \( k \) modes are non-real and exist as evanescent exponentially decaying modes, which are reflected back to space.

Using numerical simulations we can calculate the transmission spectra versus frequency for a periodic dielectric structure consisting of 10 bilayers. The transmitted frequencies are indicative of allowed energies, while the highly reflecting frequencies are the energies within the photonic band. An example is shown in Figure 7 using numerical simulations of a periodic photonic crystal similar to the structure presented in Figure 2. In order to further control the transport of light, it is possible to insert defects into a periodic dielectric structure.

1.5.3 Localized Defect Transport

It is possible to include defect cavities in such periodic photonic crystals, and for these defect cavities to support new localized resonance modes. A defect is
any cavity or layered material that breaks the periodic potential so that $V(x) \neq V(x+a)$. This is often done by inserting a material with a higher value of permittivity with respect to the alternating layers of dielectrics. The periodicity can also be broken by using the same materials that are used as alternating layers but by changing the width with respect to the other alternating layers. It is said that a defect supports a localized mode[14], because such a mode exists in the photonic band gap and is surrounded by evanescent modes that decay exponentially in the structure. This also leads to exponential decaying of the spatial electric field of the defect mode as it enters the photonic crystal in either direction as seen in Figure 8a. The defect layer with different permittivity is placed between two Bragg mirrors. If a defect layer is designed to support a mode within the photonic band gap, then any incident waves corresponding to that mode that make it through the first Bragg mirror will be trapped between the two mirrors. This trapping of light quickly leads to a build up in electric field intensity within the defect and thus the emergence of a localized spatial mode that leads to high transmittance. Because the cavity is a finite region, the frequencies of light that are able to be trapped are discrete which leads to a narrow resonant transmittance. The transmitting mode is surrounded by evanescent modes that do not transmit, and so we call it localized. There are high
reflecting walls surrounding the narrow transmission frequency that is associated with this spatial resonant mode, which can be very important for applications that require protection from incidence for other frequencies that are not the desired frequency. This is considered a Fabry-Perot filter[14] because it filters out all other frequencies (within a broad range).

Figure 8. Two different cases are presented of a 14 bilayer structure with a nonlinear lossy defect marked as yellow. In a.) we see the localized defect resonant mode preserved resulting in transmission of the frequency of light that is incident upon the structure. In b.) we see the destruction of the mode via nonlinear losses and almost total reflection of the mode. The reflection is indicated by the high concentration of electric field energy located on the side of incidence, and the transmittance in the latter case is indicated by the relatively high electric field intensity within the yellow defect layer. The wave is incident upon the structure from the right hand side in both cases and these are steady-state solutions using numerical simulation techniques.
CHAPTER 2. PHASE CHANGE MATERIALS AND SIMULATIONS

In this chapter we explore the differences between two of the most studied types of phase change materials, and we go on to explore the numerical analysis of such materials integrated into photonic structures.

2.1 Phase Change Materials

As mentioned above, in this study we explore the use of phase change materials for asymmetric transport and limiting action. Phase change materials (PCMs) are materials that go through changes in optical properties due to some external parameter variations such as temperature, irradiance or pressure. There are two main types of PCMs which we explore below.

2.1.1 Germanium-Antimony-Tellurium (GST)

The first consists of an alloy of germanium-antimony-tellurium, which undergoes an amorphous to crystalline phase change upon laser irradiance [20]. The two different phases are stable at room temperature and can be switched back and forth via laser pumping. This type of PCM has been most commonly used as means for recording memory in DVDs and CDs[13]. A medium power laser can change a region on the material from its amorphous to crystallized state by inducing
heating (the phase change happens around 150°C for GST) and record information in this way. In order to reset this region if the laser power is increased even further it leads to the melting of that local region and fast cooling leaves the material back in its amorphous state, thus allowing for the rewriting of memory [13]. Both the amorphous and crystalline state are stable at low temperatures. When GST transitions to its crystalline phase it goes through a dramatic reduction in resistivity of 8 orders of magnitude. GST does not switch back to its amorphous state when cooled after heating, which is why it is a great material for memory storage. GST also undergoes changes in its real refractive index when heated in its amorphous stage, which can be useful for applications that require small perturbations of the real refractive index, such as the hypersensitive limiter presented by Makri et al. [21].

2.1.2 Vanadium Dioxide (VO₂)
A profoundly different PCM comes from the oxides of vanadium, specifically vanadium dioxide (VO$_2$). Vanadium dioxide is of specific interest to us because it experiences an abrupt transition from a dielectric to a metal at a low temperature of 68 °C, which is close to room temperature[9]. This phase change is associated with an increase in imaginary permittivity of four orders of magnitude [8] and a factor of 10,000 conductivity increase from its dielectric to metallic phases. Unlike GST, VO$_2$ has only one stable phase per temperature: below 68°C the dielectric phase is stable, above 68°C only the metallic phase is stable.

Recent studies have pointed to lattice vibrations (phonons) as the mechanism for such an abrupt thermally induced phase change[9]. These phonons change vanadium dioxide from an amorphous “distorted monoclinic crystal structure” to a tetragonal crystal structure [9]. “As the IMT(insulator metal transition) occurs, the bandgap collapses

**Figure 9.** In this figure we see the crystallization that leads to the phase change of vanadium dioxide from a dielectric(left) to a metal(right). As you can see the lattice on the right is vibrating and the electrons are free to move about, as is indicative of a metal [9].
and nanoscale islands of the metallic phase begin to form in the surrounding insulating VO$_2$, which then grow and connect in a percolative process”[12].

Vanadium dioxide can passively switch back to its dielectric state when the heating is removed and its temperature drops below phase change to 68°C and so this makes it particularly useful for switching applications because its state can be changed so abruptly depending on the temperature of the VO$_2$. Vanadium dioxide has many temperature dependent properties, most notably its permittivity, conductivity, resistivity, and specific heat. These temperature dependent properties make VO$_2$ an excellent candidate for an optical limiter or an optical diode. The abrupt nonlinear change in material properties make executing realistic simulations extremely difficult because of the computational complexities associated with such phase changes. At the same time, providing realistic simulations of devices that incorporate VO$_2$ prior to the actual building of the device is very important, because of the high cost and precision associated with fabrication of a device that contains VO$_2$.

2.2 Electromagnetic Modelling

We use COMSOL Multiphysics which is a commercially available numerical simulation package in order to provide realistic demonstrations and executions of various photonic designs. Using this software we can realistically
Figure 10. A 3-D representation of the geometry of a periodic photonic crystal using quarter wavelength dielectrics. Incident (I), reflected (R), and transmitted (T) waves are shown. Using our simulation methods we measure transmittance and reflectance from an entry port (where the incident wave, I is coming from above) and an exit port (where the transmitted wave is heading above, T). The edge faces in the above structure are assumed to be periodic thus rendering this 3-D structure simplified to a 1-D more realistic case.

simulate various 3D periodic photonic crystal structures with temperature dependent material properties. In this study we use COMSOL’s radio frequency module and depending on the need, we utilize the electromagnetic waves, frequency fomain(steady-state) or transient(time-dependent) studies in order to provide steady-state or time dependent solutions. In practice using COMSOL can be broken down into the broad sections of Geometry, Materials, Definitions, Electromagnetic Waves, Heat Transfer in Solids, Multiphysics, Mesh and Study.
2.2.1 Geometry

In COMSOL we define the physical structure and various layer widths for our dielectric and defect layers. In this study, we always utilize a layer width of 
\[ d = \frac{\lambda}{4n} \]
where \( \lambda \) is the operating resonant wavelength of 10.5 \( \mu m \) and \( n \) is the real part of the refractive index of the material at room temperature; this is also the width we use for the defect layer. By utilizing a quarter-wavelength stack we can maximize the width of our photonic band [14] thus taking advantage of the broad range of reflective properties that created by the photonic band. This makes it so the resonant transmitting mode is surrounded by a maximally wide reflective region, thus providing a broad range of frequencies for which the asymmetric limiter can protect for. This is particularly useful in communications applications. Often times sensors receive communication of one specific wavelength but desire to block other frequencies that would otherwise create a noisy signal. Also, in the context of military applications, an enemy that is seeking to damage a sensor will have a very narrow resonant frequency to target for jamming or destruction of a communication sensor.

2.2.2 Materials

In COMSOL we select various regions and assign materials to each region. These materials are marked by their specific material properties which we can
define as a scalar or a function. In this study, the most relevant material properties that we define are relative permeability, relative permittivity, thermal conductivity, density and heat capacity. Relative permeability and permittivity obviously affect the speed and behavior of the electromagnetic wave within each layer. Thermal conductivity, emissivity, convection, radiation, material density and heat capacity are essential for providing a realistic model of the heat (produced by the incident light) transfer through the various layers.

In our periodic dielectric structure we define silica (SiO\(_2\)) and silicon (Si) for our alternating dielectrics and vanadium dioxide (VO\(_2\)) as the defect layer. In the VO\(_2\) layer we utilize temperature dependent permittivity and heat capacity.

2.2.3 Maxwell’s Equations and Heat Transfer Equations

Using COMSOL we can solve Maxwell’s equations and the heat transfer equations for either steady state or time-dependent solutions, both of which we utilize in this study. The only difference between these two solutions is that in the steady-state Maxwell’s equations and the heat transfer equations are evaluated when \(time \to \infty\) when the solution is no longer changing with respect to time. By utilizing COMSOL’s Frequency-Transient study we can capture all the data points as the system is evolving and approaching steady state. The essence of COMSOL’s Multiphysics capabilities is captured in it’s robust ability to solve the following equations self-consistently and converge upon a solution for both time-domain and
steady state simulations. COMSOL utilizes the Newton-Raphson numerical solver technique.

Maxwell’s Equations and Heat-Transfer Equations:

\[
\nabla \times \vec{H} = j_0 + \varepsilon(z) \frac{\partial \vec{E}}{\partial t}
\]

(30)

\[
\nabla \times \vec{E} = -\mu_0(z) \frac{\partial \vec{H}}{\partial t}
\]

(31)

\[
Q = \frac{1}{2} Re(\vec{j} \cdot \vec{E}), \quad \vec{j} = \sigma \vec{E}, \quad \sigma = \sigma_0 + \omega \varepsilon''(\theta)
\]

(32)

\[
\varepsilon(z, \theta) = \varepsilon'(z, \theta) - i \varepsilon''(z, \theta)
\]

(33)

\[
\rho_D C_p^D(\theta) \frac{\partial \theta}{\partial t} - \nabla \cdot (k_D \nabla \theta) = Q(\theta) + q_0 + q_r
\]

(34)

In equations 30 and 31 \( \vec{E} \) is the electric field, \( \vec{H} \) is the magnetic field, \( \varepsilon(z, \theta) \) is the real and imaginary permittivity as a function of temperature and \( \mu_0 \) is the permeability. In the equations 32, 33, and 34, \( \sigma \) is the material electrical conductivity, \( j \) is the current density, \( \rho_D(\text{kg/m}^3) \) is the mass density, \( k_D(\text{W/m}\cdot\text{K}) \) is the thermal conductivity, \( C_p^D \) is the heat capacity which is a function of temperature for
the defect layer and temperature is represented by $\theta$. $\sigma$ is the material electrical conductivity $Q$ is the total heat generated, $q_0$ is thermal convection and $q_r$ is thermal radiation.

First, an incident continuous coherent wave creates an electric field profile within the structure which then leads to the heating of the structure being studied (Equation 31 and 32). In the case of the periodic photonic structure that we study, the heat is generated directly from the absorptive loss within the lossy defect layer (Equation 32). The amount of electromagnetic power that is absorbed as heat is represented as an increased temperature with the highest temperatures in the lossy defect layer, because it is within that layer that almost all of the heat is being generated. We also incorporate heat lost from either side of the structure (where the structure meets air) via heat radiation and convection (Equation 34). The Multiphysics aspect of COMSOL is the direct linking of Maxwell’s equations with the heat transfer equations. From the above equations we can see that the heat generated $Q$ is proportional to the electric field intensity within that layer multiplied by the loss within that layer $Q \equiv (|E|^2 \cdot \varepsilon'(\theta))$.

The electromagnetic solver solves for the whole structure in each of the individual meshing regions; if the incident power of a wave is not strong enough to cause significant heating (and thus optical parameter changes for materials), then the system is said to be in thermal and electromagnetic equilibrium. At this point
the simulation has converged on a solution and transmittance, reflectance and absorbance data is reported. In another case, upon solving for each meshing region the electric field intensity (due to input power) multiplied by the loss in the structure produces significant heat $Q$ ($Q \equiv |E|^2 \cdot \varepsilon''(\theta)$) and causes a temperature change based on the heat transfer Equation 34 where $\theta$ is temperature. We can see that the amount of heat produced $Q$ leads directly to a change of temperature. This change of temperature ($\Delta \theta$) is due to the field intensity and loss within a given meshing element.

After $\Delta \theta$ is calculated this is the completion of one iteration. Next this new temperature is now considered the starting temperature for the next iterative step. The new temperature is first plugged in to the temperature dependent material properties which is the permittivity $\varepsilon(z, \theta)$ and heat capacity $C_p^D(\theta)$ for VO$_2$ and according to these functions new material properties are established for each layer. Then using these new optical properties Maxwell’s equations are solved for again increasing the time step further. If more heat is continuously produced then more and more iterations will be needed and this feedback between heat transfer equations and Maxwell’s equations will continue until the temperature of the material stops changing, at that point the electric field profile within the structure will stop changing too meaning that the solver has reached a steady state solution. Note that Maxwell equations depend on the thermally dependent permittivity in the
magnetic field curl equation and so with each iteration (if the temperature is changing) then the spatial electric field intensity as well as the transmittance, reflectance and absorption will be changing.

Also, we define periodic boundary conditions for the edges of our layers so the wave that is entering the structure will not ‘see’ the boundaries which is more realistic in an experimentally realized structure that essentially reduces to a 1-D stack of dielectrics as shown in [Figure 2].

2.2.4 Mesh and Study Techniques

One of the most computationally important aspects of running numerical simulations in COMSOL is the mesh. The mesh consists of all the individual regions that the structure is divided into for which COMSOL solves both Maxwell’s and heat transfer equations; it is the discretization of space for the use of solving the differential equations necessary in both the electromagnetic and heat transfer linked physics. As mentioned previously, the Newton-Raphson numerical method is used to solve the physics in COMSOL which involves solving Maxwell’s curl equations using an initial guessed solution that is based upon the surrounding meshing elements. By having more elements per layer this leads to more accurate initial guesses for the smaller meshing elements because the spatial variation between any given adjacent meshing elements is smaller. The mesh is often defined as an amount of meshing elements per layer of material. In
simulations of devices that utilize materials with thermally dependent nonlinear optical properties, the density must be increased significantly from 30 per layer up to 1000 per layer. This helps the numerical solver that is built into COMSOL by decreasing the size of the spatial difference between the meshing elements that must be solved for in order for COMSOL to converge on a solution.

When running simulations in COMSOL we determine the allowable error, the type of simulation (transient vs stationary), and the type of numerical solver to utilize. Some other useful features in COMSOL that we utilize are temperature probes and electric field profile probes. We can put a point probe anywhere in the structure and take temperature data at that point, which is essential in tracking the thermally induced phase change of VO$_2$. Results using such a temperature probe are presented in the next chapter. We also can use a probe line in order to measure the electric field profile within the structure; this is essential for analyzing the asymmetry of a structure.

The next chapter will consist of the actual results for a study of an asymmetric limiter.
CHAPTER 3. RESULTS

This chapter contains the core of the study, the results are presented for a photonic asymmetric limiter.
3.1 Summary

We consider the scattering problem for an asymmetric composite photonic structure with a component experiencing a thermally driven phase transition. Using a numerical example, we show that if the heating is caused by the incident light, the transmittance can become highly asymmetric within a broad range of light intensities. This

Figure 11 (a). The proposed multi-layered photonic structure (MLPS) consisting of quarter-wavelength alternating dielectric layers with an embedded phase-change material (PCM) defect layer, D, that is placed such that it breaks the mirror symmetry of the photonic structure. (b) The variation of $\varepsilon'$ and $\varepsilon''$ of the PCM, vanadium dioxide (VO$_2$), with temperature $\theta$, at $\lambda_0 = 10.5$ μm. (c) The calculated temperature increase at the center of the PCM layer in our MLPS versus incident irradiance, for forward (open black symbols) and backward (open red symbols) incident radiation, compared with that of a stand-alone (SA) PCM layer of the same thickness (solid red and black symbols). Note the value of the critical irradiance that triggers the phase-transition at $\theta_c$ is different for forward and backward incident radiation.
effect can be utilized for directional light transmission (isolation), asymmetric optical
limiting, or power switching.

3.2 Background

Control of the directionality of electromagnetic radiation constitutes a continuous
challenging theme of research for both physicists and engineers. On the fundamental level,
this challenge is directly related to the restrictions imposed by the reciprocity principle,
which applies to light propagation in media without spontaneous magnetic order and in the
absence of a magnetic field bias [24]. Although the magneto-optical approach to
asymmetric or unidirectional light propagation has been dominant in optics and the
microwave regime [31,32], it has some inherent problems, which have stimulated a great
interest in alternative ways to break the reciprocity principle and to achieve strong
asymmetry between forward and backward light transmission. One popular non-magnetic
approach is to use temporal modulation, which is an active scheme to break the reciprocity
[31,32]. In this study, we restrict ourselves to passive techniques.

Passive non-magnetic approaches to light-propagation asymmetry usually rely on
nonlinear spatially asymmetric structures. The nonlinear effects can be different for the
forward (F) and backward (B) propagating light, which can result in intensity-dependent
propagation asymmetry [33]. In all cases, at low input light intensity, the transmittance
becomes symmetric due to the reciprocity principle.

Yet another non-magnetic approach to transmission asymmetry is based on thermal
effects produced by incident light in spatially asymmetric structures involving components
with temperature-dependent material (optical) parameters [34-36]. Specifically, input light
of the same intensity incident from opposite sides of an asymmetric structure can produce different heating effects and, therefore, the transmission coefficient for forward and backward propagation can also be different. The advantages of the approach based on light-induced heating over using asymmetric structures with conventional nonlinear materials can include: (i) a much lower input light intensity required to trigger a change in the temperature-dependent material parameters, and (ii) a change in the complex index being larger than 10% (which corresponds to the maximum changes for conventional nonlinear effects [37]). The effect of asymmetric heating is dependent on thermal conductivity and other heat transfer mechanisms [38,39]. This provides additional flexibility in design but, on the other hand, the thermal transport significantly complicates numerical simulations.

The most effective manifestations of heating related effects in light propagation occur if the optical material exhibits a nearly abrupt, heat-induced phase transition, rather than a gradual change in the refractive index or absorption or both. A well-known example of such phase-change materials (PCMs) is vanadium dioxide (VO$_2$) which undergoes a monoclinic insulating to a rutile metallic phase transition near room temperature, $\theta_c = 68 ^\circ$C [40-42]. Asymmetric light transmission associated with heat-induced phase change in the VO$_2$ component of a trilayer gold/VO$_2$/sapphire structure was successfully demonstrated in Ref. [43]. One limitation of such a simple structure is that, at optical frequencies, the heat-induced change in the complex refractive indices of known PCMs may not be sufficient for a few-layer structure to act as an effective limiter, switch, or isolator for low-intensity light. Furthermore, a few-layer structure provides limited ability to tune the irradiance or fluence required to trigger the phase transition, while the desired values of the limiting/switching threshold can vary significantly in different applications.
To address the above limitations, we propose to incorporate a PCM into an asymmetric multi-layered photonic structure (MLPS). In the simplest realization, the half-wavelength-thick PCM layer is used as a defect layer which supports a localized mode within a periodic MLPS, as shown in Fig. 1(a). In other words, we consider an asymmetric optical microcavity filled with a PCM. The asymmetry of the microcavity is essential for light-induced transmission asymmetry. Using time-domain simulations for Maxwell’s equations coupled with heat-transfer equations, we show that for a range of incident light fluences, the PCM-filled microcavity displays a highly asymmetric transmittance. Specifically, the asymmetric photonic structure in Fig. 1 is perfectly transmissive in the forward direction at the microcavity resonance frequency within a broad range of the input light intensity. By contrast, in the backward direction, the same structure is totally reflective for the same input light intensities and within the frequency range covering the entire photonic band gap. For a given PCM layer (VO₂ in our case), the lower and upper limits of the incident light intensity within which the structure displays unidirectional transmittance can be changed significantly by the proper choice of the layered structure geometry. The frequency of the unidirectional (forward) resonant transmittance can also be changed significantly.

3.2 Underlying basic principle of asymmetric transport

Before analyzing the asymmetric transport within the MLPS of Fig. 1, we demonstrate the underlying principle by studying the transport properties of a toy model. Let us consider three thin, delta-like layers at positions \( z_1 = 0, \ z_3 = 1, \) and \( 0 \leq z_2 \leq 1, \)
(arbitrary units). The Helmholtz equation for this system is \[ \left( \frac{d^2}{dz^2} + k^2 \left[ n^2 \sum_{i=1}^{3} \delta(z - z_i) + n_0^2 \right] \right) E = 0 \], where \( z \) is the position along the propagation distance, \( E(z) \) is the electric field amplitude at position \( z \), \( n \) is the refractive index of the thin layers, \( n_0 = 1 \) is the refractive index of air, \( k = \frac{2\pi}{\lambda_0} \) is the wave vector of the incident wave and \( \lambda_0 \) is the wavelength in free space. The Helmholtz equation can be easily solved analytically under appropriate scattering boundary conditions: for incident waves in the forward direction, we assume the electric field amplitude is:

\[ E(z < z_1) = e^{ikz} + r_F e^{-ikz} \]  \( \text{and} \)  \[ E(z > z_3) = t_F e^{ikz} \]  \hspace{1cm} (35)

Similarly, for incident waves in the backward direction we shall have:

\[ E(z > z_3) = e^{-ikz} + r_B e^{ikz} \]  \( \text{and} \)  \[ E(z < z_1) = t_B e^{-ikz} \]  \hspace{1cm} (36)

\( r_{F/B} \) and \( t_{F/B} \) denote the amplitudes of the reflected and transmitted waves, respectively, for the incident waves in the forward/backward directions. Imposing continuity of the field and allowing the discontinuity of its first derivative at positions \( z_1, z_2, \) and \( z_3 \), we can obtain the spatial electric field distribution \( E(z) \). Figures 12(a)-(b) show two electric(E)-field intensity profiles denoted by black (red) for forward (backward) incident waves with the same incident field amplitude for situations when: (a) \( z_2 = 0.5 \) (mirror symmetry is preserved) and (b) \( z_2 \neq 0.5 \) (mirror symmetry is violated), respectively. In the former case,
the two field intensities are mirror images of one another; i.e., a transformation \( z \leftrightarrow -z \) maps one solution to the other. In contrast, Fig. 12b shows that the left and right field intensities are completely different from one another. The field intensity difference \( \Delta(z_2) \), between the field intensity at \( z_2 \) due to forward and backward propagating incident waves is given by:

\[
\Delta(z_2) \equiv |E_F(z_2)|^2 - |E_B(z_2)|^2 = 16kn(-2\cos(k) + kn\sin(k))\sin(-k + 2kz_2)B(k, n, z_2),
\]

(37)

where \( B^{-1}(k, n, z_2) = 0.25|e^{2ik}e^{2ikz_2}(k^2n^2(kn - 2i)) - (kn + 2i)(k^2n^2(1 - e^{z_2} + e^{2z_2}) + 4e^{z_2}(1 - ikn))|^2 \). The forward and backward transmittances are, nevertheless, the same; i.e., \( T_F \equiv |t_F|^2 = |t_B|^2 \equiv T_B \).

The forward/backward E-field intensity (asymmetry \( \Delta(z_2) \)) can lead to directional transport when the thin dielectric defect at position \( z_2 \) is substituted by a layer that has intensity-dependent material parameters. In this study, we shall assume that the defect layer consists of a PCM that shows a strong and abrupt change in its complex permittivity. These abrupt permittivity variations are in contrast to gradual Kerr-type or two photon absorption nonlinearities. The PCM layer experiences different absorbance, with difference \( \Delta A = |A_F - A_B| \sim \Im(m(\varepsilon_D)\omega|\Delta(z_2)| \neq 0 \) [35], for forward/backward propagating incident waves with the same incident irradiances. Consequently, for forward/backward incident waves, the PCM will develop different temperatures \( \Delta \theta \equiv |\theta_F - \theta_B| \) which will differently affect its imaginary permittivity; i.e., \( \varepsilon''_D(\theta_F) \neq \varepsilon''_D(\theta_B) \). Note that there is also a change in
real permittivity of the PCM with temperature, but it is not as dramatic as the change in the imaginary component (see Fig. 11(b)). The most dramatic difference appears when, say, $\theta_F < \theta_B = \theta_C$. In this case, $\varepsilon''_D(\theta_F) \ll \varepsilon''_B(\theta_B)$, and thus the forward and backward transmittances will be dramatically different. In Figs. 12c,d we plot an example of the spatial electric-field intensity distributions for the case of a stand-alone (SA) structure of VO$_2$ on a silica substrate. It is important to point out that the field asymmetry in the above cases is moderate (i.e., two or three-fold) and thus any directional asymmetry in insulator-metal transition (IMT) happens for a small range of incident electric field intensities, see solid symbols in Fig. 1c. Moreover, the field intensity values at $z_2$ are comparable with the incident field intensities. Therefore, in order to heat up the PCM up to $\theta_C$ and trigger the IMT, the incident irradiance must be relatively high.

3.3 Multi-Layered Photonic Structure

The multi-layered photonic structure (MLPS) consists of alternating layers of high-index ($H$) and low-index ($L$) quarter-wavelength-thick dielectric films which produces alternating transmission and reflection bands. Inside the MLPS we place a defect layer $D$ at a position away from the mirror-symmetry plane of the structure, see for example Fig. 11(a). The proposed asymmetric MLPS is represented by the form $(LH)^m[D](LH)^n$ where $m \neq n$ corresponds to $m(n)$ bilayers on the left (right)-hand side of the defect layer. Furthermore, we assume that the defect layer is made of a phase-change material (PCM) that undergoes an IMT when the temperature of the layer reaches a critical value, $\theta_C$.

The MLPS has been designed such that it supports a resonant defect mode (due to the presence of the defect layer) which is exponentially localized around the PCM defect layer.
and is located at the center of the band gap at $\lambda_0 \sim 10.5 \, \mu m$. We note that this wavelength corresponds to the middle of the long-wave infrared atmospheric transparency window. In our numerical example below, we have chosen silicon with $\varepsilon_{Si} = 11.7$ and silicon dioxide with $\varepsilon_{SiO_2} = 5.038$ as the high- and low-index dielectric materials. Furthermore, we have assumed that these materials are lossless around 10.5 $\mu m$ wavelength [44-46].

The VO$_2$ PCM layer undergoes an IMT at $\theta_c \approx 68 ^\circ C$ which changes both its real and imaginary permittivity as a function of temperature, $\theta$. In our simulations, the complex permittivity $\varepsilon(\theta) = \varepsilon'(\theta) - i\varepsilon''(\theta)$ of the VO$_2$ defect layer was obtained directly from the experimentally reported values for heating (solid symbols), measured at $\lambda_0 = 10.5 \, \mu m$ (see Fig.11b) [46]. For simulation purposes, the values of $\varepsilon'$ and $\varepsilon''$ from 20 to 72 $^\circ C$ were linearly interpolated, and outside this range we assumed them to be constant.

The latent heat released during the IMT of the VO$_2$ layer leads to a temperature dependence of the specific heat capacity, $C_p^0$, which can be approximately related with the slope of the change in electrical conductivity with temperature: $C_p^0(\theta) = C_p^0 + \frac{H_L}{\Delta \sigma_t} \frac{d\sigma}{d\theta}$ [35].

$C_p^0 = 690 \frac{J}{kgK}$ corresponds to the specific heat capacity of VO$_2$ before the phase transition and we have assumed the latent heat constant, $H_L \approx 5.042 \times 10^4 \, J/kg$ [48,49]. Finally $\Delta \sigma_t$ is the total conductivity jump during the phase transition consistent with the behavior of $\varepsilon''(\theta)$ of VO2 (see Fig. 11b).

3.4 Transient and Steady-State Modeling

The wave propagation along the $z$-direction is described by:
where: $\varepsilon(z) = \varepsilon'(z) - i\varepsilon''(z)$ is the position-dependent permittivity that varies from layer to layer and $\vec{j}_0 = \sigma_0(z)\vec{E}$ is the electric current. Equations 38 have been solved together with the transient heat-transfer rate equations, Eq. (39), which give the temperature variation of the PCM layer in the presence of continuous wave (CW) incident radiation (see Fig. 1b):

$$\rho_D C_p^D(\theta) \frac{\partial \theta}{\partial t} - \nabla \cdot (k_D \nabla \theta) = Q(\theta) + q_0 + q_r, \quad (39)$$

where $\rho_D = 4670 \text{ kg/m}^3$ and $k_D = 4 \text{ W/m} \cdot \text{K}$ denote the mass density, and thermal conductivity, of the defect layer [50,51]. The heat production $Q$ per unit volume of the PCM layer is $Q = \frac{1}{2} \text{Re} \left[ \varepsilon \left( \vec{j}_0 \cdot \vec{E} \right) + \omega \varepsilon''(z) \text{Re} \left( \vec{E} \cdot \vec{E} \right) \right]$. For computational convenience, we recast the above expression as $Q = \frac{1}{2} \text{Re} \left( \vec{j} \cdot \vec{E} \right)$, where $\vec{j} = \sigma\vec{E}$, $\sigma = \sigma_0 + \omega \varepsilon''$. 

\[ \nabla \times \vec{H} = \vec{j}_0 + \varepsilon(z) \frac{\partial \vec{E}}{\partial t}, \quad \nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}, \quad (38) \]
The term \( q_0 = h(\theta_{\text{ext}} - \theta) \) denotes the thermal convection occurring from the edges of the dielectric layer to the surrounding air, and \( h = 10W/m^2K \) and \( \theta_{\text{ext}} = 293.15K \) corresponds to heat flux coefficient and temperature of the air. Finally, \( q_r = \varepsilon_r \sigma_r (\theta_{\text{ext}}^4 - \theta^4) \) describes the heat transfer via thermal radiation from the edges of the photonic structure to the surrounding air assuming grey-body approximation of the silica.
layer. The parameters $\varepsilon_r = 0.79$ and $\sigma_r = 5.7 \times 10^8 W \cdot m^{-2} K^{-4}$ correspond to the thermal emissivity coefficient of SiO$_2$ and the Stefan-Boltzmann constant, respectively.

3.5 Results and Analysis

Equations (38) and (39) were solved self-consistently using a finite-element software package to numerically evaluate the temporal behavior of transmittance $T(t)$, reflectance $R(t)$ and absorbance $A(t) = 1 - T(t) - R(t)$ of the asymmetric MLPS, and the temperature $\theta(t)$ at the PCM defect layer. From these calculations, we extracted the asymptotic (steady-state) values as $t \to \infty$ of these quantities $T_\infty, R_\infty, A_\infty$ and $\theta_\infty$ as well as the steady-state electric field profiles. The calculations were carried out for both cases when light enters the MLPS in the forward ($F$) and backward ($B$) directions, respectively. Figures 12e,f show the steady-state electric-field intensity profiles of a MLPS with $m = 10$ and $n = 4$ bilayers at the resonant frequency ($\nu_0 = 2.86 \times 10^{13}$ Hz) corresponding to irradiances of $I = 2.66 \times 10^{-5}$ W/cm$^2$ (blue lines), $I = 1.06 \times 10^{-2}$ W/cm$^2$ (black lines) and $I = 0.39$ W/cm$^2$ (dashed red lines).

We utilize the exponential form of the resonant defect mode in order to achieve $\theta_C$ inside the PCM (and thus induce the IMT), for values of incident field irradiances that are exponentially lower in comparison to the single stand-alone structure (see Fig. 11c). Another consequence of the MLPS is the increase of the forward-backward electric-field intensity asymmetry $\Delta(z_2)$. For low irradiances (solid blue and black lines), resulting in $\theta_F, \theta_B < \theta_C$, the MLPS demonstrates high transmittance ($T_\infty \approx 0.6$) at the defect-mode frequency, see Fig. 13(a). This is associated with resonant transport via the defect mode. At the same time, the field intensity at the PCM layer is higher for waves incident in the
backward direction (Fig. 12f) than for waves incident in the forward direction (Fig. 12e). This leads us to the conclusion that an incident wave in the backward direction will heat up the PCM to its $\theta_C$ at smaller irradiances than its forward propagating incident counterpart. Indeed, for higher irradiances (dashed red lines), light entering the structure in the forward direction (i.e., from left) does not induce critical heating and thus the PCM remains in the dielectric phase (Fig. 11c). In this case, the presence of the resonant defect mode (dashed red line in Fig. 12e) leads to high transmission, similar to the case of low irradiances (solid black and blue lines). When the wave with the same high irradiance enters the MLP S in the backward direction (i.e., from the right), it causes heating of the VO$_2$ layer above $\theta_C$ (see Fig. 1b) which drives the PCM to the metallic phase. The dramatic increase in $\varepsilon_\theta''(\theta)$ greatly reduces the quality factor of the resonant mode and increases the impedance mismatch of the resonant mode with the incoming wave. As a result, the reflection increases dramatically with a simultaneous decrease in both the transmission and absorption. This qualitative difference in the transport of the MLPS for forward and backward directional incident waves occurs for a broad range of incident electric field irradiances—as compared to the case of a single bilayer structure—and will be confirmed via detailed multi-physics simulations in the rest of the paper.

Figures 13 a-c display some typical transient behaviors of the transmittance, $T(t)$, for a CW incident signal propagating in the forward (black lines) and backward (red lines) directions with small (a), moderate (b) and large irradiances (c), respectively. When the irradiance of the CW signal is small ($I = 0.033$ W/cm$^2$), the heating of the VO$_2$ defect layer is negligible for light incident in both directions. As a result, the PCM remains in the
dielectric phase irrespective of the direction of the incident light. Thus, the resonant defect

![Graphs](image)

**Figure 13.** (a)-(c). Transient evolution of transmittance, $T(t)$, with increasing irradiances, $I = 0.033$ (a), 0.361 (b) and 4.78 (c) W/cm$^2$ for the cases when light is incident in the forward (black symbols) and backward (red symbols) directions towards the MLPS (for $m = 6$, $n = 2$ bilayers) with asymmetrically located PCM defect layer. Note that for all the simulated results the background temperature is the ambient temperature, 293.15 K. (d)-(e) Transmittance and absorbance under the steady-state scenario with increasing irradiances in the case of MLPS, evaluated for cases when light enters the structure in the forward (F) and backward (B) directions. The dashed curve indicates the case of a single SiO$_2$-VO$_2$ SA defect layered structure with the same defect thickness as the case of the MLPS. Note the reduction in $A$ with increasing incident $I$ in the case of MLPS when light is incident in the forward direction (i.e., from right).

mode is unaffected and the photonic structure remains (almost) transparent at the resonant frequency, as indicated in Fig. 13(a). When the input irradiance is increased, the strong asymmetry in the distribution of the electric field intensity leads to an asymmetric IMT of the VO$_2$ defect layer. We found that for a range of irradiances, $0.08 < I < 0.4$ W/cm$^2$, the forward propagating incident wave does not cause any significant heating of the VO$_2$ defect
layer, and thus the photonic structure remains transparent at all times (black curve in Fig. 13(b)). In contrast, for the same range of irradiances, a backward propagating incident wave causes critical heating of the PCM layer at a much earlier time, which triggers the IMT and a corresponding jump in the value of $\varepsilon''_D(\theta)$ (see Fig. 11(c)). This, in turn, suppresses the resonant mode and creates an impedance mismatch with the incident CW wave. Consequently, the transmittance, $T_B$, of the backward propagating incident radiation drops abruptly by almost three orders in magnitude (see red curve in Fig. 13(b)). Further increase of the irradiance leads to the triggering of the IMT for forward propagating incident waves as well. This IMT occurs, though, at later times (i.e., at $t_F \approx 0.3$ sec) with respect to the one associated with backward propagating incident radiation (i.e., at $t_B \approx 0.03$ sec). In this case, the MLPS acts as a double-sided reflector, see Fig. 13(c). Note that both time scales can be exponentially scaled-down by increasing the number of bilayers on the left and right side of the PCM [39].

An overview of the directional behaviour of the steady state transmittance $T_\infty$ versus irradiance is shown in Fig. 13(d) for the case of the MLPS (continuous lines) and for the case of a SA VO$_2$-SiO$_2$ bilayer (dashed lines). In this figure, the forward ($T_F$) and backward ($T_B$) transmittances are shown with black and red lines. The results are obtained by solving Eqs. (38, 39) in the limit when $t \to \infty$ (steady state). The drop in transmittance occurs for irradiances that are more than an order of magnitude smaller than that of a stand-alone structure. At the same time, the range of irradiances for which asymmetric transmission occurs is dramatically increased for the MLPS (as compared to the single bilayer).

In Fig. 13e we also report the steady state absorbance, $A_\infty$. For a forward propagating
incident wave, it initially increases and then decreases (red open circles). The initial increase of $A_\infty$ is associated with the increase of $\varepsilon''(\theta)$, while its subsequent decrease is associated with the suppression of the resonant defect mode and the high reflectivity induced by the impedance mismatch. This is in contrast to the SA layered structure, where an absorption as high as $A_\infty = 0.6$ is obtained. This results in overheating of the single PCM layer, which may lead to its destruction. Therefore, the asymmetric MLPS can act also as a highly asymmetric limiter [38, 52-53].

Using time-domain simulations, we have demonstrated that a spatially asymmetric resonant microcavity filled with a PCM (VO$_2$ in our case) acts as a limiting photonic diode displaying unidirectional transmittance within a wide range of input light intensity. The same layered structure can also be seen as a highly asymmetric optical limiter with the forward limiting threshold different from the backward one by orders of magnitude. Above the respective (forward or backward) limiting threshold, the structure becomes highly reflective, which prevents overheating and significantly increases power-handling capabilities. The above features can be very desirable for infrared unidirectional valves and/or directional power limiters or switches [52-54].
CONCLUSION AND FUTURE WORK

By capitalizing on asymmetric electric field profiles, we have demonstrated a way of utilizing phase change materials for the purpose of asymmetric transport. We have developed an asymmetric limiter and tested it using realistic numerical simulations. In the future, we plan to pursue an experimental realization of such an asymmetric limiter and further increase the sensitivity by optimizing the number of layers on either side of the defect. We also plan to utilize the amorphous-amorphous optical property changes of GST in order to accomplish asymmetric transport. This has been theorized in a symmetric structure in ref. 21. by Makri et al. They utilize a nodal point in the electric field to make a thin metal layer transparent to the incident wave for one resonant frequency. For all other frequencies, the metal is ‘seen’ by the wave and the wave is totally reflected. In ref. 21 they do not consider integrating a directional dependence which our early studies indicate may be successful at providing a window for asymmetric resonant transport. The way we initiate the directionality is similar to the previous study in that an asymmetric periodic photonic crystal produces asymmetric spatial electric field profiles. By taking advantage of the various ways phase change materials can be used in conjunction with established periodic photonic crystal theory, we can create more dynamic and robust ways to control the transport and directionality of electromagnetic waves.
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