Non-Hermitian Wave Transport with Applications to Limiters and Isolators

by

Eleana Makri

Faculty Advisor: Prof. Tsampikos Kottos

A dissertation submitted to the faculty of Wesleyan University in partial fulfillment of the requirements for the Degree of Doctor of Philosophy in Physics
Abstract

Non-Hermitian wave transport has attracted a lot of attention during the last ten years, mainly because of its applications in on-chip photonic circuitry. The underlying principle of this subfield of wave transport is the violation (induced or spontaneous) of the time reversal symmetry via lossy mechanisms. When other symmetries are also involved in the design of such circuits (e.g. chiral, mirror, etc.), novel phenomena, such as topologically protected states, might emerge. In this thesis, we utilize time reversal symmetry and its violation along with other symmetries in order to propose a new type of reflective photonic limiters and asymmetric valves. The former family of devices are useful for protection of sensitive sensors from high power/fluence incident radiation, while the latter guides light in a directional fashion, thus providing the basic element for on chip isolators, circulators etc.
Acknowledgements

First and foremost, I would like to express my gratitude to my research advisor Prof. Tsampikos Kottos for his support, patience and genuine care over the past years. His knowledge, insight and physical intuition have helped shape me as a scientist and I will always be grateful on how invested he has been on my work and my scientific development.

Second, I would like to thank all former and current members of the PPQW group. I am especially grateful to Suwun Suwunnarat, Huanan Li, Roney Thomas, Hamid Ramezani, Ali Basiri, Nick Bender, Yujie Cai, and Zhi Ming. You have all taught me a lot during these years and it has been a privilege to work with you.

Moreover, I would like to thank my coauthors Ilya Vitebskiy, Ulrich Kuhl, Fabrice Mortessagne, Nicholaos I. Limberopoulos, and Andrey Chabanov for giving me the opportunity to collaborate with them, for all our stimulating discussions and for their crucial contributions to my work.

In addition, I am grateful to all the faculty, staff, and students of the Wesleyan Physics Department for providing me with such a warm and friendly work environment. I especially wish to thank Prof. Fred Ellis and Prof. Reinhold Blumel for taking the time to be part of my thesis committee.
I am indebted to the Wesleyan graduate student community, within and without the Physics Department. I owe special thanks to Mahboobeh Chitsazi, Nooshin Shatery Nejad, Wengang Zhang, Bardia Hejazi, Stefan Kramel, Nadeepa Jayasundara, Yun Seong Nam, Will Setzer, Swechhya Shrestha, Kayla Anatone, Nicole Arulanantham, and Ani Throssell. You have all made my days in Middletown much brighter.

I would also like to extend my gratitude to my close friends for being a constant source of encouragement throughout this endeavor even from far away, especially Maria Irakleidi, Maria Tsantaki, Maria Sygletou, Michalis Papachatzakis, George Sopasis, Popi Kyriakaki, Maria Manioraki, Eva Zacharioudaki, Katerina Saranti, and Marirena Christoulaki. A heartfelt thank you goes to Anna-Maria Taki, Kostis Roubedakis, Yiannis Iatrakis and Polina Moutsaki for many unforgettable meet-ups in the northeast.

Last but not least, I would like to thank my family. I am particularly grateful to my mother for her continuous support throughout my studies in spite of unfavorable circumstances, as well as my brothers, Nikos, Yiannis and Stefanos. I would like to single out Yiannis for accidentally inspiring me to pursue a career in Physics by exposing me to popular science books during my early teenage years.

Finally, I would like to express my gratitude for partial financial support from the Wesleyan Physics Department, the Office of Naval Research via Grant No. N00014-16-1-2803, the Air Force Office of Scientific Research via a MURI Grant No. FA9550-14-1-0037, and the National Science Foundation via Grant No. EFMA-1641109.
Contents

1 Introduction 1

2 Transport in one-dimensional systems 4

2.1 Helmholtz equation ........................................... 5

2.2 Transfer Matrix Formalism .................................... 6

2.3 Transfer Matrix for Layered Media .......................... 8

2.4 One-Dimensional Photonic Crystals: Band-gap and the Quarter Stack . 9

2.5 Defect Modes ..................................................... 13

2.6 Backward propagation for nonlinear media .................. 16

2.7 Transfer matrix for the 1-D tight-binding model ............. 18

2.8 Scattering matrix for the 1-D tight-binding model ........... 20

3 Reflective Photonic Limiters 22

3.1 Passive limiters .................................................. 23
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>Concept of Reflective Photonic Limiter</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Reflective Power Limiters</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Reflective Energy Limiters</td>
</tr>
<tr>
<td>3.3</td>
<td>Experimental Realization of a Reflective Photonic Limiter</td>
</tr>
<tr>
<td>3.4</td>
<td>Hypersensitive transport in photonic crystals with accidental spatial degeneracies</td>
</tr>
<tr>
<td>4</td>
<td>Waveguide Limiters</td>
</tr>
<tr>
<td>4.1</td>
<td>The SSH model - chiral and charge conjugation symmetries</td>
</tr>
<tr>
<td>4.2</td>
<td>Spectral and eigenstate implications of chiral and charge conjugation symmetries</td>
</tr>
<tr>
<td>4.3</td>
<td>Edge states</td>
</tr>
<tr>
<td>4.4</td>
<td>Waveguide photonic limiters based on topologically protected resonant modes</td>
</tr>
<tr>
<td>4.5</td>
<td>Reflective limiters based on self-induced violation of charge conjugation symmetry</td>
</tr>
<tr>
<td>5</td>
<td>Optical isolators</td>
</tr>
<tr>
<td>5.1</td>
<td>Resonant photonic structure with phase-change material</td>
</tr>
<tr>
<td>5.2</td>
<td>Non-Reciprocal Designs Based on Phase-Change Materials and Simulations</td>
</tr>
<tr>
<td>5.3</td>
<td>Semi-analytical model based on coupled oscillators with a temperature-dependent damping coefficient</td>
</tr>
<tr>
<td>6</td>
<td>Conclusion</td>
</tr>
</tbody>
</table>

v
A  Selected Publications  66
List of Figures

2.1 One-dimensional layered structure. ................................. 9
2.2 Bragg mirror. ......................................................... 11
2.3 Dispersion relation for photonic crystals. ......................... 12
2.4 Bragg mirror with a defect. ......................................... 14
2.5 Transmission spectrum of a Bragg mirror with a defect. ......... 15
2.6 Scattering field profile of a midgap defect resonant mode. .. 17
2.7 Scattering for the one-dimensional tight-binding model. ....... 19

3.1 Behavior of an ideal limiter. ......................................... 24
3.2 Reflective photonic limiter at low and high incident light intensity. .... 27
3.3 Scattering field profile of the midgap defect resonant mode for low and high incident light intensities. ......................... 28
3.4 Transmittance, reflectance, and absorbance spectra for a photonic limiter at different incident light intensity regimes. .......... 30
3.5 Metallodielectric photonic structure. .................................. 34

4.1 SSH model. ................................................................. 40

4.2 Dispersion relation for the SSH model. ................................. 41

4.3 Experimental setup for the C-CROW microwave limiter. ............ 46

4.4 Microwave limiter with charge conjugation symmetry. ............. 48

4.5 Limiter with charge conjugation symmetry in the infrared domain. .. 49

5.1 Temperature dependence of the optical parameters of VO$_2$. .......... 51

5.2 Transmission spectrum for a photonic structure consisting of two ring
resonators coupled to a straight waveguide. .............................. 52

5.3 Transmission, phase, and temperature spectra for a coupled ring res-
onator photonic structure incorporating a VO$_2$ ring resonator. ......... 54

5.4 Density plot of the asymmetry in transmittance vs. frequency and inci-
dent irradiance. .............................................................. 55

5.5 Mechanical oscillator analogue of the CRPS system. .................. 59

5.6 Transmittance, phase, and temperature spectra for our mechanical oscil-
lator analogue. Temperature dependence of the damping coefficient. ... 62
Chapter 1

Introduction

Non-Hermitian wave physics has been established during the last years as an arena where new physical phenomena with potential technological implications can exist. The basic underlying idea is that losses, which until recently have been considered an “anathema”, can be harnessed and turned into a useful ingredient of the design of new structures. Mathematically, the presence of loss or its conjugation effect (i.e. gain) is associated with the powerful notion of the violation of time-reversal symmetry.

It is fair to say that, among all areas of physics where non-Hermitian wave transport can be applicable, optics has constituted the tip of the spear, both for the development of fundamental concepts and for their implementation for technological advances. Along these lines, parity-time symmetric optics provided intriguing examples associated with unidirectional invisibility [1], power oscillations [2], lasers and coherent perfect absorbers [3, 4], hypersensitive sensors [5], etc. These novel phenomena came to light because of the interplay of judiciously tailored spatial and temporal symmetries within a photonic circuit. It soon became obvious that other type of symmetries (chiral, mirror symmetries etc.) can also be utilized in our quest for systems with novel functionalities.

This dissertation operates under the framework of non-Hermitian photonic transport
and capitalizes on time reversal symmetry and its induced or spontaneous violation. We first exploit this concept in order to realize a new class of photonic limiters, which are devices aiming to protect sensitive optical components or tissues from incident pulses of high irradiance or fluence.

Moreover, we have utilized a self-induced time reversal symmetry violation, which is imposed to a photonic structure from the incoming light when the incident power is above a critical value, in order to propose a photonic circuit that supports asymmetric transport. The circuitry relies on the implementation of high quality factor resonators made of a phase-change material that undergoes a thermally induced insulator-to-metal phase transition.

This dissertation is based on a number of published results associated with the above themes which are included in Appendix A. Below, a brief description of the content of each of the following chapters is provided.

In Chapter 2, we review the analytical and computational tools and methods that were used in order to obtain the results of this dissertation, such as the transfer and scattering matrix formalisms. We also briefly outline some physical concepts from the theory of photonic crystals which are essential for our designs.

Chapter 3 introduces the concept of photonic limiters and their applications. Our designs of reflective power and energy limiters are presented and an experimental realization of our proposal is briefly discussed. Finally, an alternative design of a metallodielectric photonic structure that can act as a reflective limiter is presented.

The design of reflective photonic limiters is extended to microwave applications in Chapter 4. Our setups rely on coupled resonator optical waveguide arrays exhibiting non-trivial topological features. Therefore, we start by introducing the essential concepts of topological photonics and continue with the discussion of our proposals which rely on features of topologically protected resonant modes.
In Chapter 5, we propose an optical design incorporating a phase-change material that exhibits asymmetric transport. We discuss the results of the simulation and capture the essential features of the system with a simpler semi-analytical model.

Finally, the conclusions of this dissertation are provided in Chapter 6. Overall, with this work we aim to take another step towards a direction where the field of photonics can view loss as a feature that can be exploited as a “blessing” rather than an “anathema”.
Before entering the main theme of this dissertation, in this Chapter we present the toolbox we used for our results. We start by deriving the Helmholtz equation from the 1-D electromagnetic wave equation. We then introduce the transfer matrix formalism, which is a useful tool for the numerical (and in some cases analytical) solution of the Helmholtz equation in linear stratified media. We define the scattering quantities of transmittance, reflectance and absorbance, which will be used extensively in the following Chapters. We proceed by examining the properties of 1-D photonic crystals, discussing the emergence of the band-gap and we briefly review the properties of defect modes. Moreover, we discuss a method for the solution of the nonlinear Helmholtz equation which completes our presentation of the ideas and techniques that we used for stratified media. We then define the transfer matrix formalism for one-dimensional discrete systems, which we utilize later to calculate transport properties of coupled res-
onator optical waveguide arrays. Finally, we introduce the scattering matrix formalism in the 1-D tight binding model, which can be a useful alternative to the transfer matrix formalism.

2.1 Helmholtz equation

Let us consider electromagnetic wave propagation through an one-dimensional medium, i.e. a medium that is infinite and uniform in the x-y direction and for which the permittivity only changes in the z direction. For simplicity, we start by only considering propagation along the z-axis, i.e. normal incidence. In this case, the transverse electric and transverse magnetic polarization directions are equivalent. The electromagnetic wave equation which governs wave propagation in this medium in terms of the electric field is:

\[
\frac{\partial^2 E(z, t)}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E(z, t)}{\partial t^2}.
\]  

(2.1)

Here, \(c = 1/\sqrt{\mu\epsilon}\) is the speed of light in the medium and \(\mu\) and \(\epsilon\) are the permeability and permittivity of the medium respectively. We can express all solutions of Eq. (2.1) as superpositions of monochromatic plane waves of the form \(E(z, t) = E_0 e^{i(k_n z - \omega t)}\), where \(\omega\) is the frequency and \(k_n\) the wavenumber in the medium. Replacing this form of solution to Eq. (2.1) and subsequently eliminating the time dependence by defining \(E(z) = E(z, t)e^{-i\omega t}\), we get

\[
\frac{\partial^2 E(z)}{\partial z^2} = -\frac{\omega^2}{c^2} E(z),
\]  

(2.2)

which is the time-independent Helmholtz equation in one dimension. From Eq. (2.2) and the functional form of the monochromatic plane wave, it can be deduced that the wavenumber is related to the frequency by the dispersion relation \(c = \omega/k_n\). It is useful to express the wavenumber in the medium in terms of the wavenumber in vacuum \(k, k_n = nk\), where \(n\) is the refractive index of the medium, which is in principle
z-dependent. We can now rewrite Eq. (2.2) as

\[ \frac{\partial^2 E(z)}{\partial z^2} + n(z)^2 k^2 E(z) = 0. \]  

(2.3)

The second order ordinary differential equation (2.3) has solutions in a medium of the form

\[ E(z) = F f(z) + B b(z), \]  

(2.4)

where \( f(z), b(z) \) are the two linearly independent solutions corresponding to the forward and backward propagating waves respectively and \( F \) and \( B \) are the corresponding wave amplitudes. In regions of uniform refractive index \( n, f(z) = e^{inkz} \) and \( b(z) = e^{-inkz} \).

The amplitudes \( F \) and \( B \) can be calculated by applying the appropriate boundary conditions, thus completely determining the stationary field for a scattering problem. For the general case of an inhomogeneous medium, the solution of Eq. (2.3) can be more complicated, but the electric field can always be calculated either analytically or numerically, as long as the form of \( n(z) \) and the boundary conditions are specified.

### 2.2 Transfer Matrix Formalism

The transfer matrix (TM) formalism is a powerful tool for the study of wave scattering in optical, acoustic, quantum and other types of systems [6]. By definition, it is the matrix that when multiplied with the wave amplitudes on the left side of a scattering sample provides us with the wave amplitudes on the right of the scattering domain.

In the case of one-dimensional systems, the electric field has the form of Eq. (2.4), with one component propagating to the left and the other propagating to the right. Therefore, the dimensions of the TM are \( 2 \times 2 \). If we denote the forward and backward propagating wave amplitudes as \( F^-, B^- \) on the left of the sample and \( F^+, B^+ \) on the right we have

\[ \begin{pmatrix} F^+ \\ B^+ \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} F^- \\ B^- \end{pmatrix}, \]  

(2.5)
where $M_{ij}$ are the matrix elements of the TM $M$.

Let us consider a scattering problem where our sample is illuminated from the left with a monochromatic wave of amplitude $F^-$. Part of the incident wave may be reflected back by the sample with reflected amplitude $B^-$. The rest of the incident energy is going to be transmitted through the sample and, assuming no losses inside the medium, transmitted on the right side with amplitude $F^+$. In order to study the transport properties of this system, we can define the transmission and reflection amplitudes for left incidence as $t_L = \frac{F^+}{F^-}$ and $r_L = \frac{B^-}{F^-}$ respectively. These quantities contain phase information and are, therefore, complex. Using these amplitudes, we can also define the real and more commonly used quantities of transmittance and reflectance as the absolute value squared of the corresponding complex amplitudes, i.e. $T_L = |t_L|^2$, $R_L = |r_L|^2$. If the media within the scattering domain are absorbing, another useful quantity is the absorbance, which is defined as $A_L = 1 - T_L - R_L$. Assuming scattering boundary conditions associated with a left-incident wave ($B^+ = 0$), and using Eq. (2.5), we can derive the complex amplitudes in terms of the TM elements:

\begin{align*}
  t_L &= \frac{\det M}{M_{22}} \quad \text{(2.6)} \\
  r_L &= -\frac{M_{21}}{M_{22}}. \quad \text{(2.7)}
\end{align*}

For right incidence, the incident, reflected and transmitted wave amplitudes are given by $B^+$, $F^+$ and $B^-$ respectively. In this case, $F^- = 0$. Using a similar method as for the case of left incidence, we obtain:

\begin{align*}
  t_R &= \frac{1}{M_{22}} \quad \text{(2.8)} \\
  r_R &= \frac{M_{12}}{M_{22}}. \quad \text{(2.9)}
\end{align*}
2.3 Transfer Matrix for Layered Media

In this dissertation, we study several problems involving wave scattering in stratified media. Because of their one dimensional geometry, they constitute a specific class of inhomogeneous media for which the TM is relatively easy to compute numerically.

Let us consider a medium constituted of \( N \) slabs of various dielectric media as shown in Fig. 2.1. The \( i \)th layer has thickness \( d_i \) and is made up of a material with refractive index \( n_i \). Each of the layers is homogeneous and is infinite in the \( x, y \) plane. In accordance with our earlier discussion, the electric field inside the \( i \)th layer will be \( F^i e^{i n_i k z} + B^i e^{-i n_i k z} \) for a monochromatic wave with vacuum wavenumber \( k \). In order to construct the TM connecting the forward and backward field amplitudes between different parts of the structure, we impose the condition of the continuity of the electric field and its derivative across interfaces between layers and take into account the accumulation of phase due to the propagation through each layer. Due to the composition rule of the transfer matrix [6], the total transfer matrix of the structure connecting the input and output waves will be given by [7]

\[
M_T = P_R K(L + 1, L) \prod_{i=1}^{N} (Q(i)K(i, i - 1))P_L, \tag{2.10}
\]

where the matrices utilized above are:

\[
P_R = \begin{pmatrix} e^{-i k n_R z_R} & 0 \\ 0 & e^{i k n_R z_R} \end{pmatrix}, \quad P_L = \begin{pmatrix} e^{i k n_L z_L} & 0 \\ 0 & e^{-i k n_L z_L} \end{pmatrix}, \tag{2.11}
\]

where \( z_L, z_R \) are the positions of the left and right ends of the structure in the chosen coordinate system and

\[
Q(i) = \begin{pmatrix} e^{i k n_i d_i} & 0 \\ 0 & e^{-i k n_i d_i} \end{pmatrix}, \quad K(i, i - 1) = \frac{1}{2n_i} \begin{pmatrix} n_i + n_{i-1} & n_i - n_{i-1} \\ n_i - n_{i-1} & n_i + n_{i-1} \end{pmatrix}, \tag{2.12}
\]

where we have considered \( n_0 = n_L \) and \( n_{N+1} = n_R \). The matrix \( K(i, i - 1) \) describes a kick occurring at the interface between the layers with indices \( i, i + 1 \), \( Q(i) \) is a matrix.
describing the free propagation inside the $i^{th}$ layer, and $P_L, P_R$ contain the phases of free propagation in air.

### 2.4 One-Dimensional Photonic Crystals: Band-gap and the Quarter Stack

In condensed matter physics, some categories of solids are known to have a crystalline structure, i.e. the atoms, molecules or ions composing them are arranged in space in a certain periodic fashion. This arrangement leads to the periodicity of the potential experienced by the electrons in the solid. The shape of the potential is crucial for the electronic properties of the solid, including the emergence of energy ranges that the electrons can or cannot occupy, namely the electronic bands and gaps respectively.

Photonic crystals (PCs) are a classical optical wave analogue of crystalline solids. The periodic potential due to the atoms or molecules is replaced by a periodic arrangement of materials with different permittivities, leading to a periodic permittivity function. This, in turn, can lead to a frequency spectrum with allowed and forbidden frequency ranges where light propagation can or cannot occur. In other words, PCs can support propagating waves (and thus are transmissive) only within certain frequency windows, while
reflecting the waves with frequencies that belong to the gap, also known as band-gap or stopband. Therefore, the design of the band-gap structure enables control of the allowed propagating frequencies, making PCs a handy tool in applications that require manipulation of light, such as frequency filters or mirrors. Although PCs can be implemented in one, two or three dimensions by appropriately manipulating the permittivity function of the optical structure, in this thesis, we only utilize one-dimensional PCs for applications, thus, we will focus on studying their properties in one-dimension. The properties mentioned, however, can be generalized for PCs of higher dimensionality.

The simplest case of a one-dimensional photonic crystal is the periodic multilayer film, which is known as a Bragg mirror (also referred to as Bragg reflector or grating). Bragg mirrors are constituted of alternating layers of two materials with different refractive indices. The arrangement is periodic, therefore, each of the layer types has the same thickness throughout the structure. An example of such a grating is shown in Fig. 2.2. The structure is composed of two different dielectric materials with homogeneous refractive indices $n_1$ and $n_2$ and respective layer thicknesses $d_1$ and $d_2$. The unit cell of this periodic structure is composed of one of each of those layers and has width $a = d_1 + d_2$. The primitive vector of the one-dimensional lattice is then $a\hat{z}$. We assume that the dielectric slabs extend to infinity in the $x$ and $y$ directions and also that the periodicity of the structure in the $z$ direction extends to infinity. The total wavevector can be written as $\vec{k} = \vec{k}_{x,y} + k_z\hat{z}$. The translational symmetry of the photonic crystal is continuous in the $x, y$ plane, allowing the component $k_{x,y}$ to take any value. However, the symmetry in the $z$ direction is discretized due to the periodicity of the crystal. This allows us to restrict $k_z$ to the first Brillouin zone due to the Bloch theorem.

We start by studying the dispersion relation of a trivial case where $n_1 = n_2 = n = 3.16$. In this example, the dispersion relation of the medium is simply $\omega = ck/n$. A plot of the nondimensional frequency vs. wavenumber is shown in Fig. 2.3(a) for the first Brillouin zone. The length of the unit cell was chosen arbitrarily since the medium is uniform. For $n_1 \neq n_2$, the dispersion relation can be found through the transfer
matrix of the unit cell $M_u(\omega)$. In particular, we know that, due to Bloch’s theorem, the forward/backward propagating waveforms at sites $z, z+a$ are connected by the relation $\Psi_{F/B}(z+a) = e^{\pm ika}\Psi_{F/B}(z)$. However, since the propagated waves at position $z+a$ can be found by the action of $M_u(\omega)$ onto the waves at position $z$, we can deduce that the eigenvalues of $M_u(\omega)$ are $e^{\pm ika}$. Therefore, the dispersion relation can be evaluated by calculating the trace of $M_u(\omega)$ and equating it to the sum of its eigenvalues, which yields the expression $2 \cos(ak) = \text{Tr}(M_u(\omega))$. In Fig. 2.3(b), we plot the dispersion relation of an infinite PC with $n_1 = 1, n_2 = 3.16$, like the one used in [8]. We also used $d_1 = d_2 = a/2$. We note the opening of a gap between the lines indicating the bands in the second case, in contrast to the coalescing light lines in the first one.

In order to understand why the photonic bandgap emerges in PCs, we first consider the states corresponding to the case $k_E = \pi/a$, i.e. the states that occur right at the band edge and for which the frequency coalesces in the trivial PCs. These two modes have a wavelength of $\lambda_E = 2\pi/k_E = 2a$ and, due to symmetry considerations, each mode will have their nodes and antinodes centered in layers of the same material. According to the variational theorem [9], the lowest frequency modes correspond to the modes for
which the following equation is satisfied:

\[ \omega_0^2 = \frac{c^2 \int |\nabla \times \vec{E}(\vec{r})|^2 d^3\vec{r}}{\int \varepsilon(\vec{r}) |\vec{E}(\vec{r})|^2 d^3\vec{r}} \]  

(2.13)

where \(\omega_0\) is the minimum frequency. This implies that, for the lowest frequency modes, the denominator of Eq. (2.13) is maximized. Therefore, for the lowest frequency modes, the local field intensity is maximum at the layers with the largest permittivity/refractive index. Due to the fact that the two modes are orthogonal, the other mode has its maximum field intensity at the layer with the lower refractive index. Since this field distribution cannot satisfy Eq. (2.13), the frequency of this mode has to be larger.
than that of the first. Another way to explain this qualitatively is the fact that the
other mode experiences a lower effective refractive index in the structure, which yields
a larger frequency. This discrepancy on the modal field distributions of the bottom and
top bands give them the respective names dielectric band and air band.

The width of the band-gap is characterized by the gap-midgap ratio, $\Delta \omega / \omega_c$, where
$\Delta \omega$ is the difference between the two band edges and $\omega_c$ is the frequency at the middle
of the bandgap. In general, this quantity increases with the contrast in the refractive
index of the chosen materials. It can be shown that the width of the bandgap can be
maximized for a given set of materials by choosing the thickness of the layers to satisfy
the condition $n_1d_1 = n_2d_2 = \lambda/4$, where $\lambda$ is the desired midgap wavelength. In this
case, it can be shown for the gap-midgap ratio that $\frac{\Delta \omega}{\omega_c} = \frac{4}{\pi} \sin^{-1} \frac{|n_2-n_1|}{n_2+n_1}$ [10]. This type
of Bragg mirror is know as the quarter-wave stack. Due to their property of maximizing
the bandgap, thus maximizing the reflection at the center of the bandgap, quarter-wave
stacks are very commonly used in applications.

2.5 Defect Modes

If the PC is illuminated with an electromagnetic wave with a frequency within the
bandgap, no extended states can be excited since there exist no real wavevectors satisf-
ying the dispersion relation. In order to investigate what happens in this case, we can
examine the bands at the vicinity the gap, following the steps outlined in [9]. Let us
consider the upper band in Fig. 2.3(b) $\omega_u(k)$. A Taylor expansion to 2nd order around
$k = \pi/a$ yields $\omega_u(k) \approx \omega_u(\pi/a) + \frac{\omega''_u(\pi/a)}{2} (k - \pi/a)^2$, where the 1st order term is taken
as zero due to time-reversal symmetry considerations. Setting $\beta = \frac{\omega''_u(\pi/a)}{2} > 0$, we
can conclude that, at the vicinity of the edge of the Brillouin zone, the correction to
the frequency is $\Delta \omega_u = \omega_u(k) - \omega_u(\pi/a) = \beta(k - \pi/a)^2$. For frequencies that belong to
the band, $\Delta \omega_u > 0$, thus $k \in \mathbb{R}$. Conversely, for frequencies in the bandgap, $\Delta \omega_u < 0$
leading to $k \in \mathbb{C}$. Therefore, the bandgap frequencies correspond to evanescent modes
of the form $E \sim e^{\pm \kappa z}$, where $\kappa = \text{Im}\{k\}$. If we send a wave with frequency within the bandgap to a semi-infinite PC, the wave will decay at a length scale of $1/\kappa$ and the value of $\kappa$ tends to be larger for frequencies at the vicinity of the midgap. Obviously these modes exhibit singular behavior as $z \to \mp \infty$ respectively. Therefore, they are physically unacceptable modes of excitation for an infinite PC.

If we break the translational symmetry of the PC by introducing a defect layer, then evanescent modes can be supported by the structure, since their exponential growth can be halted due to the termination of the crystal caused by the defect. Therefore, defects can enable the excitation of evanecent modes in a PC, provided that these evanescent modes respect the properties and shape of the defect. These modes are said to be localized at the defect. The term “localized” describes the shape of the stationary waves within the structure, as well as the fact that their group velocity is zero. The localization length is inversely proportional to the magnitude of $\kappa$, therefore, a frequency near the midgap generally results in tighter localization.

Let us consider the structure illustrated in Fig. 2.4. This structure consists of two quarter-stack PCs, each made up of 6 unit cells of the PC described in Fig. 2.2, and a dielectric defect layer with refractive index $n_d$ and of width $d_d$ placed in the middle.
Figure 2.5: Figure adapted from [11]. Transmission spectrum of the structure illustrated in 2.4. The materials used for the Bragg grating are SiO$_2$ and Si$_3$N$_4$ and the defect is a layer of GaAs. The specific material parameters are given in [11]. We assume that the constituent materials do not have losses.

The structure is mirror symmetric, i.e. the PC is deposited in a way that the order of the layers before and after the defect is flipped.

If this multi-layered structure is illuminated with a monochromatic plane wave at the frequency of the defect mode, say from the left, the corresponding evanescent mode is going to be excited. The $z$-propagating light that will reach the defect is going to be trapped between the two Bragg mirrors, bouncing back and forth. Therefore, the defect acts like a cavity. The supported modes in a finite cavity are quantized and are given by $n_d d_n = l\lambda/2$, where $l$ is an integer and $\lambda$ denotes the light wavelength in vacuum.
It can be deduced that, in order to design a quarter stack with a defect mode at the midgap, the optical length of a defect can be selected to be \( n_d d_n = \lambda/2 \), where \( \lambda \) is the midgap wavelength. The defect mode will then be the fundamental mode of the cavity corresponding to \( l = 1 \).

The transmittance spectrum of the structure in Fig. 2.4 is shown in Fig. 2.5 at the vicinity of the bandgap. We observe the edges of the transmittance bands and the transmittance dropping at the bandgap. At the midgap, we note the emergence of the defect mode which supports resonant transmission. We also include the experimental result for the transmittance spectrum at low-intensity. This result was obtained for the structure used for the experiment of Ref. [11], where the Bragg mirrors are composed of SiO\(_2\) and Si\(_3\)N\(_4\) and the defect is GaAs.

In Fig. 2.6, we show the steady state scattering field profile of the defect mode inside the same structure. The envelope of the field intensity experiences an exponential growth towards the defect and an exponential decay towards the end of the structure, as expected by a localized mode. The shape of the defect mode suggests enhanced sensitivity of this mode to the properties of the defect layer. Now that the properties of defect modes have been discussed, we are ready to outline how they can be used for designs of Reflective Photonic Limiters in Chapter 3.

### 2.6 Backward propagation for nonlinear media

In our discussion of the TM formalism, we have assumed linear media, i.e. media with linear permittivities. In reality, however, all material have a nonlinear optical response. The nonlinear components of the polarization density are the terms in its Taylor expansion of order 2 and above. When the magnitude of these terms is significant compared to the linear response term due to either large nonlinear coefficients or large electric field amplitudes, the TM method cannot be readily applied. The reason behind
this is that the calculation of the total TM for a structure assumes prior knowledge of the exact form of \( n(z) \) or \( \epsilon(z) \), but for nonlinear media, \( \epsilon(z) \) is a function of the local electric field, which is what we aim to calculate with the TM.

In order to address this problem, consider a Kerr-type nonlinearity for which the permittivity has the form \( \epsilon = \epsilon_1 (1 + \chi|E|^2) \). Since we cannot apply the TM formalism, we need to directly solve the nonlinear Helmholtz equation, which for our example has the form

\[
E''(z) + \epsilon_1 (1 + \chi|E|^2) k^2 E(z) = 0.
\]

However, there is a complication. One common issue that arises from the study of nonlinear systems is multistability, which is the presence of multiple solutions for a well-defined boundary value problem. In scattering problems, this leads to multiple possible outputs for a given input. To ensure that we obtain all possible solutions, we reverse the boundary condition: we start with the knowledge of the output field, and from this we integrate the nonlinear Helmholtz equation backwards to obtain the input field. This method is referred to as the backward map [12].

The backward map method is what we use for our nonlinear calculations in Ref. [13]. Af-
ter establishing the output boundary condition, we proceed by performing the backward integration numerically using two methods: for normal incidence, we directly integrate the nonlinear Helmholtz equation using the 4th order Runge-Kutta method. For oblique incidence, one needs to consider both transverse electric and transverse magnetic polarizations. This is done by an implementation of the transfer matrix formalism where the nonlinear material was broken into very small quasi-layers. Starting with the electric field amplitude given by the boundary condition, the local nonlinear permittivity was calculated and used to construct a transfer matrix which was used for the backward propagation to the previous quasi-layer. The electric field was then obtained, and by this the nonlinear permittivity was calculated and used for the next step of the backward propagation. This iterative process was used until the integration was completed for the total length of the nonlinear layer.

2.7 Transfer matrix for the 1-D tight-binding model

In Section 2.2, we introduced the transfer matrix formalism as a method to tackle photonic scattering problems and integrate the Helmholtz equation. In this section, the implementation of the the transfer matrix formalism for discretized systems is presented. In particular, we consider a one-dimensional array of resonators at the tight-binding approximation (i.e. the electric field is considered to be mostly confined within the resonators), such as the one shown in Fig. 2.7. The details of the tight-binding approximation are reviewed extensively in [6]. The wave amplitudes at the resonators $\psi_n$ are described by the stationary Schrödinger equation [6]

$$(\nu - \nu_n)\psi_n = t_{n-1}\psi_{n-1} + t_n\psi_{n+1}. \tag{2.14}$$
Figure 2.7: Scattering for the one-dimensional tight-binding model. The closed system is represented by the white unfilled circles. To perform a scattering experiment, the system is coupled to semi-infinite leads at the two ends of the chain, represented by the black circles. The incident wave $F^{-} e^{ikn}$ is sent from the left lead, while the reflected wave is $B^{-} e^{-ikn}$. The transmitted wave which reaches the lead on the right is given by $F^{+} e^{ikn}$.

From this equation, we can easily construct a matrix connecting the vector containing the wave amplitudes of two adjacent sites to the subsequent one:

$$
\begin{pmatrix}
\psi_{n+1} \\
\psi_{n}
\end{pmatrix}
= \begin{pmatrix}
\nu - \nu_{n} \\
1
\end{pmatrix}
\begin{pmatrix}
\psi_{n} \\
\psi_{n-1}
\end{pmatrix}
\equiv M_{n}
\begin{pmatrix}
\psi_{n} \\
\psi_{n-1}
\end{pmatrix}
$$

(2.15)

Each of the wave amplitudes can be expressed as a superposition of forward and backward propagating waves. This yields $\psi_{n} = F^{-} e^{ink} + B^{-} e^{-ink}$ for the waves on the left of the scattering setup. This allows the vector on the right handside of Eq. (2.15) to be written as

$$
\begin{pmatrix}
\psi_{n} \\
\psi_{n-1}
\end{pmatrix}
= QR_{n} \begin{pmatrix}
F^{-} \\
B^{-}
\end{pmatrix};
Q = \begin{pmatrix}
1 & 1 \\
- e^{-ik} & e^{ik}
\end{pmatrix};
R_{n} = \begin{pmatrix}
e^{ikn} & 0 \\
0 & e^{-ikn}
\end{pmatrix}.
$$

(2.16)

Therefore, the full transfer matrix which describes the process in Eq. (2.15) and connects the forward and backward propagating amplitudes before and after the site $\psi_{n}$ can be written as

$$
M_{n} = R_{n+1}^{-1} QR_{n} M_{n}.
$$

(2.17)

This formalism can be applied iteratively in order to connect the forward and backward propagating wave amplitude between multiple sites in a tight-binding model, similarly to the way we applied the optical transfer matrix in multilayered media. In particular,
it is easy to see that for a system similar to that shown in Fig. 2.7 with $\ell$ sites, the full transfer matrix can be written as

$$M^{(\ell)} = M_\ell M_{\ell-1}...M_1 = R_{\ell+1}^{-1} Q^{-1} M_\ell M_{\ell-1}...M_1 Q R_1.$$  

(2.18)

The properties discussed in Section 2.2 still hold, including the relations between the transmission and reflection amplitudes to the full transfer matrix elements $M^{(\ell)}$.

One concept that we use for scattering calculations of discrete systems is that of leads, i.e. auxiliary semi-infinite periodic chains that allow the discrete system of interest to be coupled to a continuum. For example, in the system shown in Fig. 2.7, the calculation of the full transfer matrix involves the coupling elements between the sites 0 and 1, as well as between the sites 8 and 9, which are the coupling strengths of the system to the two leads. In addition, leads determine the possible waves that can be sent into the system by appropriately manipulating their dispersion relation connecting the frequency $\nu$ to the wavenumber $k$. Leads are also extremely useful for the implementation of the scattering matrix formalism which is discussed in the following section.

### 2.8 Scattering matrix for the 1-D tight-binding model

An alternative way to look at scattering problems is the scattering matrix (S-matrix) formalism [6]. The S-matrix has broad applications in both quantum and classical wave physics. It acts on the initial state of a system and returns its final state after a scattering event. In an optical setup, following the notation of the scattering event described in Section 2.2, the S-matrix connects the input to the output wave amplitudes as follows:

$$\begin{pmatrix} B^- \\ F^+ \end{pmatrix} = S \begin{pmatrix} F^- \\ B^+ \end{pmatrix}. \quad (2.19)$$
After a bit of algebra, we can easily calculate the S-matrix elements with respect to the transmission and reflection amplitudes. We get
\[
S = \begin{pmatrix}
  r_L & t_R \\
  t_L & r_R
\end{pmatrix}.
\] (2.20)

For a system of optical or microwave resonators in the tight-binding regime, the S-matrix is a $M$-dimensional matrix, where $M$ is the number of channels at which the system is coupled to leads. The $S^{(i,j)}$ matrix elements of the S-matrix are the scattering coefficients (transmission and reflection amplitudes) describing the waves that enter the scattering setup at the $j$th channel and exit it at the $i$th. In order to calculate the matrix for a one-dimensional system, we start by considering the $N$-dimensional closed system Hamiltonian $H_B$, where $N$ is the number of lattice sites/resonators in the one-dimensional array. Each lead is assumed to be a periodic one-dimensional chain of resonators with resonant frequency $\nu_0$ and intersite coupling $t_L$. The dispersion relation of each of the leads is then $\nu = \nu_0 + 2t_L \cos k$. The S-matrix describing this system is then given by [14]
\[
S = -\mathbb{1}_M + 2i \frac{\sin k}{t_L} W^T \frac{1}{H_{\text{eff}} - \nu \mathbb{1}_N} W,
\] (2.21)
where $W$ is a $N \times M$ matrix whose elements $w^{(i,j)}$ are equal to the coupling strengths of the $i$th site with the $j$th lead, and $H_{\text{eff}} = H_B + \frac{e^{ik}}{t_L} WW^T$ is the effective Hamiltonian of the scattering problem. We utilize Eq. (2.21) in [15] for the theoretical calculation of transmittance, reflectance and absorbance.
Reflective Photonic Limiters Using Non-Hermitian Stratified Media

After the proposal and subsequent realization of the laser around 1960 [16,17], the need for protection of tissues and sensitive optical components against high intensity radiation soon became evident. This urgency immediately triggered the research endeavors for the development of structures that could act as high-power filters. These designs, coined optical power limiters [18], offered protection against laser beams through the usage of optical nonlinearities [18,19].

The research in the field of photonic limiters has yielded devices that can be grouped into two basic categories: dynamic and passive limiters [20]. Dynamic limiters consist of many intercommunicating modules. The limiting action of dynamic limiters is achieved by active processes within the apparatus. The most common example is an iris, whose aperture is dynamically adjusted based on the feedback provided by an auxiliary sensor. An important drawback of these devices is their response time, which tends to be impeded by the communication time between modules. Passive limiters, on the other hand, are limiters for which the limiting action is triggered by the optical properties of
the materials used in their design. These materials are typically nonlinear, similar to the ones used in the earliest limiters. The main benefits of passive limiters are their simplicity, the lack of the energy requirements associated with the operation of an active device, and their fast response time, which matches the optical response time of their constituent materials. Due to those benefits, the present discussion will be focused on passive limiters.

In this Chapter, we start by discussing the basic operational properties and features of existing passive limiters, including the introduction of a figure of merit. We also present the common downfalls and limitations of existing limiters. We then proceed to the discussion of our proposal and we summarize our contributions.

### 3.1 Passive limiters

The operational characteristics of an ideal limiter are illustrated in Fig. 3.1. For incoming light of low intensity, the ideal limiter should transmit the incident beam linearly. At a value of incident light intensity which is referred to as the *limiting threshold*, the limiter should reduce the signal strength to an output intensity of a constant value. In realistic photonic systems, this decrease in transmittance is gradual and not as abrupt as shown in Fig. 3.1. Obviously, this saturation value should be bounded by the minimum light intensity which has the potential to damage the protected optical component or tissue, i.e. the *sensor damage threshold* indicated by the horizontal dashed line in Fig. 3.1. As the incident intensity increases, one expects that the limiter will be damaged due to thermal effects or electrical breakdown of its constituent materials. The *limiter damage threshold* is the value of input intensity for which the limiter is rendered destroyed. Based on these quantities, one can designate a figure of merit for limiters which is referred to as the *dynamic range*. This quantity is defined as follows [21]:

\[
\text{dynamic range} = \frac{\text{limiter damage threshold}}{\text{limiting threshold}}. \quad (3.1)
\]
Figure 3.1: Behavior of an ideal limiter. The limiter transmits low-intensity incident radiation linearly. Above the limiting threshold, the limiter yields a constant transmitted intensity, which should be below the sensor damage threshold. A limiter also has its own damage threshold for high values of input intensity.

The objective of the design of new photonic limiters is the extension of the dynamic range in order to encompass a broader range of potential applications.

The main obstacle for extending the dynamic range of most passive limiters is their principal mechanism of operation: typically, limiters block excessive radiation by absorbing it, which often leads to their destruction due to overheating, effectively keeping their damage threshold at low values. A common design of a passive limiter consists of a single slab of material with complex permittivity $\epsilon = \epsilon' + i\epsilon''$, where $\epsilon''$ experiences a steep increase with the incident light intensity. When the incident light intensity is low, the limiter is transparent due to a negligible value of $\epsilon''$. If, however, the limiter is illuminated with a input signal of high intensity, $\epsilon''$ assumes a large value and the limiting material becomes opaque, leading to high absorption. The absorbed energy can lead to overheating that renders the limiter permanently opaque and in need of a repair or replacement. There is a plethora of physical mechanisms that can cause a steep increase in $\epsilon''$, including two-photon absorption [22], nonlinear scattering [23], reverse saturable absorption [24] and many others. For an extensive overview of limiting mechanisms, the reader can refer to [20,21].
There are two intensity features of a laser beam that can induce damage: (a) its instantaneous intensity and (b) the total energy carried by the pulse. In the former case, the damage is induced by the high peak intensity of a laser pulse, which can cause electrical breakdown of a dielectric material and can occur very fast. In order to safeguard from this type of damage, we need protective devices for which the increase in $\epsilon''$ is caused by the instantaneous intensity of the field [20]. We refer to this type of devices as *power limiters* and they can have response times as short as femptoseconds [25]. In case (b), damage can occur due to prolonged exposure to a laser source of high fluence, for example in the case of a CW laser source that illuminates a target for an extended period of time. This scenario involves a cumulative effect due to the total energy emitted by the laser beam, which gradually overheats the optical system [20]. This case calls for a limiter with an $\epsilon''$ that increases with temperature. The devices that offer protection from this type of damage are referred to as *energy limiters*. Sometimes $\epsilon''$ can be increased by a combination of mechanisms.

There is an alternative pathway for the realization of passive limiters. In this case, the emphasis is placed on the design of photonic structures which extend the dynamic range of existing limiting materials. This idea often relies in the implementation of photonic crystals that incorporate nonlinear constituent materials. The limiting mechanism can be a shift in the spectral location of the bandgap at high intensities due to Kerr nonlinearity [26], modulation instability [27], or metallodielectric designs [28]. Some drawbacks of these designs are the lack of broadband protection, since they often rely on simple shifts of the bandgaps, and high absorption which can lead to a low limiter damage threshold.

In the material presented in this dissertation, we utilize the latter approach for the design of photonic limiters, while addressing the common problems associated with them. In particular, we introduce the concept of *reflective photonic limiters* based on resonant transmission using judiciously designed photonic crystals. The dominant mechanism of protection is reflection instead of the usual absorption, which results in self-protecting
devices. Another advantage of these designs is that the protection yielded is broadband. We explore the idea for both a power [13] and an energy [29] limiter. In collaboration with scientists at the Sensors Directorate of the Air Force Research Laboratory, a prototype reflective photonic limiter was designed [11]. In addition, we consider a setup with a metallo-dielectric structure [8]. In the remainder of this Chapter, we present the design of reflective photonic limiters and summarize our published contributions to the field. All the relevant journal publications are included in the Appendix A.

### 3.2 Concept of Reflective Photonic Limiter

In Chapter 2, we discuss how a multi-layered structure can be designed to support a localized mode by incorporating a defect layer. In the present Section, we show how the properties of this mode can be exploited for the design of a novel type of photonic limiters. In particular, we propose the design of reflective photonic limiters based on resonant transmission through a localized defect mode. A schematic of a reflective limiter is illustrated in Fig. 3.2. A layer with nonlinear optical response is embedded as a defect between two identical Bragg mirrors. The frequency of the defect mode lies within the photonic bandgap of the multi-layered structure. At low-level incident radiation (Fig. 3.2(a)), the structure is transmissive at the frequency of the defect mode. At high level incident radiation, (Fig. 3.2(b)), the setup blocks the excessive radiation by reflecting it back. Below, the concept is discussed in the framework of both power and energy limiters.

#### 3.2.1 Reflective Power Limiters

In Fig. 3.3(a), we present the scattering field profile for the defect resonant mode for low light intensity of the structure illustrated in Fig. 3.2, which was calculated using the transfer matrix formalism. Now, let us assume an implementation of this structure
for which the defect layer is made up of a material which is lossy at high intensities and almost lossless at low intensities. Materials with these optical properties were discussed in the introduction of this chapter as the ideal constituent materials for passive photonic limiters: they have low linear losses, therefore allowing high transmission at low light intensities, however they have high nonlinear losses which enable them to block high-level incident radiation. Therefore, if the incident pulse is of low intensity, the field distribution inside the structure of Fig. 3.2 will look as shown in Fig. 3.3(a): the losses at the defect layer remain negligible, thus corresponding to $\epsilon'' \approx 0$. The associated transport properties are shown in Fig. 3.4. The structure supports resonant transmission, while reflectance and absorbance are zero.

Conversely, for strong incident pulses, the local field intensity at the defect is going to increase, alongside the magnitude of the imaginary part of the permittivity due to nonlinearities (see Fig. 3.3(b), where $\epsilon'' = 5$). The envelope of the field no longer has the shape of a localized mode. Instead, we observe a near-exponential decay from the incident interface to the end of the structure. Hence, the localized mode is now destroyed, which leads to the suppression of the resonant transmission. Indeed, in Fig. 3.4 we present transmittance, reflectance and absorbance vs. frequency for different loss strengths at the defect. Transmittance naturally drops as the value of $\epsilon''$ goes up. Absorbance starts increasing as losses are introduced, however, for larger loss strengths,
absorption starts to drop and is eventually suppressed.

This seemingly counterintuitive behavior can be explained if we realize that the existence or destruction of the localized mode depends on the balance between two dissipative mechanisms \cite{30}. On one hand, the resonant mode experiences radiative losses from the boundaries of the structure, which are related to its linewidth $\Gamma_R$. On the other hand, as $\epsilon''$ increases, so do the ohmic losses at the defect $\Gamma_d$. These losses can be estimated to be

$$\Gamma_R \approx \frac{1}{\xi} e^{-2L/\xi}, \quad \Gamma_d \approx k\epsilon'' d/\xi,$$

where $L$ is the total length of the multi-layered structure $\xi$ the localization length of the defect mode, and $d$ the thickness of the defect layer. When $\Gamma_R \gg \Gamma_d$, the system is in the underdamping regime. The defect mode has a high quality factor and supports resonant transmission. Transmittance is close to 1 as reflectance and absorbance go to 0. As the value of $\epsilon''$ increases, absorption starts increasing and transmittance drops, while
reflectance increases. The trend will continue until we reach the value of $\varepsilon''$ associated with the critical coupling regime, $\varepsilon''_{CC}$, which occurs when the Ohmic losses at the defect $\Gamma_d$ become equal to the radiative losses $\Gamma_R$. At this point, the absorbance reaches its peak value. While $\varepsilon'' < \varepsilon''_{CC}$, since the radiative losses are still the dominant dissipation mechanism, the dwell time of photons in the structure is long enough for them to be absorbed by the defect. However, as the value of $\varepsilon''$ increases to values above $\varepsilon''_{CC}$, we enter the overdamping regime where the radiative losses are overcome by the Ohmic losses. This leads to a decrease of the photon dwell time. Therefore, the photons do not dwell in the structure long enough to be absorbed, causing a decrease of absorbance and transmittance, while reflectance increases. Eventually, we reach a domain where $\Gamma_R \ll \Gamma_d$, where the reflectance becomes the dominant mechanism and transmittance and absorbance vanish.

The resulting limiter has the benefit of an extended dynamic range compared to a single-material passive photonic limiter for two reasons. On one hand, the shape of the localized mode enhances its sensitivity to the optical properties of the defect, effectively lowering the limiting threshold. On the other hand, we note that in Fig. 3.3(b), we observe a drop of the local field intensity of about two orders of magnitude between the incident interface and the defect layer. This drop protects the limiting material not only by effectively reducing the absorption, but also by offering protection from electrical breakdown: in a “traditional”, singe material photonic limiter, the limiting material would experience the full incoming pulse. Consequently, the damage threshold of the multi-layered limiter rises to much higher intensities. Due to the fact that for this limiter the dominant mechanism is reflection instead of the usual absorption, we refer to this type of limiters as reflective photonic limiters.

Before proceeding to the summary of our results on reflective photonic limiters, we want to stress some important conditions for the design of the multi-layered structure. First, as mentioned before, the linear losses of the material used for the defect layer need to be negligible to ensure high transmittance in the low-intensity regime, while the nonlinear
Figure 3.4: (a) Transmittance, (b) reflectance, (c) absorbance vs. frequency at the vicinity of the bandgap for different values of $\epsilon''$. Transmittance at the frequency of the defect mode drops with the increase of $\epsilon''$. Absorbance behaves non-monotonically: it increases as losses are introduced, but as the loss strength increases it starts dropping again. Reflectance becomes the dominant mechanism for large values of $\epsilon''$, corresponding to large light intensities.

losses need to be high. In addition, the constituent materials of the Bragg mirrors need to be lossless and highly linear in order to ensure high transmission and to avoid nonlinear shifts of the bands. Finally, the Bragg mirrors need to have a high damage threshold. This last condition enables the reflective photonic limiter to withstand the large local field intensities close to the incident interface in the high incident intensity regime as shown in Fig. 3.3(b).

The first application of the design of a reflective photonic limiter is in our article on the concept of a reflective power limiter in Ref. [13]. This design consists of a pair of identical Bragg mirrors, each made up of a total of 40 layers of two alternating dielectrics. A power limiter requires an instantaneous lossy nonlinearity, therefore,
a defect layer with permittivity of the form $\epsilon_{\gamma} = \epsilon[1 + i\gamma|E(z)|^2]$ was used between the Bragg mirrors. The defect supports a resonant localized mode at the bandgap. Transmittance, reflectance, and absorbance vs. input power were calculated for the defect mode of this structure and a comparison with a single nonlinear layer (single material limiter) was presented. Limiting action is found to occur at much smaller values of input power for the multi-layered structure than for the single layer, with the transmittance dropping rapidly. Meanwhile, the high-power absorbance is much lower for the full layered structure compared to the single layer, leading to a self-protecting device. The multi-layered structure exhibits broadband reflection at high powers and this result was confirmed for normal and oblique directions of incidence. Finally, we performed a theoretical analysis for which the nonlinear defect was approximated by a delta function $\epsilon_{\gamma}(z) = \epsilon[1 + i\gamma|E(z)|^2]\delta(z)$. The analytical results were in good agreement with the numerical calculation. The article [13] is included in full in the Appendix A.

### 3.2.2 Reflective Energy Limiters

In the previous subsection, the design of a reflective power limiter based on resonant transmission via a defect localized mode was introduced. Here, we discuss how this design can be modified for the implementation of an energy limiter. In particular, by introducing a defect layer with $\epsilon''$ that increases with temperature, the multi-layered structure illustrated in Fig. 3.2 can exhibit energy limiting properties, depending on how the thermal relaxation time of the system compares to the pulse duration of the incident signal.

For the reflective energy limiter presented in Ref. [29], we proposed a multi-layered structure consisting of two identical Bragg reflectors, each composed of 20 dielectric bilayers and a defect layer whose imaginary part of permittivity $\epsilon''$ is temperature-dependent. The instantaneous value of $\epsilon''$ is assumed to be independent of the field
intensity in this case, however, for a pulse with large total fluence, the permittivity is slowly affected due to the increase in temperature $T$ because of absorption. We assumed a simple linear relationship between $\epsilon''$ and temperature. The temperature as a function of time is calculated by introducing a rate equation

$$\frac{dT(t)}{dt} = \frac{1}{C}[A(T)W(t) + \kappa(T_0 - T)],$$

where $C$ is the heat capacity, $A(T)$ is the absorbance, $W(t)$ the light intensity, $\kappa$ the cooling rate of the system and $T_0$ the environment temperature. We calculated transmittance, reflectance and absorbance vs. pulse duration self-consistently with the rate equation for two different values of $\kappa$. For $\kappa = 0$, transmittance drops quickly, absorbance drops after an initial increase and reflectance quickly reaches values near unity. For small values of $\kappa$, the behavior is similar, but due to the cooling the system eventually reaches a steady state regime, which leads the transport characteristics of the energy limiter to stay constant for any pulse duration above a certain threshold. In both cases, we have an improvement of the limiting features compared to the standalone limiting layer. This improvement is quantitatively similar to that of the power limiter. Large values of $\kappa$ effectively describe situations where the thermal relaxation time is smaller that the heating rate of the system, in which case limiting action will not be triggered. Ref. [29] is provided in the Appendix A.

### 3.3 Experimental Realization of a Reflective Photonic Limiter

In Ref. [11], a prototype reflective limiter was experimentally realized within our collaboration with researchers of the Sensors Directorate of the Air Force Research Laboratory. The design consists of a half-wave defect of GaAs sandwiched between 2 identical Bragg mirrors, each made up of 6 alternating quarter-wave layers of SiO$_2$ and Si$_3$N$_4$. GaAs was chosen because it is a material that has good transparency at low powers at the
frequency range of interest, which was around 1.6\(\mu\)m. In addition, GaAs is known to have a high two photon absorption coefficient in this frequency range when in crystalline form. Due to the fact that the GaAs used in the experiment would be amorphous, a characterization was first performed where it was confirmed that the material exhibits nonlinear absorption in the amorphous form as well. The multi-layered structure was deposited onto a borosilicate glass substrate using the plasma-enhanced chemical vapor deposition method. Then, a Spectra-Physics Solstice Ti:Al_2O_3 laser with a 150 fs width Gaussian pulse and a repetition rate of 1 kHz was used to obtain transmittance and reflectance at different values of peak irradiance and within a wavelength range of about 1.60 – 1.64\(\mu\)m. The structure was experimentally confirmed to exhibit reflective limiting. Ref. [11] is supplied in the Appendix A.

### 3.4 Hypersensitive transport in photonic crystals with accidental spatial degeneracies

In Ref. [8], we present the design of a metallodielectric multi-layered structure which has the potential to be used as a reflective photonic limiter. The design of the structure is shown in Fig. 3.5. It consists of two quarter-wave Bragg gratings, which in this case are not identical. However, the two different gratings are designed to have the same band-gap structure. Each grating consists of 5 bilayers of dielectrics. The materials used for the grating on the left have refractive indices \(n_1, n_2\), where \(n_1 > n_2\), while the ones used on the grating on the right have \(n_3 < n_4\). The structure starts and ends with the large \(n\) material of each grating. At the interface, we introduce a cavity, which is made up of a quarter-wave layer of \(n_1\) and a quarter-wave layer of \(n_4\), with a very thin metallic nanolayer embedded in between. The defect cavity supports a localized mode which provides resonant transmission in the middle of the photonic bandgap (see blue line in Fig. 3.5(b)), with a nodal point of the associated electric field distribution which spatially coincides with the position of the metallic nanolayer (see blue line in Fig. 
Figure 3.5: Figure taken from [8]. On the left, the metallodielectric multi-layered structure is illustrated, along with the field profile of the defect mode for the unperturbed system (blue) and for a system with an increase of 2% on the permittivity of the white layer adjacent to the metallic nanolayer. (a) Field profiles at the vicinity of the metallic nanolayer (green). The mode of the unperturbed system exhibits a nodal point at the same position as the metallic layer. This is no longer true for the perturbed system (b) Associated transmittance spectra for the unperturbed (blue) and perturbed (red) systems.

3.5(a)). This coincidence of the position of the metallic nanolayer with the nodal point is what we call accidental spatial degeneracy and is the feature that enables resonant transmittance for the unperturbed system.

If a small perturbation in the refractive indices of either of the layers of the central cavity is introduced, for example due to heating because of the tiny absorption caused by the metallic nanolayer, then the nodal point of the electric field will be shifted (Fig. 3.5(a) red line). The nodal point of the mode no longer coexists with the metallic nanolayer. In fact, the mode will now experience losses due to the significant value of the electric field at the position of the metallic defect. This will initially increase the absorption and will
eventually trigger the underdamping-to-overdamping mechanism discussed previously in this Chapter, leading to the suppression of the resonant transmission and absorption and causing the reflection of the incident wave.

In Ref. [8], the sensitivity of the transmission of the proposed metallodielectric structure was studied. The transmittance exhibits high sensitivity to even tiny perturbations of the permittivity of the layers of the central cavity. For a perturbation of $\sim 1\%$ of the permittivity of the layer on the left, transmittance drops to $\sim -70$ dB and absorbance to $\sim -40$ dB around the frequency of the defect mode.

In order to confirm the effectiveness of this design as a reflective limiter, we consider illuminating the structure with a pulse of intensity $W_I(t)$. The tiny absorption of the metallic nanolayer will introduce heating of both the nanolayer and the adjacent dielectric layers of the central cavity. We assume that the dielectric on the left of the metallic nanolayer is temperature-dependent, increasing linearly with temperature. In order to obtain the temperature at the dielectric layer, we introduced the following heat rate equation

$$\frac{dT}{dt} = \frac{1}{C} A(T) W_I(t),$$

where $C$ is the heat capacity and $A(T)$ is the temperature dependent absorption coefficient. The structure was confirmed to exhibit reflective limiting properties. Ref. [8] is included in full in Appendix A.
Waveguide limiters based on topologically protected resonant modes

In Chapter 3, the concept of photonic limiters was introduced, with the emphasis placed on limiters used for sensor protection in the optical and near-infrared part of the electromagnetic spectrum. When used for free-space applications, the limiting action usually occurs by guiding the beam through a continuous medium with limiting properties. However, limiters in a closed waveguide environment are of interest as well, especially for applications associated with receiver protectors in radar systems. An example of such a limiter can consist of resonators inside a waveguide which are evanescently coupled, or it might involve smaller set-ups with single ring resonators coupled to bus waveguides [31]. The former can have applications in the microwave and radio wave regimes in free space, while the latter can be useful for on-chip protection.

In the microwave and radio wave regimes, limiters are widely used in transmitter-receiver (TR) systems, such as radars and communication satellites [32,33]. In this framework, they are often called receiver protectors and their purpose is to prevent damages inflicted on sensitive electronic components by high-power signals. In TR systems, these signals
are often sourced within the system: the transmitter emits powerful signals to space, while the receiver needs to be sensitive to weak signals. The transmitter and the receiver share the same antenna, which can lead to problems: if the emitter sends out a strong signal, part of it will be reflected from the antenna due to impedance mismatch. If the reflected wave reaches the sensitive receiver, the latter can be destroyed. For this reason, receiver protectors are used to shield the receiver from the signals emitted by the transmitter. Some examples of receiver protectors include TR tubes and PIN diodes. Unfortunately, many solutions have the disadvantage of high power consumption [34], while PIN diodes exhibit spike leakage problems due to the time-delay between the signal reception and the diode turning on [35].

Meanwhile, the field of topological photonics has emerged as a rapidly growing research field during the last decade [36–38]. The inspiration for this field is rooted into the discovery of topological insulators, which are bandgap materials with the unique property of being insulating in the bulk, while supporting conducting states at their interface known as edge or surface states [39]. These states allow for electron transport which is very robust to impurities. The basic premise of topological photonics is to explore topological phases of matter in optical setups and harness the robustness of edge states for a new class of optical devices [37]. Some important results in topological photonics include a photonic analogue of the quantum Hall effect [38], mode selectivity [40], unidirectional edge state propagation without backscattering [41], topological lasers [42,43], and more. Non-Hermitian setups have also been explored as a fertile platform for the observation of topological effects [40,42–45].

In this Chapter, we apply ideas from topological photonics to the design of reflective waveguide photonic limiters. We start by reviewing some fundamental concepts in the field of topological photonics, including how edge states can emerge as a consequence of various symmetries and how topologically protected defect modes are supported by edge states in one-dimensional CROW arrays with topological features. Using all these concepts, we present experimentally and theoretically the principle of operation of a
reflective CROW array microwave limiter [46]. We also present an alternate design based on self-induced $CT$ symmetry violation, with suggestions for its implementation in the microwave/radio wave domain and on-chip in the infrared part of the electromagnetic spectrum [15]. Due to the robustness of the defect mode to positional imperfections of the waveguides, these limiters have the benefit of spectral protection of the transmitted frequency at low powers.

4.1 The SSH model - chiral and charge conjugation symmetries

The Su-Schrieer-Heeger (SSH) model [47] describes a 1-D lattice with alternating hopping amplitudes, i.e. a dimer chain. Originally introduced to describe the properties of polyacetylene, it is the simplest model describing a topological insulator. We look into an implementation of an SSH array in electromagnetics, where the lattice sites are represented by identical dielectric resonators with frequency $\nu_0$ positioned at alternating distances $d_1$ and $d_2$. The distance between two resonators determines the evanescent coupling strengths, which are $t_1$ and $t_2$ respectively. We assume that the electric field is confined inside the resonators for the most part, therefore establishing a tight-binding regime [6,40]. Due to the field confinement, only the coupling between nearest neighbors is considered. The bulk is then described by the following Hamiltonian [6]

$$H_b = \sum_n \nu_n |n\rangle \langle n| + \sum_n t_n (|n\rangle \langle n+1| + |n+1\rangle \langle n|),$$

(4.1)

where $\nu_n = \nu_0$ in the resonant frequency of the $n^{th}$ resonator, and $t_n = t_1$ or $t_2$ is the strength of the evanescent coupling between the resonators with indices $n$ and $n+1$. The SSH lattice is bipartite, i.e. one can define two sublattices based on the dimer structure as seen in Fig. 4.1 and there are no couplings between sites belonging to the same sublattice. Sublattice A consists of the odd-indexed sites (blue sites in Fig. 4.1) while sublattice B is defined by the even-indexed resonators (white sites).
The original version of the SSH model is Hermitian, since it is inspired by chemical and condensed matter physics, i.e. there are no losses at the resonators in the photonic implementation. In this case, we can define a new auxiliary Hamiltonian by $H = H_b - \nu_0 \mathbb{1}$, since $\nu_0$ is a common number added to all diagonal elements, thus, the only consequence of subtracting it is a shift of the spectrum. This Hamiltonian anticommutes with the chiral symmetry operator $C$, i.e.

$$\{H,C\} = 0,$$

where the chiral symmetry operator for $N$ dimers is $C = \bigoplus_{i=1}^{N} \sigma_z$ [39]. In the Wannier basis, the operator takes the form

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & -1 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{pmatrix}.$$  

$C$ is both Hermitian and unitary, i.e. it has the property $C^\dagger C = C^2 = 1$. When this anticommutation relation holds, $H_b$ is said to have chiral symmetry. Chiral symmetry is associated with some important properties of the spectrum which is discussed in more detail later in this Chapter.

The dispersion relation of the SSH model is

$$\nu(k) = \nu_0 \pm \sqrt{t_1^2 + t_2^2 + 2t_1t_2 \cos k},$$

where $\nu$ is the frequency and $k$ the wavevector which takes values in the interval $[-\pi, \pi]$ and the lattice constant has been taken to be equal to 1. From this we can define two allowed bands at the frequency ranges $\nu_0 - t_1 - t_2 < \nu < \nu_0 - |t_1 - t_2|$ and $\nu_0 + |t_1 - t_2| < \nu < \nu_0 + t_1 + t_2$, defining a bandgap of width $2|t_1 - t_2|$. The dispersion relation for different configurations of the SSH model is shown in Fig. 4.2. We see the opening of
the bandgap occurring when $t_1 \neq t_2$ in Fig. 4.2(a),(c), while for $t_1 = t_2$, the dispersion relation reduces to our known result for the tight-binding model with uniform coupling $\nu(k) = \nu_0 \pm 2t_1 \cos k/2$ seen in Fig. 4.2(b).

In the presence of losses at the resonators, i.e. complex eigenfrequencies of any of the resonators $\nu_n$, $H$ no longer anticommutes with the Hamiltonian, so the condition of chiral symmetry is violated. However, if the real part of the resonant frequency of all resonators is still the same, $H$ anticommutes with the $CT$ operator, therefore we have

$$\{H, CT\} = 0. \quad (4.5)$$

Above, $T$ indicates the time-reversal operator. A Hamiltonian with the property shown in Eq. (4.5) is said to have particle-hole symmetry or charge conjugation symmetry. Again, this property has important implications for the spectrum of the Hamiltonian, which are discussed in the following section.
4.2 Spectral and eigenstate implications of chiral and charge conjugation symmetries

In this section, we examine the consequences of chiral and $CT$ symmetries in the spectrum and eigenstates of $H_b$. We work with the auxiliary Hamiltonian $H$ defined earlier, but, as mentioned before, the conclusions can be generalized to $H_b$ by merely adding $\nu_0$ to the spectrum.

Let us start with the case of a Hamiltonian with chiral symmetry. The eigenvalue
Chapter 4 - Waveguide Limiters

The equation is
\[ H |\psi_n\rangle = \tilde{\nu}_n |\psi_n\rangle , \]  
where \( \tilde{\nu}_n = \nu_n - \nu_0 \) is the eigenfrequency corresponding to the eigenstate \( |\psi_n\rangle \). We can insert the identity operator to the left handside of this equation and proceed as follows
\[ H \mathbb{1} |\psi_n\rangle = \tilde{\nu}_n |\psi_n\rangle \implies (HC)C |\psi_n\rangle = \tilde{\nu}_n |\psi_n\rangle . \]

Using the chiral symmetry property, this yields
\[ - (CH)C |\psi_n\rangle = \tilde{\nu}_n |\psi_n\rangle , \]
and after after acting from the left with the \( C \) we acquire
\[ H(C |\psi_n\rangle) = -\tilde{\nu}_n(C |\psi_n\rangle) . \]

Thus, if \( \tilde{\nu}_n \) is an eigenvalue of \( H \) with corresponding eigenstate \( |\psi_n\rangle \) there exists an eigenfrequency \( \tilde{\nu}_m = -\tilde{\nu}_n \) with corresponding eigenstate \( |\psi_m\rangle = C |\psi_n\rangle \). Therefore, all eigenfrequencies of \( H \) exist in pairs of opposite values.

This spectral property has consequences in the eigenstates. We start by defining the projection operators for the sublattices A and B, \( P_A = \sum_{n \text{ odd}} |n\rangle \langle n| \) and \( P_B = \sum_{n \text{ even}} |n\rangle \langle n| \). Then a unitary representation of the chiral symmetry operator is \( C = P_A - P_B \). The projection operators can be written as \( P_A = \frac{1+C}{2} ; P_B = \frac{1-C}{2} \).

If \( \tilde{\nu}_n \neq 0 \), then the states \( |\psi_n\rangle \) and \( |\psi_m\rangle \) have to be orthogonal since they correspond to different eigenvalues. We then have
\[ \langle \psi_n | \psi_m \rangle = \langle \psi_n | C | \psi_n \rangle = 0 \implies \langle \psi_n | (P_A - P_B) | \psi_n \rangle = 0 \implies \langle \psi_n | P_A | \psi_n \rangle = \langle \psi_n | P_B | \psi_n \rangle , \]
which implies that for non-zero “energy”, which in photonics translates to the frequencies of the bands, the corresponding states are supported equally on each of the sublattices.
For $\tilde{\nu}_n = 0$, the orthogonality condition no longer holds and we have $|\psi_n\rangle = \pm C |\psi_n\rangle$, i.e. an eigenstate corresponding to $\tilde{\nu}_n = 0$ is its own chiral symmetric partner. If the action of the chiral symmetry operator does not affect the state, it is implied that the “zero” energy states (midgap in photonics) have support on only one of the two sublattices.

For the case of $CT$ symmetry, we start from Eq. (4.6) and act from the left with the $CT$ operator and then use the property of $CT$ symmetry. This gives us

$$CTH|\psi_n\rangle = CT(\tilde{\nu}_n |\psi_n\rangle) = \tilde{\nu}_n^*(CT|\psi_n\rangle).$$

(4.11)

Thus, in this case, if $\tilde{\nu}_n$ is an eigenfrequency of $H$ with corresponding eigenstate $|\psi_n\rangle$, there exists an eigenfrequency $\tilde{\nu}_m = -\tilde{\nu}_n^*$ with corresponding eigenstate $|\psi_m\rangle = CT|\psi_n\rangle$. The eigenfrequencies once again exist in pairs, each consisting of two eigenvalues with opposite real and equal imaginary parts.

In this case, for $\text{Re}\{\tilde{\nu}_n\} = 0$, the consequence for the eigenstate is that $|\psi_n\rangle = \pm CT|\psi_n\rangle$. Since $CT$ is a time-reversal operation, i.e. a complex conjugation of the elements of $|\psi_n\rangle$ in the position representation, followed by the chiral symmetry operation, we conclude that the state has purely real amplitudes on one of the sublattices and purely imaginary on the other.

One special case that we use in our work is a non-Hermitian $CT$ symmetric system where the losses occur only on one sublattice. In this case, we can show that the “zero” energy state of the associated Hermitian chiral symmetric Hamiltonian is still an eigenstate of the new non-Hermitian Hamiltonian. Indeed, suppose $|\psi_0\rangle$ is an eigenstate of a chiral symmetric Hamiltonian $H$ with associated eigenvalue $\tilde{\nu}_n = 0$. We know that $|\psi_0\rangle$ has support on only one sublattice, say on sublattice A. Let us now define a non-Hermitian Hamiltonian as $H_\gamma = H + i\gamma P_B$. Then $|\psi_0\rangle$ is also an eigenstate of $H_\gamma$ with eigenvalue 0 because $H_\gamma |\psi_0\rangle = H |\psi_0\rangle + i\gamma P_B |\psi_0\rangle = 0$. Therefore, in this special case of $CT$ symmetry, the state $|\psi_0\rangle$ still has support on only one sublattice and the associated eigenfrequency is still real.
4.3 Edge states

The most crucial parameter characterizing a Hamiltonian of a topological insulator is the topological invariant, which is an integer number describing a gapped system that does not change under adiabatic deformations of the Hamiltonian in the parameter space [39]. It essentially characterizes the topological phase of the system. We are going to explore this in the example of the SSH model. In Fig. 4.1(a),(b), we show the two possible configurations of an SSH array. In configuration (a), the we have $t_1 > t_2$, while for (b) $t_1 < t_2$. The topological invariant associated with the SSH model is called the winding number. It can be evaluated by analyzing the Hamiltonian in the momentum space which is

$$\begin{align*}
H_k &= \begin{pmatrix}
0 & t_1 + t_2 e^{-ik} \\
(t_1 + t_2 e^{ik}) & 0
\end{pmatrix} = (t_1 + t_2 \cos k)\sigma_x + t_2 \sin k\sigma_y
\end{align*}
$$

(4.12)

where $\sigma_i$ are the Pauli matrices. We can then define a vector in the $x,y$ plane using as components the coefficients of the Pauli matrices above, $d_x = t_1 + t_2 \cos k$ and $d_y = t_2 \sin k$. The trajectory of the endpoint of the vector $\vec{d} = d_x \hat{i} + d_y \hat{j}$ on the $x,y$ plane as the wavevector goes through the Brillouin zone is closed due to the $2\pi$ periodicity of the Hamiltonian. Looking at the vector components, we can easily see that for the SSH model the trajectory is a circle centered at $(t_1,0)$ with radius $t_2$. The winding number can now be defined as the integer describing the number of times that the endpoint of the vector encircles the origin $(0,0)$ as the wavevector runs through the Brillouin zone.

For $t_1 > t_2$, the winding number is 0. For $t_1 < t_2$, the endpoint of the vector circles around the origin once, therefore, the winding number is 1.

Edge states are midgap states emerging at the interface between materials with different topological invariants [39]. The total number of edge states appearing at such an interface is equal to the difference between the two topological invariants of the bulk of those two materials. This is a property known as bulk-edge correspondence [36,39]. In order to understand this, one needs to realize that the topological invariant is an integer.
number that does not change under adiabatic deformations of the Hamiltonian of the system in the parameter space without closing the gap [37]. For example, let us recall the dispersion relation of the SSH model in Eq. (4.4) and the two configurations in Fig. 4.1. The width of the gap is given by $2|t_1 - t_2|$. There is no way to go adiabatically from configuration (a) (corresponding to topological invariant 0) to configuration (b) (topological invariant 1) without closing the bandgap, i.e. without having at some point $t_1 = t_2$. Thus, one edge state which closes the gap (at frequency $\nu = \nu_0$) emerges at the interface between the two configurations. Edge states are said to be *topologically protected*. This has a multiple meaning: first, their existence is guaranteed as long as the change in topological invariant at an interface remains. Second, the corresponding eigenvalues always lie in the middle of the gap, even if we introduce disorder at the coupling strengths. In an optical setup, this causes robustness of the spectral location of the mode to positional disorder in the resonator array. In higher dimensions than 1, the associated states also support very robust unidirectional propagation [37].

In the SSH model, the edge states correspond to the case $\nu_n = 0$ with consequences that we discussed in Section 4.2, most importantly the fact that the corresponding eigenstates vanish at either one of the two sublattices. In fact, it can be shown that these states are exponentially localized around the defect [39]. For example, for a topological defect state, associated with the configuration shown in Fig. 4.1(c), the mode profile (in the thermodynamic limit) takes the form [39,46]

$$\psi_n = \begin{cases} \frac{1}{\sqrt{\xi}} e^{-|n-n_0|/\xi}, & n \text{ odd}, \\ 0, & n \text{ even}, \end{cases} \quad (4.13)$$

where $\psi_n$ is the wave amplitude at the $n^{th}$ resonator, $\xi = 1/\ln(t_1/t_2)$, and $n_0$ the index of the defect resonator.
The drop in transmittance is associated with an increase of the nonlinear losses in the defect resonator: (b) A circuit with various limiters. Therefore, we have included losses to quantify the losses of the resonators. We observe that the transmittance of the standalone defect resonator, which includes a manually modulated absorbing insulator-to-metal phase transition. (d) Our measurements involve a capacitive coupling of a material that experiences a thermally induced nonlinear losses. We show the transmittance for two different patches. The linewidth of the defect resonator, which we corrected by using resonators with a slightly higher eigenfrequency. The linewidth $\gamma$ has been used in order to quantify the losses of the resonators. When the system is coupled to the antennas, the resonance frequency $\nu_0$ increases. This behavior is in distinct contrast to the case of the bandgap of the system. While the defect resonator is assumed to have the same nonlinear losses at the defect resonator. Rather, we show the transmittance spreading out evanescently and is well-separated spectrally from other resonances, making it suitable for the implementation of the tight-binding model. The experimental setup is illustrated in Fig. 4.3. It consists of 21 cylindrical resonators positioned at alternating distances $d_1, d_2$. A topological defect is introduced at the middle by repeating the separation $d_2$. The first TE mode is excited by a kink antenna fixed near the first resonator on the left. Transmittance was measured at the 13th resonator by a loop antenna.

### 4.4 Waveguide photonic limiters based on topologically protected resonant modes

In Ref. [46], we demonstrate theoretically and experimentally how a one-dimensional array of microwave cylindrical resonators can act as a reflective limiter using a topologically protected midgap mode. The proposed system is a C-CROW (chiral-CROW) array consisting of high permittivity cylindrical resonators. Each resonator has a resonant frequency at $\nu_0 = 6.655 \text{ GHz}$ and linewidth of $1.4 \text{ MHz}$ corresponding to its first TE mode. This mode is chosen because it exhibits high confinement within the resonator spreading out evanescently [48] and is well-separated spectrally from other resonances [49], making it suitable for the implementation of the tight-binding model. The experimental setup is illustrated in Fig. 4.3. It consists of 21 cylindrical resonators positioned at alternating distances $d_1 = 12 \text{ mm}$ and $d_2 = 14 \text{ mm}$, establishing a 1-D tight-binding model. At the center of the chain (site 11), a dimerization topological defect is introduced by repeating the interdisk separation $d_2$ before and after the resonator. This defect supports a topologically protected resonant mode at the middle of the bandgap of the system. While the defect resonator is assumed to have the same resonant frequency as the rest of the resonators in the array, it is also assumed to have nonlinear absorption. In order to verify the effect of nonlinear losses at high powers,
measurements are performed for realizations of the array where increased Ohmic losses are introduced to the defect resonator by an absorber patch. The setup is shielded from above by a metallic plate where a loop antenna is mounted and coupled to the 13th resonator. The first TE mode of the resonators is excited using a signal emitted by a kink antenna at the left edge of the C-CROW array (see Fig. 4.3). Transmittance is obtained through the measurements of the loop antenna, as well as reflectance from the kink antenna. Moreover, the modal field distribution in the CROW array is measured for the different loss setups. From a theoretical point of view, we calculated the scattering properties using the S-matrix formalism and the field profiles using the transfer matrix formalism. We observed the properties of the midgap defect mode and verified the fact that it is localized at the defect. We confirmed both experimentally and theoretically that this setup acts as a reflective waveguide limiter at the midgap frequency: for low losses at the defect resonator, the C-CROW array supports resonant transmission, while for strong Ohmic losses, the transmission (and eventually the absorption) are suppressed. A benefit of our setup is the robustness of transmission at low intensities to positional disorder of the resonators. Ref. [46] is included in full in Appendix A.

4.5 Reflective limiters based on self-induced violation of charge conjugation symmetry

In Ref. [15], we consider two one-dimensional $CT$-symmetric arrays: one in the microwave domain and a one in the infrared regime. Starting with the microwave configuration, the array is shown in Fig. 4.4. It is similar to that of Ref. [46], in that it consists of 21 resonators positioned at alternating distances with a dimerization defect in the 1-D tight-binding regime, with some key differences. First, the defect resonator at site 11 in this case is assumed to be made of a material with a heat or field intensity-induced permittivity modulation. The resonant frequency of the defect resonator at low intensities/temperatures is assumed to match that of the other resonators in the array.
Figure 4.4: Figure taken from [15]. The array consists of 21 dielectric resonators with a dimerization defect. The resonators at sites 10 and 13 (red) involve large Ohmic losses. The central resonator has a thermally or field intensity-modulated permittivity. At low intensities, the $CT$ symmetry protected defect mode supports resonant transmission and exhibits a staggered shape (see top subfigure). At high intensities, the $CT$ symmetry is violated due to the permittivity modulation of the central resonator and the defect mode is destroyed, suppressing the associated transmission and causing reflection of the incoming signal.

Second, the resonators adjacent to the defect on sites 10 and 12 are assumed to involve large linear losses. The real part of the resonant frequency of these resonators is still the same as that of the other resonators of the array. Due to non-Hermiticity, this array is no longer chiral symmetric, however, it satisfies the condition of $CT$ symmetry. The staggered shape of the topologically protected defect mode still applies in this case, in accordance with our discussion in the end of Section 4.2. The defect mode is a topologically protected resonant mode that supports resonant transmission at low intensities at the midgap frequency. At high intensities, however, the central resonator experiences a modulation of its permittivity due to heating or high field intensity, leading to the violation of $CT$ symmetry. This causes the destruction of the defect mode, which loses its staggered shape and is no longer shielded from the Ohmic losses in the structure. The associated resonant transmission is dramatically suppressed, even for slight variations of the resonant frequency of the defect $\delta \approx 0.5%\Delta$, where $\Delta$ is the width of the gap.
At the infrared configuration, we assume an array of 9 ring resonators positioned at alternating distances, with a dimerization defect at the central resonator (see Fig. 4.5). The defect ring resonator is made of VO$_2$, which is a material that exhibits insulator-to-metal phase transition at 342 K. At the 4$^{th}$ and 6$^{th}$ sites, the ring resonators are assumed to have significant Ohmic losses. Each of the ring resonators supports a clockwise and a counterclockwise propagating modes at the same frequency. Due to this fact, the array supports two quasidegenerate topologically protected defect modes. Numerical simulations were performed using the COMSOL Multiphysics simulation package. The system is coupled to two bus waveguides, one at each end of the array. The transport measurements, as well as the real part of the permittivity of the defect resonator are evaluated for different values of light intensity exciting the system through one of the bus waveguides. At low intensities, the array is $\mathcal{CT}$-symmetric and supports resonant transmission. At high intensity, the real part of the permittivity of the defect ring increases rapidly, although the system does not reach phase transition temperature. This causes $\mathcal{CT}$ symmetry breaking, leading to the destruction of the mode and the suppression of resonant transmission. Ref. [15] is provided in Appendix A.
Optical isolators based on asymmetric Fano resonances using phase-change materials

Controlling the directionality of light via photonic circuits is a research topic of high interest, fueled by the need of structures that provide asymmetric transport for applications where feedback into an optical component needs to be blocked, such as in the case of a laser cavity [50]. Traditional designs for optical isolators involve breaking the conditions of Lorentz reciprocity through the use of very large magnetic fields, which can be power-consuming and impossible to implement for on-chip applications. Other schemes are based on dynamic modulation [51, 52], which also has the disadvantage of being an active approach involving externally powered components.

Solutions involving passive structures have also been proposed [53–55]. These structures typically rely on optical nonlinearities incorporated within a spatially asymmetric design, resulting in scattering properties that depend on the direction of incidence, when the incident light intensity is high enough to trigger the nonlinear response. An alter-
native to optical nonlinearities is the implementation of structures involving materials with temperature-dependent optical properties [56, 57]. Again, a judiciously designed asymmetric structure incorporating such a material can experience a heating rate that depends on the direction of incidence, resulting in a direction-dependent modulation of the permittivity and eventually causing asymmetric transmission. A great candidate for this type of applications is a class of materials that exhibit a temperature-induced insulator-to-metal phase transition known as phase-change materials. This process is associated with a dramatic jump in the values of their optical parameters, namely the refractive index and their extinction coefficient. Among these materials, vanadium dioxide VO$_2$ has been well studied and is known to undergo its phase transition at a temperature of $\theta_C = 68^\circ$ C [59–61]. Experimental data of the complex permittivity values of VO$_2$ for different temperatures at a wavelength of 10.5$\mu$m are provided in Fig. 5.1. This material has also already been incorporated in structures exhibiting asymmetric transport [62].

Meanwhile, a Fano resonance is a resonant effect occurring due to the interference be-
Figure 5.2: Transmission spectrum for two ring resonators coupled to a bus waveguide. The lineshape and depth of the resonance can be adjusted by changing the separation of the two resonators.

tween two scattering pathways, one being through a continuum of states, and the other through a resonant phenomenon. The interference between the two processes results in an asymmetric lineshape caused by the sharp variations of the phase due to the resonant phenomenon component. Fig. 5.2 illustrates the transmission spectrum of a resonant structure consisting of two ring resonators with the same resonant frequency side coupled to a bus waveguide. The lineshape is adjusted by altering the separation between the two ring resonators which effectively changes the relative phases of the two components. Fano resonances have been explored in the framework of nanoscale structures [63], including the design of isolators using coupled microresonator systems [53,64,65].

In this Chapter, we discuss the design of a resonant photonic structure which relies on Fano resonances. The structure incorporates two ring resonators coupled to a straight waveguide. One of the resonators is composed of VO$_2$. The thermal sensitivity of the optical parameters of VO$_2$ results in a dependence of the profile of the losses on the field intensity, which, in turn, leads to an asymmetric response of the structure for incident
light of different direction. Optical simulations using the Comsol Multiphysics modeling software, as well as a semi-analytical model describing the basic properties of the system will be discussed.

5.1 Resonant photonic structure with phase-change material

The proposed coupled resonant photonic structure (CRPS) (see Fig. 5.3) consists of two ring resonators, each of which is designed to support a resonant mode at the mid-infrared wavelength of 10.5 \( \mu \text{m} \), corresponding to their optical mode number \( m = 27 \). The wavelength was chosen to be within the atmospheric transparency window. The ring resonator on the right is made of lossless Si with refractive index \( n_{\text{Si}} = 3.3 \), while the one on the left is made of a phase-change material (PCM) and has a complex temperature-dependent refractive index \( n_{\text{PCM}} = n'_{\text{PCM}}(\theta) + in''_{\text{PCM}}(\theta) \), which at the low-intensity regime can be taken to be \( n_{\text{PCM}} = 3.3 + i0.001 \). The two resonators are side-coupled to a bus waveguide that is also made of Si, and they are also directly coupled to each other. The thicknesses of the bus waveguide and the ring resonators are kept at 1.7 \( \mu \text{m} \). The whole CRPS lays on top of a ZnS substrate with refractive index \( n_{\text{SiO}_2} = 2.2 \). Using eigenmode analysis via Comsol multiphysics software, the quality factors of the resonant optical mode supported by the lossless silicon ring and the lossy PCM ring are evaluated as \( Q_{\text{Si}} = 2.026 \times 10^4 \) and \( Q_{\text{PCM}} = 1.855103 \) respectively.

Two crucial parameters in this setup are the coupling strength between the ring resonators and the coupling between each ring resonator and the bus waveguide. These couplings introduce the two scattering pathways which are necessary for the observation of the characteristic asymmetric transmission lineshape associated Fano-type phenomena. The couplings can be controlled by changing the center-to-center distance of the two ring resonators \( d \) and the distance between each ring resonator \( s \) and the straight
waveguide respectively. After optimization, we used $d = 35.8 \mu m$ and $s = 2.295 \mu m$. This leads to a very weak resonator-to-resonator coupling regime, which results in a resonant transmission dip of about -60 dB.

Using the parameters above and steady-state analysis of the CRPS, we obtain the transmittances from left ($T_L$) and right ($T_R$) at low irradiance (Fig. 5.3(a)). The technical details of the simulations are provided in Section 5.2. The transmittance spectrum has resonant dips of $\sim -60$ dB and $\sim -30$ dB at $\nu_1 = 28.5872$ THz and $\nu_2 = 28.6$ THz respectively (see red dashed line and black no-fill circles in Fig. 5.3(a)). In
this regime, the transmittances for incidence from the left and right are equal, since the system follows the conditions of Lorentz reciprocity at the low-irradiance limit for which $n_{\text{PCM}}$ is fixed.

However, as we increase the irradiance of the input signal, the direction of incidence has a significant effect on the transport properties of the system. Let us start with the case of incidence from the right. In this case, the electromagnetic wave will first couple to the Si resonator, which has a very high Q-factor. This leads to a strong local electric field density on the Si resonator (see right inset of Fig. 5.3a). Due to high radiative losses from the Si resonator, this results in the sharp resonant dip which is observed in the transmittance (see red solid line of Fig. 5.3a).

For incidence from the left, the wave first couples to the VO$_2$ resonator, which has small Ohmic losses at low temperatures. These linear losses result in a local increase in temperature, which, due to the thermal sensitivity of the optical properties of VO$_2$, leads
to an increase of the values of its optical parameters. The growth of $\epsilon''$ with temperature causes a further increase of the local absorption. By the time we reach a steady state temperature, this process has induced a significant alteration of the optical properties of VO$_2$ due to the change in temperature, which does not occur for incidence from the right (see Fig. 5.3c red vs. black solid line). This eventually results in a resonant frequency detuning between the two ring resonators, which causes the suppression of the resonance as shown in Fig. 5.3a. In our calculations we achieved asymmetric transmission while using a value of irradiance that results in temperatures of the PCM ring below the phase transition temperature.

In Fig. 5.4, we present the asymmetry factor $T_{AS} = |T_L - T_R|$ for a frequency range from 28.45 to 28.7 THz and for a range of incident irradiances. We observe significant asymmetry in transmittance for a broad range of frequencies for incident irradiances above 15 kW/cm$^2$. We discuss the technical details of the simulation below.

5.2 Non-Reciprocal Designs Based on Phase-Change Materials and Simulations

The material used for the PCM ring is VO$_2$, which undergoes a first order reversible transition from a monoclinic insulating phase to a rutile metallic phase (IMT) around temperature $\theta_c = 342$ K. This is accompanied by an abrupt change in both the real, $n'_{\text{PCM}}(\theta)$ and imaginary, $n''_{\text{PCM}}(\theta)$ refractive index of the VO$_2$ ring resonator. The change in real and imaginary parts of the refractive index of VO$_2$ are modeled in agreement with experimental data found in the literature for the operating wavelength [66] by the following expressions

\begin{align}
    n'_{\text{PCM}}(\theta) &= n'_0 + \frac{\Delta n'}{\exp[-(\theta - \theta_c)/\Delta \theta] + 1}, \quad (5.1a) \\
    n''_{\text{PCM}}(\theta) &= n''_0 + \frac{\Delta n''}{\exp[-(\theta - \theta_c)/\Delta \theta] + 1}, \quad (5.1b)
\end{align}
where \( n'_0 = 3.3 \) and \( n''_0 = 0.001 \), \( \Delta \theta = 5 \) K denotes the smoothing parameter over which the phase transition takes place, and \( \Delta n' = 3.323 \), \( \Delta n'' = 8.8 \) indicate the “heights” of the jumps of the optical parameters during the phase transition.

We launch a continuous wave (CW) signal towards the CRPS via the bus waveguide from the left (\( L \)) and the right (\( R \)) directions (see golden arrows at the inset of Fig. 5.3(a)). The electromagnetic wave propagation inside a medium with temperature-dependent optical parameters can be described by the following set of coupled Maxwell’s and heat-transfer equations in the steady-state regime:

\[
\nabla \times \vec{E} = i\mu \omega \vec{H}, \quad \nabla \times \vec{H} = -2\mu_0 \omega \vec{E}, \quad (5.2a)
\]
\[
-\nabla . (k(\vec{r})\nabla \theta(\vec{r})) = Q(\vec{r}, \theta), \quad (5.2b)
\]
\[
Q(\vec{r}, \theta) = \omega n(\vec{r}, \theta)'n(\vec{r}, \theta)''Re\{\vec{E} \cdot \vec{E}\}. \quad (5.2c)
\]

\( \vec{E} \) and \( \vec{H} \) denote the electric and magnetic field vectors, \( \mu = \mu_0 \) is the permeability of free space, and \( n(\vec{r}, \theta)' \), \( n(\vec{r}, \theta)'' \) are functions describing the spatial and temperature dependence of the real and imaginary parts of the refractive index. The heat transfer within the CRPS is described by Eq. (5.2b), where the parameter \( k(\vec{r}) \) is the thermal conductivity which, at the VO\(_2\) ring, takes the value \( k_{\text{VO}_2} = 4 \) W/mK. We assume a constant surrounding temperature boundary condition of \( \theta_0 = 293.15 \) K at the edges of the surrounding SiO\(_2\) substrate. The energy dissipated at the lossy VO\(_2\) ring resonator leads to heating, which is given by \( Q(r, \theta) \) within the PCM ring resonator, see Eq. (5.2c). The generated heat, in turn, leads to an increase in the temperature of the VO\(_2\) ring resonator, which modifies its optical parameters \( n' \) and \( n'' \) as shown in Eqs. (5.1). The Maxwell’s equations and the heat-transfer equation are solved in a self-consistent manner until a steady-state temperature is reached within the system.
5.3 Semi-analytical model based on coupled oscillators with a temperature-dependent damping coefficient

In order to obtain a deeper understanding of the asymmetric transport properties of the CRPS, we model it with a simpler system consisting of coupled oscillators. In particular, we model the bus waveguide as an infinite chain of identical masses $m$ coupled by springs with constant $K$ (see Fig. 5.5). The ring resonators are represented by two equal masses, each of which is coupled with one mass in the infinite array with a spring of constant $K_l$. The two masses are also coupled to each other with spring constant $K_c$. The resonator on the left is also assumed to have losses due to friction $\mu$. Later, we are going to incorporate a temperature dependence in analogy with the temperature dependent optical parameters of the VO$_2$ resonator in the CRPS system. The system is described by the following set of equations

$$m\ddot{X}_n + K_0(X_n - X_{n-1}) + K_0(X_n - X_{n+1}) + K_l(X_n - X_{n-1})\delta_{n,0} + K_l(X_n - X_{n+1})\delta_{n,1} = 0,$$

$$m\ddot{X}_{0'} + K_c(X_{0'} - X_{1'}) + K_l(X_{0'} - X_0) + \mu\dot{X}_{0'} = 0,$$

$$m\ddot{X}_{1'} + K_c(X_{1'} - X_{0'}) + K_l(X_{1'} - X_1) = 0.$$  

(Eq. 5.3a) is the equation of motion describing the $n^{th}$ mass in the infinite array, Eq. (5.3b) is the equation of motion for the lossy resonator $0'$, and Eq. (5.3c) is the equation of motion for the mass $1'$. Dividing with the mass $m$, the equations can be rewritten as

$$\ddot{X}_n = -\omega_0^2(2X_n - X_{n-1} - X_{n+1}) - \omega_l^2(X_n - X_{0'})\delta_{n,0} - \omega_l^2(X_n - X_{1'})\delta_{n,1},$$

$$\ddot{X}_{0'} = -\omega_c^2(X_{0'} - X_{1'}) - \omega_l^2(X_{0'} - X_0) - \gamma\dot{X}_{0'},$$

$$\ddot{X}_{1'} = -\omega_c^2(X_{1'} - X_{0'}) - \omega_l^2(X_{1'} - X_1),$$

where $\omega_0 = \sqrt{K_0/m}$, $\omega_l = \sqrt{K_l/m}$, $\omega_c = \sqrt{K_c/m}$, and $\gamma = \mu/m$ is the damping coefficient of the oscillator on site $0'$. 


We proceed to analyze the transport properties of the system. We assume that $X_n(t) = a_n e^{-i \omega t}$. After replacing in the equations above, we get

\begin{align}
-\omega^2 a_n &= -\omega_0^2 (2a_n - a_{n-1} - a_{n+1}) - \omega_1^2 (a_n - a_{0'}) \delta_{n,0} - \omega_1^2 (a_n - a_{1'}) \delta_{n,1}, \quad (5.5a) \\
-\omega^2 a_{0'} &= -\omega_0^2 (a_{0'} - a_{1'}) - \omega_1^2 (a_{0'} - a_{0}) + i \gamma \omega a_{0'}, \quad (5.5b) \\
-\omega^2 a_{1'} &= -\omega_0^2 (a_{1'} - a_{0'}) - \omega_1^2 (a_{1'} - a_{1}). \quad (5.5c)
\end{align}

In order to obtain the scattering properties of this setup, we consider an excitation through the infinite chain of coupled masses. The dispersion relation giving the frequencies of propagating waves on the infinite chain is $\omega = 2\omega_0 \sin(\frac{k}{2})$. For left incidence, the appropriate scattering boundary conditions are

\begin{align}
a_n &= I_L e^{i kn} + R_L e^{-i kn}, \quad n \leq 0, \quad (5.6a) \\
a_n &= T_L e^{i kn}, \quad n > 0. \quad (5.6b)
\end{align}

Applying these boundary conditions to Eqs. (5.7) using $n = 0, n = 1$ for Eq. (5.5a), we
obtain the following system of equations:

\[-\omega^2(I_L + R_L) = -\omega_0^2[2(I_L + R_L) - I_Le^{-ik} - R_Le^{ik} - T_Le^{ik}] - \omega_1^2(I_L + R_L - a_{0'}),\]  
(5.7a)

\[-\omega^2T_Le^{ik} = -\omega_0^2(2T_Le^{ik} - I_L - R_L - T_Le^{2ik}) - \omega_1^2(T_Le^{ik} - a_{1'}),\]  
(5.7b)

\[-\omega^2a_{0'} = -\omega_1^2(a_{0'} - a_{1'}) - \omega_1^2(a_{0'} - (I_L + R_L)) + i\gamma\omega a_{0'},\]  
(5.7c)

\[-\omega^2a_{1'} = -\omega_1^2(a_{1'} - a_{0'}) - \omega_1^2(a_{1'} - T_Le^{ik}).\]  
(5.7d)

From Eqs. (5.7a, 5.7b) we can obtain the transmitted and reflected wave amplitudes from the left in terms of the incident wave amplitude \(I_L, \gamma, a_{0'}, a_{1'}\):

\[T_L = \frac{a_{0'}\omega_1^2\omega_0^2 + a_{1'}\omega_1^2(e^{-ik}\omega_0^2 + \omega_1^2) - 2iI_L\omega_1^2\sin k}{2\omega_1^2\omega_0^2 + e^{ik}\omega_1^2 - 2i\omega_1^2\sin k},\]  
(5.8a)

\[R_L = \frac{a_{0'}\omega_1^2(\omega_0^2 + e^{ik}\omega_1^2) + a_{1'}\omega_1^2\omega_0^2e^{ik} - I_Le^{ik}\omega_1^2(\omega_1^2 + 2\omega_0^2\cos k)}{2\omega_1^2\omega_0^2 + e^{ik}\omega_1^2 - 2i\omega_1^2\sin k}.\]  
(5.8b)

Using Eqs. (5.8, 5.7c, 5.7d), we can obtain transmittance from the left as \(T_L = |T_L/I_L|^2\).

For incidence from the right, the associated boundary conditions are

\[a_n = I_Re^{-ikn} + R_Re^{ikn}, \quad n \geq 1,\]  
(5.9a)

\[a_n = T_Re^{-ikn}, \quad n < 1.\]  
(5.9b)

Following the same procedure as before, we can compute the transmitted and reflected amplitudes as

\[T_R = \frac{a_{0'}\omega_1^2(\omega_0^2 + \omega_1^2e^{ik}) + a_{1'}\omega_1^2\omega_0^2e^{ik} - 2I_Re^{ik}\omega_1^2\sin k}{2\omega_1^2\omega_0^2 + e^{ik}\omega_1^2 - 2i\omega_1^2\sin k},\]  
(5.10a)

\[R_R = \frac{a_{0'}\omega_1^2\omega_0^2 + a_{1'}\omega_1^2(\omega_0^2e^{-ik} + \omega_1^2) - I_Re^{-ik}\omega_1^2(\omega_1^2 + 2\omega_0^2\cos k)}{2\omega_1^2\omega_0^2 + e^{ik}\omega_1^2 - 2i\omega_1^2\sin k}.\]  
(5.10b)

We can obtain \(a_{0'}, a_{1'}\) by solving the following equations

\[-\omega^2a_{0'} = -\omega_1^2(a_{0'} - a_{1'}) - \omega_1^2(a_{0'} - T_R) + i\gamma\omega a_{0'},\]  
(5.11a)

\[-\omega^2a_{1'} = -\omega_1^2(a_{1'} - a_{0'}) - \omega_1^2(a_{1'} - I_Re^{-ik} - R_Re^{ik}).\]  
(5.11b)
Using Eqs. (5.10, 5.11a, 5.11b) we can obtain the transmittance for incidence from the right, $T_R = |T_R/I_R|^2$.

We now introduce the temperature dependence of the damping coefficient $\gamma$. We will use the functional dependence

$$\gamma(\theta) = \gamma_{\text{min}} + \frac{\gamma_{\text{max}} - \gamma_{\text{min}}}{\exp[-(\theta - \theta_c)/\Delta] + 1},$$

(5.12)

where $\gamma_{\text{max}}$ and $\gamma_{\text{min}}$ are the maximum and minimum values of the damping coefficient respectively and $\Delta$ the smoothing parameter. The functional dependence of the damping coefficient with temperature is illustrated in Fig. 5.6(d). Note that the functional dependence of the damping coefficient in our analytics is analogous to the functional dependence of the imaginary part of the refractive index for the phase transition material that was used in our optical simulations. In our analytics, we do not consider a change of resonant frequency of the oscillator which would correspond to the change in real refractive index with temperature in the optics. To proceed, we need to incorporate a model describing the heating rate of the system due to the frictional losses. The change in temperature at the mass with friction involves two terms: (a) a heating term, which is the time averaged power dissipated due to friction that causes the heating and (b) a cooling term which describes the dissipation of heat. The heat rate equation is then

$$\frac{d\theta(t)}{dt} = \kappa(\theta(t) - \theta_0) + \overline{P(t)},$$

(5.13)

where $\kappa$ is the thermal conductance, $\theta_0$ is the temperature of the surroundings, and $\overline{P(t)}$ is the averaged power over one period of oscillation that is dissipated to heat. $\overline{P(t)}$ can be calculated as [67]

$$\overline{P(t)} = \gamma(\theta)\langle \dot{X}_\varphi^2 \rangle = \gamma(\theta) \frac{1}{T} \int_0^T \text{Re}\{X_{\varphi}(t)\}^2 dt = \frac{\gamma(\theta)\omega^2}{2} \left(\text{Re}\{a_{\varphi'}\}^2 + \text{Im}\{a_{\varphi'}\}^2\right).$$

(5.14)

Eq. (5.14) can now be replaced in Eq. (5.13). For a given incident wave amplitude $I_{L/R}$ and wavenumber $k$, we can numerically find the root of Eq. (5.13), which corresponds to the steady-state of the system. This allows us to calculate the transport properties
in this regime, $T_L, T_R$. For some parameter ranges, the heat rate equation has multiple roots, thus exhibiting multistability. In these cases, we considered as the steady state solution the maximum temperature obtained that corresponded to a root and for which the second time derivative of the temperature is negative.

In Fig. 5.6, we present the steady state transmittances, phases and temperatures of the oscillator $0'$ vs. frequency for three different values of incident wave amplitude $I_{L/R}$, 0.1, 5, and 12.5. The parameters used are $\omega_0 = 1$, $\omega_c = 0.005$, $\omega_l = 0.8$, $\gamma_{\text{min}} =$
\( \kappa = 0.02, \gamma_{max} = 10, \theta_c = 342, \theta_0 = 293, \Delta = 5. \) The phase transition temperature is indicated with a dotted line in Fig. 5.6(c). In Fig. 5.6(d), we have also included a plot of the functional dependence of \( \gamma(\theta) \) at the temperature range that appears in our calculations. For a small incident amplitude of \( I_{L/R} = 0.1 \) (black/red lines and symbols), the system exhibits symmetric transport. For an incident amplitude of \( I_{L/R} = 5 \) (turquoise/violet), we observe that, for incidence from the right, the system has only slightly deviated from the \( I_R = 0.1 \) case around the resonant frequency. However, for incidence from the left, a significant asymmetry in transmission is already exhibited, driven by the heating of the resonator, which is already at a temperature around \( \theta_c \).

Notice that the transmission is asymmetric for a range of \( \omega \) from 0.75 to 1. For even larger values of \( I_{L/R} \) (blue/green lines), we observe that, although the transmittance around the resonant frequency is still asymmetric, the range of frequencies where we observe the phenomenon narrows down. This can be understood by the plot of the temperature vs. \( \omega \). As the incident amplitude increases, the temperature goes around or above the phase transition temperature for a broad range of frequencies, even for incidence from the right. This will eventually lead to a destruction of the resonance due to the detuning of the two resonators.

Although the toy model that we have developed here is simple, it retains the basic physics principles that are responsible for the observed asymmetry in the transport properties of the photonics circuit of the previous section. It is analytically tractable and can potentially be explored further in order to better understand the transport near the phase-change transition point. In this case, the phase stability of the high-temperature regime and the hysteresis effects due to the heating and cooling cycles have to be carefully analyzed in order to obtain a better picture of the transport properties of our setup. The investigation along these lines will cast more light on the physics of photonic structures based on PCM and will be the subject of future work.
Conclusion

In this dissertation, we explored how the violation of time reversal symmetry can be exploited for the design of a new class of power/energy photonic limiters and for the design of photonic circuits that exhibit asymmetric transport. Specifically, we studied the scattering problem of a periodic multi-layered structure with an embedded defect with a light intensity-induced complex permittivity modulation. At low values of incident light intensity, the structure was found to support high resonant transmission at the frequency of the localized defect mode. For input power exceeding a certain threshold, the losses induced at the defect cause a destruction of the localized mode due to a transition from the underdamping to the overdamping regime, rendering the photonic structure highly reflective within a broad frequency range. The limiting action of such a structure has been demonstrated in the framework of both power and energy limiters. Moreover, in collaboration with the Sensors Directorate of the Air Force Research Laboratory, a prototype of such a photonic structure was realized and experimentally verified to exhibit reflective limiting action.

In addition, we have utilized ideas from topological photonics in order to design photonic limiters with scattering characteristics that are robust against certain types of
fabricational imperfections. We have investigated the scattering properties of a chiral symmetric CROW array in the microwave domain. The system consists of a bipartite lattice of resonators that are evanescently coupled with a dimerization topological defect. When the defect exhibits field intensity induced losses, the system demonstrates reflective limiting action. When a CROW array with charge conjugation symmetry is considered, the defect mode turns out to be hypersensitive to small variations of the resonant modes of the individual resonators. These schemes can be used for the realization of a new, topologically protected family of limiters.

Finally, we consider the scattering problem of two ring resonators coupled to a bus waveguide, with one of them being composed of a phase change material. Using numerical simulations, we show that if the incident light intensity exceeds a certain threshold, the induced heating alters the optical properties of the system, resulting in a highly asymmetric transmittance profile within a broad range of light intensities. The underlying physics has been explained using a toy model consisting of an array of coupled oscillators.

Reflective limiters are a new promising category of photonic limiters. In this work, we exploit the fact that a transition from underdamping to overdamping due to self-induced violation of time reversal symmetry can cause the destruction of a resonant mode. However, other physical mechanisms could be employed for a realization of such type of structure, e.g. self induced Anderson localization (due to randomness created by the field intensity) or field induced dynamical detuning. Some of these ideas could also be explored towards the direction of asymmetric transport. Our research has immediate appeal for protection of sensitive sensors like antennas, receivers in radar systems and telecommunication satellites from high power radiation.
In this Appendix, we provide the articles that we have published in peer-reviewed journals and are relevant to the content of this thesis in chronological order. The references within these publications are all self-contained. A list of those publications is provided below.

1. **Concept of a reflective power limiter based on nonlinear localized modes**

2. **Reflective optical limiter based on resonant transmission**

3. **Hypersensitive transport in photonic crystals with accidental spatial degeneracies**

4. **Experimental Realization of a Reflective Optical Limiter**

5. **Waveguide photonic limiters based on topologically protected resonant modes**

6. **Reflective limiters based on self-induced violation of CT symmetry**
Concept of a reflective power limiter based on nonlinear localized modes

Eleana Makri, Hamidreza Ramezani, and Tsampikos Kottos
Department of Physics, Wesleyan University, Middletown, Connecticut 06459, USA
Ilya Vitebskiy
Air Force Research Laboratory, Sensors Directorate, Wright Patterson AFB, Ohio 45433, USA
(Received 23 August 2013; published 19 March 2014)

Optical limiters are designed to transmit low-intensity light, while blocking the light with excessively high intensity. A typical passive limiter absorbs excessive electromagnetic energy, which can cause its overheating and destruction. We propose the concept of a photonic reflective limiter based on resonance transmission via a localized mode. Such a limiter does not absorb the high-level radiation, but rather reflects it back to space. Importantly, the nearly total reflection occurs within a broad frequency range and direction of incidence. The same concept can be applied to infrared and microwave frequencies.

The continuing integration of optical devices into modern technology has led to the development of an ever increasing number of novel schemes for efficiently manipulating the amplitude, phase, polarization, or direction of optical beams [1]. Among these manipulations, the ability to control the intensity of light in a predetermined manner is of the utmost importance, with applications ranging from optical communications to optical computing [2,3] and sensing. As laser technology progresses, novel protection devices (optical limiters) are needed to protect optical sensors and other components from high-power laser damage [4–9].

Here we focus on the most popular, passive optical limiters. The simplest realization of a passive optical limiter is provided by a single nonlinear layer with the imaginary part $\epsilon''$ of its permittivity being dependent on the light energy density $W$. At low incident energy densities, the value $\epsilon''(W)$ is relatively small, and the nonlinear layer is transparent. As the light intensity increases, the value $\epsilon''(W)$ also increases, and the nonlinear protective layer turns opaque. The physical reason for the increase in $\epsilon''(W)$ as a function of $W$ can be different in different nonlinear optical materials. It can be two-photon absorption, photoconductivity, heating, or a combination of the above mechanisms. Specific examples of such nonlinear optical materials can be found in Refs. [7–9] and references therein. In more sophisticated schemes, the nonlinear layer can be a part of a complicated optical setup. The problem though is that in all cases, the nonlinear limiter absorbs the excessive power, which might cause overheating or even destruction of the device (a sacrificial limiter). Our goal is, using the existing nonlinear materials, to design a photonic structure that would reflect the excessive power back to space, rather than absorbing it. A free-space realization of such a reflective limiter is supposed to reflect a high-intensity radiation within a broad frequency range and regardless of the direction of incidence.

Our proposal is based on the phenomenon of resonant transmission through a localized (defect) mode. The localized mode frequency lies inside a photonic band gap of the underlying photonic structure. The simplest realization of our approach is illustrated in Fig. 1, where a nonlinear defect layer is sandwiched between two linear lossless Bragg mirrors. If the light energy density $W$ is low, the imaginary part $\epsilon''(W)$ of the permittivity of the nonlinear defect layer can be neglected, and the defect can support a localized mode. As a consequence, at low light intensities, the layered structure in Fig. 1 will be transmissive in the vicinity of the localized mode frequency. If the incident light intensity grows, so does the value $\epsilon''(W)$. Eventually, the increase in $\epsilon''(W)$ decouples the two Bragg mirrors in Fig. 1, and the entire stack becomes highly reflective, not opaque, as in the case of a standalone nonlinear layer. In other words, the high-intensity light will be reflected back to space, rather than absorbed by the limiter. Even this simple design can provide protection from high-level radiation within a broad frequency range and for an arbitrary direction of incidence. For a given material of the nonlinear defect layer, the incident light intensity at which the structure in Fig. 1 becomes highly reflective can be controlled by the proper design of the Bragg mirrors. A problem with the simple design of Fig. 1 is that the low-intensity transmittance occurs only in the vicinity of the localized mode frequency. This problem can be addressed by using more sophisticated photonic structures, for instance, those involving two or more coupled defect layers, as is done in the case of optical filters [10].

To illustrate our idea, we consider a pair of identical Bragg mirrors, each consisting of two alternating layers with real permittivities $\varepsilon_1$ and $\varepsilon_2$, placed in the intervals $-L \leq z \leq 0$ and $d_f \leq z \leq L + d_f$. The width of each layer is $d$. A nonlinear lossy layer of width $d_f$ is placed between the two mirrors at $0 \leq z \leq d_f$; its complex permittivity $\varepsilon_f = \varepsilon(1 + i\gamma|E(z)|^2)$ is field dependent. In the particular case of $\varepsilon = \varepsilon_1$ and $\gamma = 0$, we have a standard Bragg mirror with a band gap around the frequency $\omega_g = c/(n_d d)$ ($c$ is the speed of light). The defect layer creates a localized mode with the frequency $\omega$, lying within a photonic band gap. At this frequency, the entire stack displays resonance transmission accompanied by a dramatic field enhancement in the vicinity of the defect layer. The enhanced field, in turn, causes the respective increase in the imaginary part of the defect layer permittivity $\varepsilon_f$. The latter will eventually result in decoupling of the two Bragg reflectors and rendering the entire structure in Fig. 1 highly reflective.

We first consider normal incidence. In this arrangement, a time-harmonic electric field of frequency $\omega$ obeys the
From these values we evaluate the forward intensity light and (b) nearly total reflectivity of a high-intensity light. This setup provides (a) a resonant transmission of a low intensity light and (b) nearly total reflectivity of a high-intensity light.

Helmholtz equation:

$$\frac{d^2 E(z)}{dz^2} + \frac{a^2}{c^2} E(z) = 0,$$  \hspace{1cm} (1)

Eq. (1) admits the solution

$$E_0E(z) = \hat{E}_f \exp(ik_z z) + \hat{E}_b \exp(-ik_z z)$$  \hspace{1cm} (2)

just after the nonlinear layer

where the wave vector \( k = n_0 c/\epsilon \). The transmittance, reflectance, and absorption, e.g., for a left incident wave, are then defined as

$$T = |E_f/E_0|^2; \quad R = |E_b/E_0|^2; \quad A = 1 - T - R,$$  \hspace{1cm} (3)

respectively [11]. They can be calculated numerically using a backward map approach [12].

The amplitudes of forward and backward propagating waves on the left \( z < 0 \) (right \( z > L + d_r \)) domains outside of the Bragg mirror are related to the ones before (after) the nonlinear layer by the relations:

$$\begin{pmatrix} E_f' \\ E_b' \end{pmatrix} = M^{NL} \begin{pmatrix} E_f \\ E_b \end{pmatrix}; \quad \begin{pmatrix} E_f \\ E_b \end{pmatrix} = M^{NL -1} \begin{pmatrix} E_f' \\ E_b' \end{pmatrix},$$  \hspace{1cm} (4)

where \( M^{NL} (M^{NL -1}) \) are the 2 \times 2 transfer matrices of the optical structures associated with the domain \( 0 < z < L \) \( L \leq z < 0 \) \( d_r < z < L + d_r \). Above we have expressed the field before (after) the nonlinear layer as \( E'' = E_f' \exp(ik_z z) + E_b' \exp(-ik_z z) \) \( E'' = E_f \exp(ik_z z) + E_b \exp(-ik_z z) \). The field \( E'(z = 0) \) and its derivative \( dE'/dz(0) \), \( z = 0 \) of the nonlinear layer are then evaluated using \( M^{NL} \) from (Eq. (2)) together with the boundary conditions (associated with a left incident wave) \( E'_L = 0 \) and \( E'_R = 1 \). Using \( E''(z = 0) \) and \( dE''(z = 0)/dz \) as boundary conditions we have integrated backwards Eq. (1), with the help of a fourth-order Runge-Kutta algorithm, and obtained the field \( E''(0) = 0 \) and its derivative \( dE''(z = 0)/dz \) at the other end \( z = 0 \) of the nonlinear layer. From these values we evaluate the forward \( E_f'' \) and backward \( E_b'' \) propagating amplitudes. Utilizing Eq. (2) together with \( M^{NL} \) we finally find the amplitudes \( E_f' \) and \( E_b' \), which allow us to calculate \( T, R, \) and \( A \). Note that for a backward map with boundary condition \( E_f'' = 1 \) we have \( |E_f''| = 1/T \).

It is convenient to work with the rescaled variable

$$\hat{E}(z) = \sqrt{\epsilon} E(z).$$

In this representation, Eq. (1) becomes

$$\frac{d^2 \hat{E}(z)}{dz^2} + \frac{a^2}{c^2} \hat{E}(z) = 0,$$  \hspace{1cm} (5)

where \( \hat{\epsilon} \neq 0 \) \( \hat{\epsilon} \neq 0 \), while \( \hat{\epsilon} = 0 \) \( \hat{\epsilon} = 0 \). In other words, in this representation, the nonlinear layer has a fixed absorption rate which is equal to unity, the outgoing field boundary associated with the backward map varies as \( \hat{E}_f'' = \sqrt{\epsilon} T \) while the incident light energy density is \( \hat{\mathcal{W}}_0 = |\hat{E}_f''|^2 \) at a resonant frequency \( \omega_0 = 8.15 \). The parameters of the one-dimensional photonic band-gap structure are indicated at the text. We observe that for moderate values of \( \mathcal{W}_0 \), both \( T \) and \( A \) are suppressed and the system becomes reflective, i.e., \( R \approx 1 \). Inset: \( T, R, A \) for a single nonlinear layer (normal incidence). This system, for moderate \( \mathcal{W}_0 \) values, does not reflect but mainly absorbs the incident energy.
CONCEPT OF A REFLECTIVE POWER LIMITER BASED . . .

matrices $M^{(L/R)}_8$ and $M^{(E)}_8$ [see Eq. (2)] are defined as $M^{(L/R)}_8 = 1/\xi_{12}^{L/R}$, $M^{(E)}_8 = -\xi_{12}^{L/R}$, $M^{(L/R)}_2 = -\tau_1^{L/R}/\xi_{12}^{L/R}$, and $M^{(E)}_2 = 1/\xi_{12}^{L/R}$. We remark that bistabilities are present for a very narrow parameter range of the system and therefore can be omitted from our considerations below.

Next we calculate the field amplitudes just before and after the delta defect by utilizing the transfer matrices Eq. (5) associated with the linear segments. For a left incident wave, we have at $z = 0^-$

$$E_r^L = E_t^L \frac{r_2 E_r^L}{t_2}$$

$$E_t^L = E_t^L \frac{E_t^L r_2}{t_2}$$

while at $z = 0^+$ just after the delta defect we have

$$E_r^L = \frac{t_2 E_r^L}{1 - |t_2|^2}$$

$$E_t^L = \frac{t_2 r_2 E_t^L}{1 - |t_2|^2}.$$  

Using Eqs. (4) and (5) together with the continuity of the field at $z = 0$ and the suitable discontinuity of its derivative we write the incident and reflected field amplitudes in terms of the transmitted wave amplitude

$$E_r^L = \left(1 - \left(\frac{t_2}{t_2} - \frac{1 - t_2}{t_2}\right)\gamma |E_r^L|^2\right)E_r^L,$$

$$E_t^L = \left(1 - \left(\frac{t_2}{t_2} - \frac{1 - t_2}{t_2}\right)\gamma |E_t^L|^2\right)E_t^L,$$

where $\gamma$ is the transmission amplitude in the absence of the $\phi$-like layer, $\tau_1$ is the transmission amplitude when $\gamma = 0$, and $\xi = \frac{2i \tau_1 \gamma}{t_2}$. From Eq. (6) we deduce the transmission, reflection and absorption amplitudes. For the transmission and reflection amplitude we get that

$$t = \tau_2 - i\tau_2 \gamma |E_r^L|^2,$$

$$r = \left(\frac{t_2}{\tau_2} - \frac{1 - t_2}{\tau_2}\right) \left|\gamma |E_r^L|^2\right|.$$  

The transmittance, reflectance, and absorption can then be calculated as $T = |t|^2$, $R = |r|^2$, and $A = 1 - T - R$. From Eq. (7) we observe that increasing $\gamma$ (we note that the energy density of the incident light $\omega_I \sim \gamma$) results in an increase of the denominator of the transmission amplitude and therefore to a decrease of $T$ (for very large $\gamma$ values it becomes zero). At the same time the reflection amplitude, becomes $r \rightarrow (\frac{t_2}{\tau_2} - \frac{1 - t_2}{\tau_2}) \left|\gamma |E_r^L|^2\right|$ corresponding to perfect reflection, i.e., $R \rightarrow 1$. Consequently in this limit we have zero absorption, $A = 0$.

Figure 3 demonstrates the effect of $\omega_I$ on a resonant localized mode for the case of symmetrically placed Bragg mirrors on the left and right side of a $\phi$-like defect. The alternate layers of the Bragg mirrors have permittivity $\epsilon_1 = 4$ and $\epsilon_2 = 9$ while the permittivity of the defect layer is $\epsilon = 1.5$.

The transport characteristics of the Bragg mirrors $t_2 = t_2^0$ and $r_2 = r_2^0$ have been calculated numerically and used as inputs in Eqs. (7). We find (see Fig. 3) that the overall behavior of $T$, $R$, and $A$ is similar to the one observed in the simulations of Fig. 2.

For comparison, we also report (inset of Fig. 3) the behavior of $T$, $A$, and $R$, for a single nonlinear layer (without any Bragg mirrors), vs the incident light energy density $\omega_I$. They are calculated analytically using the continuity of the field and the discontinuity of its derivative at the position of the $\phi$ defect. Specifically, $T = \frac{\omega_0}{\omega_0} \left(1 + \gamma |E_r^L|^4\right) I/4$ and $A = k \omega_0 |E_r^L|^4 I^2 T$.

We find that for moderate $\omega_I$ values the single nonlinear layer is mainly absorptive (inset of Fig. 3) while the structure of Fig. 1 is mainly reflecting the incident light back to space (main panel of Fig. 3).

We have also investigated the efficiency of the proposed limiter in the case of oblique incidence. A representative example in the case of an incident angle $\phi = 6^\circ$ is shown in Fig. 4. The Bragg mirror considered in this example consists of two layers with permittivities $\epsilon_1 = 9$, $\epsilon_2 = 16$ while the...
nonlinear impurity has permittivity $\epsilon = 16$. We find again that as the incident light energy density $W_L$ takes moderate values, the transmittance and the absorption are suppressed, and the structure becomes reflective, i.e., $R \approx 1$. This behavior has to be contrasted with the one found for the single nonlinear layer where for moderate $W_L$ values the dominant mechanism is absorption; see the inset of Fig. 4.

The effectiveness of the structure of Fig. 1 to act as a self-protecting power limiter for any incident angle calls for a generic argument for its explanation. The following heuristic argument, provides some understanding of the mechanism underlying our structure. First we recall that the defect results in the creation of a resonance mode which is localized around the impurity layer $z = 0$ and decays away from its localization center with an envelope profile $E_i(z) \sim \exp(-\alpha|z|)$ (all distances are measured in units of the width layer $d$). An incoming (say from the left) wave that carries an incident energy flux $S$ can resonate via this mode as long as the loss coefficient is $\alpha \approx \alpha L \sim \exp(-\alpha L)$, which in the case of evanescent modes is $\approx \alpha \sim 1$. This can only happen if $\alpha \approx 0$, which in turn means that the energy flux is given by the Poynting vector $S = \psi^* \psi$, in the case of evanescent modes is $\approx \alpha \sim 1$, which in turn means that the energy flux is given by the Poynting vector $S = \psi^* \psi$, which is normally the case. We have shown that such a layered structure can act as a reflective power limiter. Specifically, at low intensity of the incident light, the entire stack will be transmissive in the vicinity of the localized mode frequency. When the input power exceeds a certain level, the nonlinearity suppresses the localized mode, and the layered structure becomes highly reflective (not absorptive?) within a broad frequency range and for a wide direction of incidence. In other words, the excessively strong radiation will be reflected back to space, rather than being absorbed by the lossy nonlinear layer. This can prevent overheating and destruction of the limiter. A simple realization of such a self-protected (reflective) power limiter is provided by a lossy nonlinear layer sandwiched between two Bragg mirrors, as shown in Fig. 1. A shortcoming of such a simple design is that although the high-intensity radiation will be reflected back to space within a broad frequency range, the low-intensity transmission occurs only within a narrow frequency band in the vicinity of the localized mode frequency. This problem can be addressed by using a more sophisticated layered structure than that shown in Fig. 1.

The above approach to the realization of a reflective power limiter is perfectly scalable and can be applied to any frequency range. Of course, the structural geometry and the material choice of the nonlinear layer and the Bragg reflectors are all dependent on the frequency of interest and on the light intensity limitations. For instance, the material of choice for the nonlinear defect layer can be ZnSe (at optical frequencies) and InP or GaAs (at near-infrared frequencies). The Bragg reflectors can be made of alternate layers of silicon nitride (Si$_3$N$_4$) and silica (SiO$_2$). For a given nonlinear defect layer, the incident light intensity at which the whole structure turns from transmissive to highly reflective is strongly dependent on the number of layers in the Bragg reflectors. An experimental realization of this setup is currently under investigation.

This work is partly sponsored by the Air Force Research Laboratory (AFRL/RYDF) through the AMMATAC contract with Alion Science and Technology, and by the Air Force Office of Scientific Research LIR09RY04COR and FA 9550-10-1-0433 and by an AFOSR MURI grant FA5950-14-1-0037.

[11] The quantities $T, R, A$ are the same for a right incident wave as well, since our structure is reciprocal.

[13] In all numerical simulations in Figs. 2–4 we use units such that $c = 1, d = 1$.
[14] We assume the incident wave has amplitude $O(1)$ and that due to continuity of the wave function at the boundary $E_{\text{ref}}(z = -L) \sim O(1) \to E_{\text{ref}}(z = 0) \sim \exp(L)$.
[15] A similar argument can be used for $\alpha_+ \sim O(1)$.
Reflective optical limiter based on resonant transmission

Eleana Makri and Tsampikos Kottos
Department of Physics, Wesleyan University, Middletown, Connecticut 06459, USA

Ilya Vitebskiy
Air Force Research Laboratory, Sensors Directorate, Wright Patterson Air Force Base, Ohio 45433, USA

(Received 21 October 2014; published 27 April 2015)

Optical limiters transmit low-level radiation while blocking electromagnetic pulses with excessively high energy (energy limiters) or with excessively high peak intensity (power limiters). A typical optical limiter absorbs most of the high-level radiation, which can cause its overheating and destruction. Here we introduce the concept of a reflective energy limiter which blocks electromagnetic pulses with excessively high total energy by reflecting them back to space, rather than absorbing them. The idea is to use a defect layer with temperature-dependent loss tangent embedded in a low-loss photonic structure. The low-energy pulses with central frequency close to that of the localized defect mode will pass through. But if the cumulative energy carried by the pulse exceeds certain level, the entire photonic structure becomes highly reflective (not absorptive) within a broad frequency range. The underlying physical mechanism is based on self-regulated impedance mismatch which increases dramatically with the cumulative energy carried by the pulse.

I. INTRODUCTION

The protection of photosensitive optical components from high incident radiation has applications ranging from microwave and optical communications to optical sensing [1–3]. As a result, a considerable research effort has focused on developing protection schemes and materials that provide control of high-level optical and microwave radiation and prevent damages of optical sensors (including the human eye) and microwave antennas [4–9]. Optical limiters constitute an important class of such protection devices. They are supposed to transmit low-level radiation, while blocking light pulses with excessively high level of radiation. A typical passive optical limiter absorbs most of the high-level radiation, which can cause its overheating and destruction. The most common realization of a passive optical limiter is provided by a single protective layer with complex permittivity \( \varepsilon = \varepsilon' + i\varepsilon'' \), where the imaginary part \( \varepsilon'' \) increases sharply with the radiation level. For low-level radiation, the absorption is negligible and the protective layer is transparent. An increase in the radiation level results in an increase in \( \varepsilon'' \), which renders the protective layer opaque. As a consequence, most of the high-level radiation will be absorbed by the limiter. If the same protective layer is incorporated into a certain photonic layered structure, the entire multilayer can become highly reflective for high-level radiation, while remaining transmissive at certain frequencies if the radiation level is low. Such a photonic reflective limiter can be immune to overheating and destruction by high-level laser radiation, which is our main objective.

The physical reasons for the sharp increase in \( \varepsilon'' \) with the radiation level can be different. For instance, it can be photoconductivity, two-photon absorption, heating, or any combination of the above mechanisms. In our previous publication [10] we considered the particular case of a strong nonlinear dependence of \( \varepsilon'' \) of the protective layer on light intensity. This can be attributed, for instance, to a two-photon absorption. We showed that incorporation of such a nonlinear layer in a properly designed low-loss layered structure makes the entire assembly act as a reflective power limiter. In this paper, we consider a more practical particular case where the increase in \( \varepsilon'' \) is due to heating of the protective layer. We show that, depending on the pulse duration as compared to the thermal relaxation time, the properly design layered structure incorporating such a protective layer can act as a reflective energy limiter or as a reflective power limiter. Specifically, for short pulses, such a layered structure acts as an energy limiter, reflecting light pulses carrying excessively high energy. By comparison, for sufficiently long pulses, the same layered structure will act as a power limiter. In either case, most of the incident radiation will be reflected back to space, even though a standalone protective layer would act as an absorptive optical limiter.

The proposed architecture consists of a (protective) defect layer embedded in a low-loss Bragg grating. In contrast to the reflective power limiter introduced in Ref. [10], the defect layer does not have to be nonlinear, but it must display strong temperature dependence \( \varepsilon''(T) \) of the imaginary part of its permittivity. If the total energy carried by the pulse is low, \( \varepsilon''(T) \) remains small enough to support a localized mode and the resonant transmittance associated with this mode. If, on the other hand, the energy carried by the pulse exceeds a certain level, the defect layer becomes lossy enough to suppress the localized mode, along with the resonant transmittance. The entire stack turns highly reflective, which is consistent with our goal. We refer to this limiter as a reflective energy limiter in order to distinguish it from the nonlinear reflective power limiter introduced in Ref. [10]. Finally, if the pulse duration significantly exceeds the thermal relaxation time of the defect layer, the entire layered structure will again act as a reflective power limiter with the cutoff light intensity determined by the thermal relaxation time of the defect layer, not by the nonlinearity in \( \varepsilon'' \), as was the case in Ref. [10].

The organization of the paper is as follows. In Sec. II we clarify the different mechanisms underlying a reflective energy limiter (the theme of the present study) and a reflective power limiter (the theme of Ref. [10]). In Sec. III, a conceptual design...
For the reflective energy limiter is presented, along with the mathematical formalism used in our calculations. In Sec. IV, we analyze the role of thermal conductivity. The latter plays an important role if the pulse duration is comparable or exceeds the pulse intensity and duration. In this respect, it does not behave as a simple linear system.

To summarize, in Ref. [10] we assumed a strong instantaneous nonlinearity of the defect layer, but no time-cumulative effects (like heating). This is why the optical limiter in Ref. [10] is only sensitive to the pulse peak intensity and not to its duration. In contrast, in this paper we assume a negligible instantaneous nonlinearity, while the heating is essential for the performance of the limiter. In practice, there might be a combination of the two mechanisms. The bottom line, though, is that no matter what causes the rise in $\epsilon''$, the layered structure in Fig. 1 will act as a reflective optical limiter. In contrast, a standalone defect layer would absorb most of the high-level radiation.

We will demonstrate the concept of energy limiter, high-lighted above, using a simplified model. In our modeling we will omit any dispersive phenomena of $\epsilon''$ originating from the material considered (temporal dispersion). We did this for a reason: Indeed, the change in $\epsilon''$ due to heating required for the limiter to perform is usually at least two or three orders of magnitude, which is much greater compared to the typical temporal dispersion of optical materials. We will also assume that the pulse duration is much larger than the carrier period. This justifies the use of the adiabatic approximation, which means that the heat release during one carrier period of oscillation is infinitesimally small. Finally we have assumed a simplified dependence of $\epsilon''$ from the temperature of the defect layer. More realistic schemes or dependences will only mask the demonstration of the concept with unnecessary numerical complications.

III. PHYSICAL STRUCTURE AND MATHEMATICAL MODEL

We consider two identical lossless Bragg reflectors consisting of two alternating layers. Each mirror consists of forty layers which are placed at $-L \leq z \leq 0$ and $d \leq z \leq L + d$. For the sake of the discussion we assume that the layers consist of $\text{Al}_2\text{O}_3$ and $\text{SiO}_2$ with corresponding permittivities $\epsilon_1 = 3.08$ and $\epsilon_2 = 2.1$. These values are typical for these materials at wavelengths $\lambda \sim 1$ $\mu\text{m}$. The width of layers is assumed to be $d_1 = 151$ nm and $d_2 \approx 183$ nm respectively. At $0 \leq z \leq d$ we introduce a defect lossy layer with complex permittivity $\epsilon_d = \epsilon_1' + i\epsilon_1''$. We further assume that the imaginary part of the permittivity of the defect layer depends on the temperature $T$, i.e., $\epsilon_1''(T) = c_1 T$. For simplicity, we assume linear dependence, i.e., $\epsilon_1''(T) = c_1 T$, where $c_1, c_2$ are some characteristic
At each layer inside the grating, Eq. (1) admits the solution for a left incident wave, which can be obtained by iterating backwards in the frequency domain: $k$ is the refraction index of the layer, $c$ is the speed of light in the vacuum and $n_0$ is the refractive index of air. Imposing continuity of the field and its derivative at each layer interface, as well as taking into consideration the free propagation in each layer, we get the following iteration relation:

$$
\left( \begin{array}{c} E_{l+1}' \\ E_{l+1}'' \end{array} \right) = M_l \left( \begin{array}{c} E_{l+1}^{(0)} \\ E_{l+1}^{(1)} \end{array} \right), \quad
M_l = P_l^{(0)} Q_l^{(0)} K_l^{(0)} P_l^{(1)}.
$$

(2)

where

$$
Q_l^{(0)} = \left( \begin{array}{cc} e^{ik_{l+1}d} & 0 \\ 0 & e^{-ik_{l+1}d} \end{array} \right),
$$

$$
K_l^{(0)} = \left( \begin{array}{cc} n_{l+1}/n_l & 0 \\ 0 & n_{l}/n_{l+1} \end{array} \right),
$$

$$
P_l^{(0)} = \left( \begin{array}{cc} e^{ik_{l+1}d} & 0 \\ 0 & e^{-ik_{l+1}d} \end{array} \right),
$$

$$
P_l^{(1)} = \left( \begin{array}{cc} e^{ik_{l+1}d} & 0 \\ 0 & e^{-ik_{l+1}d} \end{array} \right).
$$

(3)

At the same time the field outside the layered structure can be written as $E(z) = E_{l-1}^{(0)} \exp(ik_{l-1}z) + E_{l-1}^{(1)} \exp(-ik_{l-1}z)$ for $z < -L$ and $E(z) = E_{L}^{(0)} \exp(ik_Lz) + E_{L}^{(1)} \exp(-ik_Lz)$ for $z > L + d$. The amplitudes of forward- and backward-propagating waves on the left $z < -L$ and right $z > L + d$ domains are related via the total transfer matrix $M = p^{(0)} Q^{(0)} K^{(0)} K^{(0)} Q^{(0)} p^{(1)}$ (where $N$ is the number of layers on each grating and $n_{2N+2} = n_0$):

$$
\left( \begin{array}{c} E_{l-1}' \\ E_{l-1}'' \end{array} \right) = \left( \begin{array}{cc} M_{l-1} & M_{l+1} \\ M_{l+1} & M_{l-1} \end{array} \right) \left( \begin{array}{c} E_{l-2}' \\ E_{l-2}'' \end{array} \right).
$$

(4)

The transmittance and reflectance and the field profile, say for a left incident wave, can be obtained by iterating backwards Eqs. (2) and (4) together with the boundary conditions $E_L^0 = 0$ and $\left| E_L' \right|^2 = 1$ due to the linearity of the equations, one can always impose a value for the outgoing field and calculate via a backward iteration of the transfer matrices the corresponding input field [11]. Specifically we have $T = \left| E_L'/E_L'' \right|^2$, $R = \left| E_L'/E_L'' \right|^2$. These can be expressed in terms of the transfer matrix elements as $T = \left| M_{l+1}/M_{l-1} \right|^2$, $R = \left| M_{l-1}/M_{l+1} \right|^2$. The absorption coefficient $\alpha$ can then be evaluated in terms of transmittances and reflectances as $\alpha = 1 - T - R$. This is the case that the permittivity of the defect layer is replaced by $\epsilon_d = \epsilon_1$, the whole structure is periodic and displays a typical dispersion relation consisting of transparent frequency windows (bands) where light is transmitted with near-unity transmittance alternated with frequency windows (gaps) where the incident light is experiencing almost complete reflection.

When the defect is included in the middle of the grating, for zero temperature $T = 0$ corresponding to permittivity $\epsilon_d = \epsilon_2$, the layered structure supports a localized resonant defect mode [see Fig. 1(a)] with a frequency lying in a photonic band gap of the Bragg grating [see Fig. 1(b)]. For the specific setup that we consider here, we find that a resonant mode is localized in the middle of the gap at wavelength $\lambda_0 = 1060$ nm. This defect mode is localized in the vicinity of the defect layer and decays exponentially away from the defect [see Fig. 1(a)]. In the vicinity of the localized mode frequency $\omega_r$, the entire layered structure displays a strong resonant transmission due to the excitation of the localized mode [see Fig. 1(b)]. In other words, the transmittance is $T(\omega_r) \approx T_0 \approx 1$ while the reflectance and the absorption in the absence of any losses are $R(\omega_r) \approx R_0 \approx 0$ and $A(\omega_r) \approx A_0 \approx 0$ respectively. This picture is still applicable even in the presence of small (but nonzero) dissipative permittivity $\epsilon_d' \neq 0$ [see Figs. 1(a) and 1(b)].

An alternative expression for the absorption coefficient $\alpha$ can be given in terms of the permittivity and field intensity $\left| E(z) \right|^2$ inside the defect layer. The resulting expression is derived by subtracting the product of Eq. (1) with $E'(z)$ from its complex conjugate form and then integrating the outcome over the interval $-L \leq z \leq L$. We get

$$
\int_{-L}^{L} \left( \frac{E dE}{dz} - \frac{E dE}{dz} \right) dz = 2ik^2 \int_{-L}^{L} \epsilon_r(z) \left| E(z) \right|^2 dz = 0.
$$

(5)

Substituting in Eq. (5) the expressions of the electric field at $z = -L$ and $z = L$ respectively we get

$$
\alpha = 1 - T - R = \frac{k}{|E'_0|^2} \int_{-L}^{L} \epsilon_r(z) \left| E(z) \right|^2 dz.
$$

(6)

Furthermore, we assume that $\epsilon_r(z)$ is zero everywhere inside the layered structure apart from the interval $0 \leq z \leq d$ where the defect layer is placed. In this interval it takes a uniform value $\epsilon_d(z) = 0 \leq z \leq d = d_0(z)$. These simplifications allow us to express the absorption coefficient of Eq. (6) in the form

$$
\alpha(T) = \rho(T) \omega_0 \epsilon_d(z)^2 \left( T \right),
$$

(7)

where $\rho(T) = I_0 / |E'_0|^2$ is the ratio of the integral of light intensity $I_0 = \int_{-L}^{L} \left| E(z) \right|^2 dz$ at the lossy layer and the incident light intensity. It is obvious from Eq. (7) that $\alpha(T)$ depends on both the dissipative part of the permittivity and the value of the electric field inside the defect layer. Although the former increases monotonically with the temperature $T$ and thus with the duration time of the incident pulse, this is not true for $\alpha(T)$. The latter, which is a unique function of the permittivity, remains approximately constant up to some value of $\epsilon_d'$ above which it decreases, leading eventually to a total decrease of the absorption coefficient together with a simultaneous increase.
of the reflectivity of the structure. This is related to the fact that the increase of $\varepsilon'_2$ spoils the resonant localized mode [see Fig. 1(c)], which is responsible for high transmittance. Specifically, when the losses due to $\varepsilon'_2$ overtake the losses due to leakage from the boundaries of the structure, the resonant mode ceases to exist [see Fig. 1(c)] and the structure becomes reflective, i.e., $R \approx 1$, and $T \approx 0$ [see Fig. 1(d)].

As a consequence we have that $A = 1 - T - R \approx 0$ and the system does not absorb the high incident energy of the incoming light source but rather reflects it back in space.

In the nonmonotonic shape of the envelope of the scattering field, in Fig. 1(c) is a direct consequence of the fact that the structure becomes reflective $R \approx 1$; $T \approx 0$. One has to realize that in the case where both Bragg gratings on the left and right of the defect layer are finite, the field inside each half-space is written as a linear combination of two evanescent contributions with exponentially decreasing and exponentially increasing amplitudes. Their relative weight is determined by the boundary conditions $E(z = -L) = E'_{f}(-L)$ and $E(L) = E'_{f}(L) = E''_{f}(-\mathbf{c} \times \mathbf{t})$ at the two outer interfaces of the layered structure. In the case of reflective structures these boundary conditions lead to the relation $E(-L) = E'_{f} \sim O(1)$ and $E(L) \sim 0$. It can be shown rigorously that in this case, the field on the left half-space of the structure is dominated originally by the exponentially decaying component while after some turning point $z_0$ the exponentially increasing component becomes dominant up to the defect layer. After that the field decays exponentially as in the resonant case. Similar scattering field profiles have been found in cases of active (gain) defects [12].

One can use a simple qualitative argument to estimate the condition under which $A(T)$ continues to increase. As we discuss previously, we assume that the electromagnetic energy losses occur in the lossy defect layer. The dissipated power can be estimated from Eq. (7) to be $\mathcal{Q} \propto \mathcal{E}^2(z)/\omega c \varepsilon_2$. Due to the energy conservation, the rate of energy dissipation cannot exceed the energy supply provided by the incident wave. The latter is $S_0 \propto c \mathcal{E}^2$. Taking this constraint into account we get the following upper limit on the field intensity at the defect layer location:

$$
\frac{c}{\omega} |\mathcal{E}'_f|^2 \geq |\mathcal{E}_d|^2.
$$

Comparing Eqs. (8) and (9) we can conclude that if

$$
\frac{c}{\omega} |\mathcal{E}'_f|^2 \exp(-2k'L) \ll 1
$$

then the amplitude of rising evanescent mode $\mathcal{E}(z = -L)$ at the left stack boundary is much less than amplitude of the incident wave

$$
|\mathcal{E}(z = -L)|^2 \sim |\mathcal{E}'_f|^2.
$$

The latter condition, Eq. (11), implies that the energy density inside the left grating is much smaller than the energy density of the incident wave, and hence only a small portion of the incident light energy $S_0 \propto c |\mathcal{E}'_f|^2$ will cross the stack boundary at $z = -L$. In other words, the condition Eq. (10) for high stack reflectivity (and hence low transmittance and absorption) will always be satisfied if the loss tangent $\tan\delta(T)$ of the defect layer is large enough and/or if the number of layers in the Bragg grating is large enough.

Next, we want to quantify the above arguments. To this end, we calculate explicitly the transport characteristics of our grating structure for an incident laser pulse. Although the analysis can be generalized for any incident pulse shape, in our numerical simulations below, we have assumed for simplicity that the incident laser pulse has a train form [14]

$$
W_I(t) = 0 \quad \text{for } t < t_0 = 0 \quad \text{for } 0 \leq t < t_f = 0 \quad \text{for } t \geq t_f.
$$

We want to calculate the total energy transmitted, reflected, and absorbed during the duration of the pulse. These can be expressed in terms of the time-dependent transmittance $T(t)$, reflectance $R(t)$, and absorption $A(t)$, which are the main quantities that we analyze below. All other observables can be easily deduced from them. For example, the integrated (over the period of the pulse) absorption $\bar{A}$ can be defined as

$$
\bar{A} = \int_{-\infty}^{\infty} dW(t) T(t),
$$

while similar expressions can be used for calculating the total (over the period of the pulse) transmittance $\bar{T}$ and reflectance $\bar{R}$.

Our starting point is the “rate” equation

$$
\frac{d}{dt} T(t) = \frac{1}{c} \left[ A(T) W_I(t) + \kappa T_0 - T \right]
$$

that describes the heating rate of the defect layer. Above, $C$ is the heat capacity, $W_I(t)$ is the heat flux density, and $\kappa$ is the thermal conductance of the defect layer. The first term in Eq. (14) describes the heating process of the lossy layer while the second one corresponds to heat dissipation from the defect layer to the mirror (if any) or to the air. To further simplify our calculations, we assume that the temperature changes are within a domain where both thermal conductance and heat capacity are constants and independent of temperature changes.
Substitution of the absorption coefficient from Eq. (7) into Eq. (14) leads us to the following equation:

\[
\frac{d}{dt} T(t) = \frac{1}{C} [\varepsilon''(T) \rho(T) W_f(t) + \kappa \varepsilon''(T) \rho(T)] - \kappa \varepsilon''(T) \rho(T). \tag{15}
\]

which expresses the temporal behavior of the temperature \(T(t)\) in terms of the given profile \(W_f(t)\) of the incident pulse. Everything else, e.g., \(\varepsilon''(T)\), \(A(t)\), \(T(t)\), and \(R(t)\), can be directly and explicitly expressed in terms of \(T(t)\).

In case that \(\kappa = 0\), one can further show that the outcome can be written in terms of the total incident energy \(U_f = \int_0^t W_f(t) dt\). Furthermore, using Eq. (15) we get that \(T_f = \int_0^t \varepsilon''(T_f) \rho(U) dU/C \). The associated total absorption is \(\Delta = \int_0^t \varepsilon''(T) \rho(U) dU/U_f\), while similar expressions can be derived for the other transport characteristics.

In Fig. 2 we report the outcomes of a direct integration of Eq. (15) for \(\kappa = 0\). In this case, the incident thermal energy does not dissipate outside of the defect layer, i.e., the thermal relaxation time is infinite. Therefore, time-cumulative effects are important and thus our structure acts as an energy limiter. In Fig. 2(a) we report the temporal behavior of permittivity \(\varepsilon''(T)\) as a function of the pulse duration \(t_f\). Notice that for train pulses the pulse duration \(t_f\) is directly analogous to the total incident energy \(U_f\). We will therefore alternate, in our presentation below, the dependency of \(\varepsilon''(T), R, A\) from the pulse duration with the (more natural parameter for an energy limiter) total incident energy of the pulse.

Originally \(\varepsilon''(T)\) is essentially unaffected by the incident energy and the same is true for the resonance mechanism (via the defect mode) that is responsible for high transmittance in the absorption of losses. In this domain \(T_i \approx 1, R_i \approx 0\) while there is a slow increase of the absorption \(A_i\), as it can be seen from Fig. 2(b) (solid lines). Once the incident energy (pulse duration time) exceeds some critical value, there is a rather abrupt increase in \(\varepsilon''(T)\) which results in the destruction of the resonance mode. Subsequently, the incident energy does not resonate into the structure, leading to a decaying absorption \(A_i \approx 0\), while the same is true for the transmittance \(T_i \approx 0\). At the same time, there is a noticeable growth of the reflectance, which becomes approximately equal to unity \(R_i \approx 1\). For comparison we also plot at the same figure the results of the standalone layer. We find that for large incident energies (pulse durations \(t_f\)) the absorption \(A_i(t_f)\) is higher by more than two orders of magnitude as compared to the case of the reflective energy limiter.

We have also performed the same analysis for the case where the thermal conductance \(\kappa\) is different from zero. In Fig. 3 we report the results of the numerical integration of Eq. (15) in the presence of thermal conductivity. For long pulse duration we find a steady-state behavior of the transport characteristics of the reflective energy limiter. The physical nature of the steady-state regime is quite obvious. It corresponds to the situation when the heat released in the defect layer is completely carried away by thermal conductivity. At this point, the temperature of the defect layer stabilizes and the time derivative \(dT(t)/dt\) in Eqs. (14) and (15) vanishes. The latter condition determines the steady-state values of the defect layer temperature as a function of the incident light amplitude.

In this limiting case our structure acts as a power limiter. For comparison, the results of the standalone lossy layer are also reported in this figure. We find that in the steady-state regime our structure performs superbly, resulting in absorption values which are more than two orders of magnitude smaller than the ones achieved by the standalone lossy layer.

V. CONCLUSIONS

At infrared and optical frequencies, the reflectivity of known uniform materials is well below 90%, especially so when the incident light intensity is dangerously high. So, if we want to build a highly reflective optical limiter, we
have to rely on photonic structures which would support some kind of low-intensity resonant transmission via slow or localized modes at photonic bandgap frequencies. If the incident light intensity increases, the respective localized mode must disappear, and the entire photonic structure will behave as a simple Bragg reflector. Here we considered the so-called dissipative mechanism of the localized mode suppression. At first glance, it seems counterintuitive, because the high reflectivity and low absorption are caused by the increase in the loss tangent of the defect layer in Fig. 1. A qualitative explanation for such a phenomenon is that the large value of $\varepsilon''$ in the defect layer results in decoupling of the left and the right Bragg reflectors in Fig. 1. Of course, there might be other ways to suppress resonant transmittance when the incident light intensity, or the total energy of the pulse, grow dangerously high. Still, the presented “dissipative” mechanism seems simple and practical.

A key physical requirement to the constitutive materials of the reflective photonic limiter is that the dielectric layers of the Bragg reflectors in Fig. 1 must be lossless and linear. Indeed, if at high-level radiation the Bragg reflector layers also become lossy, the optical limiter will still perform, but it will not be a reflective limiter anymore, because a significant portion of the high-level radiation will be absorbed by the grating. Fortunately, at visible and infrared frequencies there are plenty of available optical materials with negligible losses and nonlinearities that can be used for the construction of the Bragg mirrors.

ACKNOWLEDGMENTS

This work is sponsored by the Electromagnetics Portfolio of Dr. Arje Nachman via the Air Force Office of Scientific Research No. LRIR09RY04COR and by AFOSR MURI Grant No. FA9550-14-1-0037.

[13] We stress that in the case in which both Bragg gratings on the left and right of the defect layer are finite, both evanescent contributions are present in either half-space, although only one if they is dominant on either side. Furthermore, one can show that the presence of both evanescent contributions on either side of the defect layer can provide an energy flux and, hence, a nonzero transmittance.
[14] We have checked that the same qualitative behavior is obtained for other pulse shapes as well.
Hypersensitive Transport in Photonic Crystals with Accidental Spatial Degeneracies

Elena Makić1, Kyle Smith2, Andrey Chabanov2, Ilya Vitebskiy3 & Tsampikos Kottos1

A localized mode in a photonic layered structure can develop nodal points (nodal planes), where the oscillating electric field is negligible. Placing a thin metallic nanolayer at such positions will have nearly no effect on the localized mode and the resonance transmission associated with this mode. This is the well-known phenomenon of induced transmission. Here we demonstrate that if the nodal point is not a point of symmetry, then even a tiny alteration of the permittivity in the vicinity of the metallic layer drastically suppresses the localized mode along with the resonant transmission. This renders the layered structure highly reflective within a broad frequency range. Applications of this hypersensitive transport for optical and microwave limiting and switching are discussed.

One of the main technological and fundamental challenges of our days is the design of electromagnetic architectures that allow for an efficient manipulation of the amplitude, phase, polarization, or direction of electromagnetic signals. Management of these features can lead to many diverse applications ranging from optical and microwave communications1,2, sensors and power limiters3, to energy harvesting, switching, and optical computing4. In this endeavor, the enhancement and control of the interaction between electromagnetic radiation and matter is of utmost importance.

An efficient way to achieve this enhancement is via localized modes supported by defect layers embedded in a layered photonic structure. Such localized modes develop nodal points where the amplitude of the oscillating electric field is very small. Placing a thin metallic nanolayer at such positions will have nearly no effect on the localized mode and the resonance transmission associated with this mode. This is the well-known phenomenon of induced transmission (see, for example5–8, and references therein). By comparison a stand-alone metallic nanolayer of the same thickness is totally opaque at the same frequency range which explains the term “induced transmission”5. Here we argue that a small perturbation \( \Delta \epsilon \) in the permittivity of a layer(s) nearby to the metallic nanolayer, can drastically affect the localized mode and resonance transmission associated with it. Depending on the nodal point symmetry, there are three possible scenarios: (a) the nodal point (with the metallic nanolayer) coincide with the mirror plane of the layered structure before and after the perturbation; (b) the nodal point coincide with the mirror plane in the original configuration, but the perturbation destroys this symmetry; and (c) the nodal point of the localized mode with the metallic nanolayer is not a symmetry point, neither before nor after the perturbation, in which case the coincidence of the metallic nanolayer and the node of the unperturbed localized mode can be viewed as accidental spatial degeneracy (ASD). In the case (a), the symmetric alteration of the layered structure results simply in a shift of the localized mode frequency. The metallic nanolayer still coincides with the nodal point of the localized mode at the shifted frequency and hence does not affect the resonant transmission at that frequency. In the cases (b) and (c), the nodal point of the localized mode shifts away from the metallic nanolayer, which can result in a dramatic suppression of the localized mode, along with the resonant transmission. In either case, the layered structure becomes opaque at any frequency. Due to the presence of the metallic nanolayer, the abrupt transition from resonant transmission to broadband opacity can be caused by just a tiny change (few percent point) of the permittivity \( \epsilon = \epsilon' + i \epsilon'' \) of one of the dielectric layers of the defect cavity, which justifies the use of the term hypersensitivity. The above feature equally applies to the cases (b) and (c), but with one important exception, when the permittivity alteration \( \Delta \epsilon \) is self-induced by the localized mode. Typically, a self-induced change in the permittivity is associated with nonlinear effects, heating, etc. If the permittivity change \( \Delta \epsilon \) is indeed self-induced, the transition from resonant transmission to broadband opacity is very pronounced and abrupt in the case (c) of accidental spatial degeneracy as compared to the case (b), where the unperturbed layered structure is symmetric.

1Department of Physics, Wesleyan University, Middletown, CT-06459, USA. 2Department of Physics and Astronomy, University of Texas at San Antonio, TX-78249, USA. 3Air Force Research Laboratory, Sensors Directorate, Wright Patterson Air Force Base, OH-45433, USA. Correspondence and requests for materials should be addressed to T.K. (email: tkottos@wesleyan.edu)

Received: 10 December 2015
Accepted: 02 February 2016
Published: 23 February 2016

OPEN
In most applications of metallo-dielectric layered structures (see for example5,9) the abovementioned hyper-
sensitive transport characteristics of asymmetric configurations to a self-induced alteration of the refractive index
would be undesirable and counterproductive. In this paper we take an alternative viewpoint. We demonstrate
how such hypersensitive transport can be used in microwave (and optical) limiters, and we show that it can dra-
matically enhance their performance. As an example, we consider a microwave limiter based on an asymmetric
metal-dielectric layered structure supporting a localized mode with $\text{ASD}$. We show that even a small self-induced
alteration of the refractive index at the neighborhood of the maxima of the localized mode produces an abrupt
transition from resonant transmission for low-level radiation to high broadband reflectivity for high-level radi-
ation. On the other hand, if the asymmetric permittivity alteration is caused by external physical action, such as,
asymmetric mechanical stress, electric field etc., rather than being self-induced by the localized mode, the above-
mentioned hypersensitivity will not be related to the ASD, and it will be equally strong in the setting (b) and (c).
This effect can be used in switches, modulators and sensors.

It is important that the induced transmission and its hypersensitivity to the incident electromagnetic wave
intensity in asymmetric metal-dielectric photonic structures are significant only in cases where the imaginary
permittivity of the metallic nano-layer is large. This critical condition is satisfied at frequencies starting from
microwave and up to the mid infrared.

Consider a 1D photonic crystal (PC) consisting of two lossless Bragg gratings (BG), with constitutive compo-
nents different for each grating, as shown in Fig. 1. The refraction indices and thicknesses of the bilayers of the left
BG (LBG) layers are $(n_l = 3.16, l_1 = 0.3162\, \text{cm})$ and $(n_l = 1, l_2 = 1\, \text{cm})$ and those of the right BG (RBG) are
$(n_r = 1.9, l_3 = 0.6667\, \text{cm})$ and $(n_r = 4.74, l_4 = 0.2108\, \text{cm})$ respectively. The interface between the two gratings
constitutes an asymmetric cavity. The periodic modulation of the index of refraction of each grating is engineered
in a way that both of them have the same band-gap structure, which is just a matter of convenience. The cavity
consists of two different quarter-wave layers with $(n_l, l_1)$ and $(n_r, l_3)$ and a thin metallic nanolayer between them
with thickness $l_2 = 0.18\, \text{cm} < \lambda$ and permittivity $\epsilon = 4.31 + \text{i} \times 10^6\, \text{Hz}$. Under typical circumstances the
permittivity of each of the two layers of the cavity is affected differently by an external perturbation. For example
the left layer $(n_l, l_1)$ can be more sensitive to high-level radiation than the right layer. The defect cavity supports a
localized mode with a frequency $f$, located in the middle of the photonic band-gap and whose nodal point coi-
cide with the metallic nanolayer. Evidently, this cavity is asymmetric, which corresponds to the case (c) described
above.
The transmission $T$, reflection $R$, and absorption $A$ are calculated via the transfer matrix approach. The latter connects the amplitudes of forward and backward propagating waves on the left and the right domains outside of the PC. At the $j$-th layer inside the structure, and also outside of the PC, a time-harmonic field of frequency $\omega$ satisfies the Helmholtz equation:

$$\frac{d^2 E(z)}{dz^2} + \left(\varepsilon - \varepsilon_j\right) E(z) = 0$$  \hspace{1cm} (1)$$

where $\varepsilon = \varepsilon_j - n^2$ is the permittivity of the $j$-th layer ($\varepsilon = 1$ for the vacuum). At the $j$-th layer, Eq. (1) admits solutions of the form $E_j(z) = \epsilon_j^+ e^{ik_j^+ z} + \epsilon_j^- e^{-ik_j^- z}$, where $\omega = 2\pi f$ is the wavevector at the vacuum. Outside the PC, Eq. (1) admits the solution $E_1(z) = \epsilon_1^+ e^{ik_1^+ z} + \epsilon_1^- e^{-ik_1^- z}$.

The continuity of the field and its derivative at the interface between two layers (or a layer and the vacuum) can be expressed in terms of the total transfer matrix, $\mathcal{M}$ which connects the forward and backward amplitudes on the left (L) and right (R) of the PC:

$$\begin{bmatrix} E_f^{(L)} \\ E_b^{(L)} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} E_f^{(R)} \\ E_b^{(R)} \end{bmatrix} \hspace{1cm} (2)$$

where $N$ is the total number of layers. The single-layer transfer matrix, $M_j$, connects the field amplitudes of the $j$-th and the $(j+1)$-th layers i.e. $\begin{bmatrix} E_f^{(L)} \\ E_b^{(L)} \end{bmatrix} = M_j \begin{bmatrix} E_f^{(R)} \\ E_b^{(R)} \end{bmatrix}$. Thus the transfer matrix approach allows us also to construct the field $E_j(z)$ at each layer, provided that appropriate scattering boundary conditions are imposed. The latter, for a left incident wave, take the form

$$T = R = \frac{\epsilon_1^+ \epsilon_2^-}{\epsilon_1^- \epsilon_2^+}$$

We start our analysis with the investigation of the transmission spectra of each of the two mirrors. Their dispersion relations $\omega(q)$ is calculated using the transfer matrix of one bilayer $\mathcal{M}^{ab} = M_{a1} M_{b1}^*$:

$$\mathcal{M}^{ab} = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}, \hspace{1cm} A = e^{i\pi a k} \cos(l_p q) + \frac{\rho_a}{\rho_b} + \frac{\rho_b}{\rho_a} \sin(l_p q),$$

$$B = -\frac{\rho_a}{\rho_b} \frac{\rho_b}{\rho_a} \sin(l_p q),$$

where the indices $a$ and $b$ indicate the layers 1 and 2 (3 and 4) associated with the LBG (RBG).

Propagating waves in each grating correspond to frequencies $\omega = k z$ for which

$$\text{Tr}(\mathcal{M}^{ab}) = 2 \cos(l_p q) \cos(l_p q) \cos(l_p q) = \frac{\rho_a}{\rho_b} + \rho_b \rho_a \sin(l_p q) \sin(l_p q) \leq 2 \cos(q),$$

where the total width of the bilayer $\Delta = l + l_2$ defines the periodicity of the LBG (for $a = 1$, $b = 2$) or the RBG (for $a = 3$, $b = 4$). Direct inspection of Eq. (4) indicates that the dispersion relations $\omega(q)$ and $\omega(q)$ are identical as long as $\Delta = \frac{Q}{Q}$ and $\Delta = \frac{Q}{Q}$. Once turned to finite photonic structures, both LBG and RBG will share the same band gap structure of the transmission spectrum, as long as these conditions are satisfied. We consider that each BG consists of five (quarter-wavelength) bilayers.

In Fig. 1b we show the transmission spectrum $T(f)$ of our PC for $\Delta z = 0$. The position of the band-edges is nicely described by Eq. (4). Moreover a resonant mode with $T(f)$ is $\approx 1$ at resonance frequency $f\approx 7.5$ GHz, below the middle of the band gap, has been created. The resonant mode is localized at the vicinity of the defect cavity and decays exponentially inside the two mirrors due to destructive interferences from the layers (blue profile at Fig. 1). The electric field $E(z)$ has a nodal point at the position of the metallic layer (blue profile at Fig. 1a). Thus the resonant localized mode is unaffected by the presence of the lossy layer and the entire PC is completely transparent at $f = f_r$ (see Fig. 1b). Furthermore, the lack of mirror symmetry ensures that the ASD occurs only for the resonance mode $f_r$. For all other (Fabry-Perot) resonances with frequencies $f = f_r$, the electric field distribution has finite amplitude at the position of the metallic layer leading to large reflection $R(f) \approx 1$ (see discussion below) and vanishing transmission $T(f) \approx 0$.

Moreover, any small perturbation (say, due to heating), which will change the permittivity of any of the two layers of the defect cavity (say the left one) by $\Delta \varepsilon$, will engage immediately the metallic nanolayer and lift the ASD of the resonance localized mode. In other words, the electric field will no longer have a nodal point at the position of the metallic layer (see red profile at Fig. 1a). This will trigger various competing mechanisms. On the one hand, it will increase the impedance mismatch and thus it will enhance the reflection. This mechanism is present whenever the electric field interacts with the metallic layer, even for $f = f_r$. On the other hand, it will lead to an increase of absorption. One can estimate the effect of these two competing mechanisms by analyzing the transport from a single lossy $\delta$-like defect with permittivity $\varepsilon(z) = \chi \delta(z)$. In this case, we have that:
Since the layer on the right is composed of a different material, in general, we expect a different variation of its permittivity with the temperature. For simplicity, we consider that the right layer is itself (which is usually the case in practical situations) or from a combination of these two physical mechanisms.

The hypersensitivity of the transport characteristics of our composite structure to small permittivity changes allows us to implement our PC as an efficient energy limiter. These are devices that protect electromagnetic sensors from high-energy radiation, while at the same time they are transparent to low energy radiation. Typically this protection is achieved via the absorption of the incident energy from the limiter, which turns opaque. At the same time this excessive energy overheats the limiter and leads to its self-destruction. Recently, however, the concept of reflective limiters has been introduced. These structures consist of a BG with one lossy defect layer, which undergoes a uniform self-induced permittivity change. The proposal for limiting action was based on the phenomenon of resonant transmission via a localized (defect) mode. The defect mode is transmissive at low-energy incident pulses, while it becomes highly reflective at high-energy pulses. Nevertheless, this proposal suffers from one drawback; the limiting action requires several orders of magnitude change of the permittivity of the lossy defect.

Instead, our design requires changes of only a few percentage points in the permittivity of one composite layer in order to provide limiting action. Importantly, the high reflectivity for the high intensity input persists within a broad frequency range – not just within a photonic band-gap as was the case in ref. 25,26.

We consider the PC of Fig. 1. We further assume, for the sake of the discussion, that the left layer of the defect cavity has a permittivity which depends on temperature (T) variations as $\varepsilon_l(T) = \varepsilon_0 + c_T(T)$, where for simplicity we consider that $\Delta \varepsilon(T) = c_T + c'_T(T)$. Since the layer on the right is composed of a different material, in general, we expect a different variation of its permittivity with the temperature. For simplicity, we assume that the right layer is much more resilient to the changes in temperature and thus we will keep its permittivity constant $\varepsilon_0$. There are various physical mechanisms that can lead to the heating of the dielectric layer. For example, it can originate from the heating of the nearby metallic nanolayer or from the presence of a small $c''$ at the permittivity of the dielectric layer itself (which is usually the case in practical situations) or from a combination of these two physical mechanisms.
The rate equation that determines the temporal behavior of the temperature $T(t)$ at the cavity is:

$$\frac{d}{dt}T(t) = \frac{1}{C}A(T)W(t)$$

where $W(t) = E(t)$ is the incident pulse intensity, which is a given function of time, $C$ is the heat capacity, and $A(T)$ is the temperature dependent absorption coefficient of the asymmetric cavity. A numerical integration of Eq. (6) (for a given pulse profile $W(t)$) allows us to evaluate the temperature $T(t)$ and from there the permittivity variations $\Delta \varepsilon / \varepsilon_1$ which are reported in Fig. 3a as a percentage change $\varepsilon \Delta \varepsilon / \varepsilon_1$. Then $f_r(A)$ and $f_r(\varepsilon)$ are calculated using transfer matrices as a function of pulse duration $t$ (see Fig. 3b–d). We find that $f_r(\varepsilon)$ (Fig. 3b) initially increases and reaches some maximum value around $t \approx t_0$. Corresponding to very small permittivity changes $\Delta \varepsilon / \varepsilon_1 \leq 0.1\%$. Further increase of $\Delta \varepsilon / \varepsilon_1$ leads to an abrupt decay of $f_r(\varepsilon)$ for resonance frequencies to values smaller than $-30$ dB. The off-resonance values already have absorption that is below $-60$ dB. At the same time the transmission (Fig. 3c) decays while the reflection (Fig. 3d) reaches unity. Therefore our photonic structure acts as a hypersensitive reflective microwave limiter, it will turn highly reflective within a broad frequency range for very small relative permittivity changes $\approx 0.5\%$. This behavior has to be contrasted with the proposal of ref. 25 where a limiting action is triggered only when the variation (due to heating) of the refraction index $\varepsilon = \varepsilon_1 + i\varepsilon_2$ of a defect lossy layer, which is embedded in a Bragg grating, is many orders of magnitude. The outcome of these calculations is also reported in the inset of Fig. 3a by referring to $T(f_r)$, $R(f_r)$ and $A(f_r)$ at resonance frequency versus the relative change of the permittivity. For these simulations we have used a BG with the same constitutive layers as the LBG of our PC. The lossy defect layer is placed in the middle of the grating and has $\varepsilon_2 = 12.1104$, $\varepsilon_2 = 10^{-10} + T$. We see that the reflective limiting action occurs when the permittivity changes of the lossy defect layer are more than seven orders of magnitude.

In conclusion, we have introduced a photonic layered structure design with hypersensitive transport characteristics. This layered structure consists of an asymmetric dielectric cavity incorporating a metallic nanolayer and sandwiched between two Bragg mirrors. When the metallic nano-layer coincides with the nodal point of the localized mode, the system develops the phenomenon of induced transmission. However, even a small change in $\varepsilon$ and/or $\varepsilon$ in one of the dielectric layers of the asymmetric cavity abruptly suppresses the localized mode and renders the layered structure highly reflective at all frequencies – not just at frequencies of the photonic band gap. Furthermore, we have shown that these metal-dielectric structures can be used as hypersensitive microwave (or...
optical) limiters. Specifically, at low incident wave intensity, these structures support a narrow-band transmission. If the electromagnetic wave intensity and/or fluence exceed certain level, even a small self-induced change (due to non-linearities or heating effects) in the refractive index of the asymmetric layer causes an abrupt transition to a broadband reflectivity. The proposed design can be adjusted to different frequency ranges starting from micro-wave frequencies and up to the mid infrared. The main physical requirement is that the imaginary part, $\hat{\varepsilon}$, of the metallic nanolayer is large. The concept of hypersensitive layered structures can be applied not only to electromagnetic waves but also to acoustic waves, matter-waves etc.

References


Acknowledgements

We acknowledge AFOSR support from the portfolio of Dr. A. Nachman via LRIR09RY04COR Grant (I.V.) and MURI No. FA9550-14-1-0057 (E.M., T.K.). (A.C.) acknowledges AFOSR support from the portfolio of Dr. A. Sayir via FA9550-16-1-0030 Grant.

Author Contributions

E.M. and K.S. carried out most of the calculations. A.C., I.V. and T.K. formulated the problem and wrote the manuscript with input from the rest of the team. All the authors have contributed to discussion and analysis of the results.

Additional Information

Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Makri, E. et al. Hypersensitive Transport in Photonic Crystals with Accidental Spatial Degeneracies. Sci. Rep. 6, 22169, doi: 10.1038/srep22169 (2016).

This work is licensed under a Creative Commons Attribution 4.0 International License. The images or other third party material in this article are included in the article’s Creative Commons license, unless indicated otherwise in the credit line; if the material is not included under the Creative Commons license, users will need to obtain permission from the license holder to reproduce the material. To view a copy of this license, visit http://creativecommons.org/licenses/by/4.0/.
Optical limiters transmit low-intensity light, while blocking laser radiation with excessively high irradiance or fluence. A typical optical limiter involves a nonlinear material which is transparent at low light intensity and becomes opaque when the light intensity exceeds a certain level. Most of the high-level radiation is absorbed by the nonlinear material causing irreversible damage. This fundamental problem could be solved if the state of the nonlinear material changed from transparent to highly reflective (not absorptive) when the intensity becomes too high. None of the known nonlinear optical materials display such a property. A solution can be provided by a nonlinear photonic structure. In this communication, we report the experimental realization of a reflective optical limiter. The design is based on a planar microcavity composed of alternating SiO$_2$ and Si$_3$N$_4$ layers with a single GaAs defect layer in the middle. At low intensity, the planar microcavity displays a strong resonant transmission via a cavity mode. As the intensity increases, two-photon absorption in GaAs kicks in, initially resulting in the microcavity-enhanced light absorption. A further increase in light intensity, though, suppresses the cavity mode along with the resonant transmission; the entire planar microcavity turns highly reflective within a broad frequency range covering the entire photonic band gap. This seemingly counterintuitive behavior is a general feature of resonant transmission via a cavity mode with purely nonlinear absorption.

I. INTRODUCTION

Optical limiters are essential for the protection of the human eye, optical sensors, and other optical and electronic devices from high-intensity laser radiation. The existing passive optical limiters utilize nonlinear optical materials, which turn opaque when the irradiance or fluence exceeds a certain limiting threshold. Such nonlinearity can be caused by two-photon absorption [1], reverse saturable absorption [2,3], photoconductivity, and other physical mechanisms (see, for example, Refs. [4–10] and references therein). A common problem with the existing passive optical limiters is that the nonlinear optical material is directly exposed to the high-level radiation, which can result in overheating, dielectric breakdown, or other irreversible damage. One possible solution is provided by photonic structures, such as a planar microcavity or a Fabry-Perot resonator, incorporating a low-loss nonlinear optical material with the refractive index dependent on the light intensity [11–20].

The limiting action in these cases can be achieved by a nonlinear shift of the transmission window of the photonic structure. Such a shift is inherently small and thus cannot provide broadband protection from high-power laser radiation. More importantly, the nonlinear material in such schemes is directly exposed to the high-level laser radiation, which can cause irreversible damage.

To address the above problems, a new concept of a reflective photonic limiter was introduced recently in Refs. [21,22]. The proposed conceptual design, see Fig. 1, is based on the well-known phenomenon of resonant transmission via a nonlinear localized defect mode (hereinafter, a cavity mode). There are two key physical requirements to the constitutive optical materials of the planar microcavity in Fig. 1: (a) the defect layer must display purely nonlinear absorption—the linear absorption should be negligible; (b) the Bragg reflectors on both sides of the nonlinear defect layer must be linear with negligible losses, and they must have a much higher laser-induced damage threshold compared to the nonlinear material of the defect layer. At low intensity, such a planar microcavity displays a narrow-band resonant transmission in the vicinity of the cavity mode. When the irradiance or fluence exceed a certain level, nonlinear absorption suppresses the cavity mode, along with the resonant transmission; the entire layered structure acts as a Bragg mirror, reflecting most of the incident radiation back to space. Importantly, at the high reflectivity regime, only a tiny portion of the high-level input radiation reaches the defect layer, which protects the nonlinear optical material from laser-induced damage. Herein, an experimental demonstration of such a planar microcavity...
can be very different from that of the incident light. In the vicinity of the defect layer (inside the microcavity) photon absorption. Importantly, the field intensity the gallium arsenide displays a strong nonlinear (two-photon) absorption. If the irradiance is increased, so is the field intensity at the nonlinear layer location increases, as a consequence, both the real and imaginary parts of the permittivity \( \varepsilon = \varepsilon' + i\varepsilon'' \) of the GaAs will change. The change in the real part \( \varepsilon' \) is small (less than 1% in our case). For shorter wavelengths, the nonlinear shift in \( \varepsilon' \) could be much larger (see, for example, Ref. [23] and references therein), but not in our case where the photon energy is smaller than the GaAs band gap. The small shift in \( \varepsilon' \) can only result in a small shift of the transmission window. If the transmission window shift is smaller than the transmission window itself and/or smaller than the laser pulse bandwidth, the shift itself does not provide protection from high-level laser microcavity supports a cavity mode with the frequency lying in the middle of the photonic band gap of the Bragg reflector. In the spectral vicinity of the cavity-mode frequency, the planar microcavity in Fig. 1 displays a strong resonant transmission, shown in Fig. 2. The photonic band gap of the Bragg reflector is clearly visible between 1385 and 1970 nm, while the sharp peak at 1633 nm [full width at half-maximum (FWHM), 13.3 nm, 52% transmission] corresponds to the cavity mode resulting from the incorporation of a single GaAs defect layer in between the two Bragg reflectors. Fringes on either side of the photonic band gap are Fabry-Perot resonances, and for wavelengths shorter than 1385 nm, are attenuated by linear absorption from GaAs. The fact that the experimentally observed resonant transmissivity is below unity (0.52) can be attributed to structural imperfections and, in part, to a small but finite linear absorption of the GaAs layer; the latter is greatly enhanced by the planar microcavity, as depicted next in Fig. 3(a). In our simulations, linear absorption and possible structural imperfections are neglected.

If the irradiance is increased, so is the field intensity at the defect-layer location. As a consequence, both the real and imaginary parts of the permittivity \( \varepsilon = \varepsilon' + i\varepsilon'' \) of the GaAs will change. The change in the real part \( \varepsilon' \) is small (less than 1% in our case). For shorter wavelengths, the nonlinear shift in \( \varepsilon' \) could be much larger (see, for example, Ref. [23] and references therein), but not in our case where the photon energy is smaller than the GaAs band gap. The small shift in \( \varepsilon' \) can only result in a small shift of the transmission window. If the transmission window shift is smaller than the transmission window itself and/or smaller than the laser pulse bandwidth, the shift itself does not provide protection from high-level laser
We assume that, regardless of the specific physical nature of nonlinear absorption in the defect layer, the dynamic range of the planar microcavity in Fig. 1 is much higher than that of a stand-alone GaAs layer due to the simultaneous increase in the damage threshold and reduction in the limiting threshold.

radiation. In contrast, the increase in the imaginary part $\varepsilon'$ of the GaAs permittivity due to nonlinear absorption can be significant enough to completely suppress the cavity mode and render the entire structure in Fig. 1 highly reflective at the entire photonic band gap. To understand such a behavior, consider the simulated field distribution inside the planar microcavity of Fig. 1 at the frequency of transmission resonance. The computations are performed using the same standard transfer matrix formalism as in Refs. [21,22] and presented in Fig. 3.

If the irradiance is low, the GaAs layer displays negligible absorption ($\varepsilon' = 0$), and the steady-state field distribution inside the layered structure will look like that shown in Fig. 3(a). In this case, the field intensity in the vicinity of the defect layer is much higher than that of the incident wave. This will result in the enhancement of nonlinear interactions and, eventually, will lead to a decrease of the limiting threshold, compared to that provided by a stand-alone nonlinear layer (see, for example, Ref. [24] and references therein).

If the irradiance is well above the limiting threshold, the nonlinear GaAs layer becomes quasi lossy, as seen in Fig. 4. We can speculate that the governing physical mechanism responsible for the nonlinear losses in amorphous GaAs is two-photon absorption, the same as in single-crystalline GaAs, but we know too little about amorphous GaAs to try to describe quantitatively the nonlinear absorption in it and to solve numerically the nonlinear scattering problem for the layered structure in Fig. 1. Instead, we invoke the following important qualitative conclusion drawn in Refs. [21,22]. We assume that, regardless of the specific physical nature of nonlinear absorption in the defect layer, the increase in the imaginary part $\varepsilon'$ of the GaAs permittivity due to nonlinear absorption can be significant enough to completely suppress the cavity mode, providing strong resonant transmission shown in Fig. 2.

The field intensity in the vicinity of the defect layer is now much lower than that of the incident wave. The latter amounts to shielding the nonlinear layer from high-level laser radiation. This will result in the enhancement of the effective refractive index of the nonlinear layer is of the order of unity. In the example shown in Fig. 3(b) we used $\varepsilon'' = 5$, but the resonant field distribution inside the layer structure remains qualitatively the same for $\varepsilon''$ values anywhere between, for example, 1 and 10. Upon inspection of Fig. 4, those values are qualitatively consistent with the experimentally observed nonlinear absorption in amorphous GaAs.

The dynamic range of a limiter is usually defined as the ratio of the limiter damage threshold and the limiting threshold. The above-semiqualitative consideration shows that the dynamic range of the planar microcavity in Fig. 1 is much higher than that of a stand-alone GaAs layer due to the simultaneous increase in the damage threshold and reduction in the limiting threshold.
The conditions under which the photonic structure turns reflective depend on (i) the choice of nonlinear material of the defect layer, (ii) the number of bilayers in the Bragg mirrors in Fig. 1, and (iii) the laser pulse shape and duration. For instance, the multilayer can become highly reflective if the peak irradiance exceeds a certain level [13]. Alternatively, the planar microcavity can become reflective when the pulse fluence exceeds a certain level [14]. In the former case, we have a reflective irradiance limiter, while the latter case corresponds to a reflective fluence limiter.

In many cases, though, both the pulse duration and the peak irradiance are equally important. In practical terms, it implies the possibility of simultaneous protection from laser pulses with excessively high peak irradiance and/or excessively high fluence.

Let us reiterate that a planar microcavity similar to that shown in Fig. 1 will act as a reflective optical limiter if the following physical requirements are satisfied:

1. The constitutive materials of the Bragg mirrors (SiO$_2$ and Si$_3$N$_4$ in Fig. 1) have negligible losses at the frequency range of interest, and their laser-induced damage threshold is high.
2. The defect layer (GaAs in Fig. 1) displays a significant nonlinear absorption (such as two-photon absorption), but negligible linear absorption at the wavelength of interest.

If the first of the two conditions is violated, the structure in Fig. 1 would still act as an optical limiter, but it would be an absorptive limiter—not a reflective one. A violation of the second condition would result in suppression of the low-level resonant transmission, making the stack act as a Bragg reflector regardless of the incident light irradiance and fluence. As long as the above two conditions are satisfied, one can always adjust the layer thicknesses so that the low-intensity resonant transmission occurs at a desired frequency in the middle of a photonic band gap. The irradiance or fluence above which the optical limiter becomes reflective can be adjusted within a wide range by changing the number of periods (bilayers) in the Bragg mirror in Fig. 1.

The specific choice of optical materials presented in Fig. 1 satisfies the above conditions. For wavelengths longer than 800 nm, pure, crystalline GaAs has negligible absorption [25]. For the amorphous GaAs used in this study, linear absorption of a stand-alone film is not observed in the vicinity of the cavity mode.

### III. EXPERIMENTAL DETAILS

Plasma-enhanced chemical vapor deposition (PECVD) is used to deposit the Bragg reflector onto amorphous borosilicate glass. The Bragg reflector consisted of a 264.8-nm-thick SiO$_2$ layer, on top of which a 194.8-nm-thick layer of silicon nitride Si$_3$N$_4$ is deposited. This bilayer structure is deposited six successive times in one PECVD run. Following deposition of the GaAs defect layer, the order of the Bragg reflector layers is reversed; silicon nitride is deposited first, followed by silicon dioxide. According to Ref. [26], the refractive indices for silicon dioxide and silicon nitride are $n$(SiO$_2$) = 1.49 and $n$(Si$_3$N$_4$) = 2.16, respectively. These values are consistent with the photonic band-gap location in Fig. 2. The refractive index of the amorphous GaAs can be extracted from the cavity-mode location in Fig. 2. The respective value $n$(GaAs) = 3.42 is consistent with that from [27].

Gallium arsenide is deposited at room temperature by using a GCA rf diode sputtering system. The chamber is evacuated to a base pressure less than $1 \times 10^{-6}$ Torr and then backfilled with 10 mTorr of argon. After cleaning the target for 5 min, sputtering is commenced by applying a bias of 175 W rf to a 5" GaAs target placed 4 cm above the substrate with the Bragg reflector. X-ray 2θ–ω diffraction patterns are obtained for the subsequent film using a PANalytical Empyrean x-ray diffractometer equipped with a PIXcel scanning line detector. The 2θ–ω x-ray diffraction patterns lacked the presence of any peaks indicating that the GaAs film is amorphous.

Next, we proceed with the nonlinear characterization. Steady-state transmission spectra are acquired using a Cary 5000 UV-VIS-NIR absorption spectrophotometer. Reflective optical limiting behavior is characterized with the I-scan method [28]. A Spectra-Physics Solstice Ti:Al$_2$O$_3$ laser with a Gaussian, 150-fs pulse width (measured at the sample) and 1-kHz repetition rate is used to pump a TOPAS optical parametrical amplifier. The laser power is measured using an Ophir 3A-FS thermopile. Attenuation of the incident power is achieved using a pair of crossed, linear polarizers. The laser is focused onto the sample using a 100-mm focal-length lens. Light is incident at an angle of 6° to enable the collection of reflection spectra.

Transmitted or reflected light is focused onto the entrance slit of a Horiba-Jobin Yvon HR320 spectrometer, equipped with a 2500-nm blaze, 120-g/mm diffraction grating, and an extended InGaAs low-noise, single-channel detector connected to an SR830 lock-in amplifier. Four reflective 1.0-absorbance neutral-density filters are positioned in front of the entrance slit to prevent detector saturation. A 200-mm focal length, on-axis spherical mirror is used to collect reflected light and focus it onto the spectrometer entrance slit. Power-dependent reflectivity and transmissivity measurements are obtained from spectral data. The full transmission and reflection spectrum for each incident power level is obtained for both the reflective optical limiter and the free laser pulse. Reflection spectra of the laser pulse are obtained by substituting a gold mirror for the Bragg reflector. In this way, accurate transmissivity and reflectivity values are obtained, regardless of any wavelength shifts associated with nonlinear absorption in the defect layer.
IV. RESULTS AND DISCUSSION

The as-deposited silicon dioxide, silicon nitride, and gallium arsenide films are all amorphous, as determined by x-ray diffraction. Gallium arsenide is a direct band-gap semiconductor, with two-photon absorption in the near-infrared to midinfrared regions [29,30]. However, the GaAs typically studied for nonlinear measurements is crystalline. Because the underlying silicon nitride layer is amorphous, annealing the GaAs into a crystalline material would be difficult, if not impossible. Lederer and co-workers [31] studied the nonlinear absorption of GaAs optical limiters and demonstrated that although the efficacy is reduced, GaAs can still be an effective two-photon absorption material, even when amorphous. To verify that the as-deposited amorphous GaAs is capable of two-photon absorption, irradiance-dependent absorption, transmission, and reflection measurements are carried out with a laser pulse centered at 1600 nm (FWHM, 40.5 nm) for a stand-alone GaAs film of similar thickness, as illustrated in Fig. 4. These measurements correspond to the spectrum of the laser pulse. At low peak irradiance (<1 GW/cm²), the sum of the transmitted and reflected energy is approximately equal to the incident energy, indicating that for this film thickness, the linear absorption is negligible. As the peak irradiance increases to 50 GW/cm², the nonlinear absorption increases, indicating the as-deposited, amorphous GaAs does display two-photon absorption at the wavelengths of interest.

Representative irradiance-dependent, normalized transmissivity spectra for the planar microcavity are presented in Fig. 5. For an average irradiance of 0.82 W/cm² (5.18 GW/cm² peak irradiance, black line) and 7.39 W/cm² (46.3 GW/cm² peak irradiance, red line), the transmissivity decreases from a maximum of 0.42 to 0.024, respectively. The FWHM spectral widths are 12.43 and 15.71 nm, respectively. Detector noise prevents an accurate determination of the transmissivity maximum wavelength, so a wavelength range of 1630–1636 nm is reported. The accuracy of the spectrometer used to collect the data in Fig. 5 is ±1 nm; the 0.82 W/cm² transmissivity spectral width is statistically equal to the spectral width of the steady-state transmission spectrum in Fig. 2. As the average irradiance increases to 7.39 W/cm², the spectral width increases to 15.7 nm. This slight broadening of the cavity-mode transmission band is a result of modulations in the refractive index caused by two-photon absorption as described in Sec. II.

Figure 6 depicts representative nonlinear reflectance spectra for an average irradiance of 1.08 W/cm² (6.77 GW/cm² peak irradiance) and 6.02 W/cm² (37.7 GW/cm² peak irradiance). Under low irradiance, the reflectivity takes the minimal value of 0.31. The exact reflectivity maximum wavelength cannot be elucidated from the spectrum, but it is between 1630–1639 nm. At higher average irradiance, 6.02 W/cm², the reflectivity spectrum is constant, between 0.9–1.0 for all frequencies inside the band gap. It is clear from Fig. 6 that some transmission is present, but detector noise prevents the expected reflection decrease from being observed. The transition from low to almost total reflectivity indicates that the layered structure in Fig. 1 does act as a reflective optical limiter.

The full set of irradiance-dependent reflectivity and transmissivity can be found in Fig. 7. Peak irradiance values ranging from 4.24 W/cm² (0.827 W/cm² average irradiance) to 46.3 GW/cm² (7.39 W/cm² average irradiance) are studied. Near the resonant transmission maxima, the transmissivity decreases from 0.41 at 4.24 GW/cm² to 0.026 at 46.3 GW/cm². The reflectivity shows a
ment with the predictions of Refs. [21,22]. Upon visual inspection, no optical damage could be observed for the stand-alone GaAs film. The role of the layered structure is threefold. First, it makes the optical limiter reflective, rather than absorptive. Second, it shields the vulnerable nonlinear layer (GaAs) from laser-induced damage, thereby greatly increasing the dynamic range of the limiter. Finally, our planar microcavity design provides much stronger transmitted light attenuation above the limiting threshold. The above features can be attractive for various applications, including sensor protection and mode locking for the generation of ultrashort laser pulses. The current design can only perform at short-wave IR, where GaAs displays negligible linear absorption and very strong nonlinear two-photon absorption. With a judicious choice of optical materials, the same principle can certainly be replicated for other wavelength ranges.

ACKNOWLEDGMENT

The authors acknowledge support by the Air Force Office of Scientific Research (AFOSR) through Program Officer Dr. Arje Nachman, award numbers LRIR12RY11COR and LRIR14RY14COR and MURI Grant No. FA9550-14-1-0037.

V. CONCLUSIONS

An experimental demonstration of a reflective optical limiter is presented. The nonlinear planar microcavity consisting of a nonlinear GaAs layer placed between two Bragg mirrors, displays a narrow-band low-intensity resonant transmission. In the case of femtosecond pulses with high peak irradiance, the resonant transmission disappears, and the planar microcavity turns highly reflective within a broad frequency range, covering the entire photonic band gap of the Bragg mirrors. By comparison, a stand-alone GaAs layer would also act as a nonlinear optical limiter at the same frequency range, but it would be an absorptive limiter—not a reflective one. The limiting threshold of the nonlinear microcavity in Fig. 1 is expected to be much lower, while its damage threshold is expected to be much higher than that of the stand-alone GaAs layer. The role of the layered structure is threefold. First, it makes the optical limiter reflective, rather than absorptive. Second, it shields the vulnerable nonlinear layer (GaAs) from laser-induced damage, thereby greatly increasing the dynamic range of the limiter. Finally, our planar microcavity design provides much stronger transmitted light attenuation above the limiting threshold. The above features can be attractive for various applications, including sensor protection and mode locking for the generation of ultrashort laser pulses. The current design can only perform at short-wave IR, where GaAs displays negligible linear absorption and very strong nonlinear two-photon absorption. With a judicious choice of optical materials, the same principle can certainly be replicated for other wavelength ranges.

ACKNOWLEDGMENT

The authors acknowledge support by the Air Force Office of Scientific Research (AFOSR) through Program Officer Dr. Arje Nachman, award numbers LRIR12RY11COR and LRIR14RY14COR and MURI Grant No. FA9550-14-1-0037.

V. CONCLUSIONS

An experimental demonstration of a reflective optical limiter is presented. The nonlinear planar microcavity consisting of a nonlinear GaAs layer placed between two Bragg mirrors, displays a narrow-band low-intensity resonant transmission. In the case of femtosecond pulses with high peak irradiance, the resonant transmission disappears, and the planar microcavity turns highly reflective within a broad frequency range, covering the entire photonic band gap of the Bragg mirrors. By comparison, a stand-alone GaAs layer would also act as a nonlinear optical limiter at the same frequency range, but it would be an absorptive limiter—not a reflective one. The limiting threshold of the nonlinear microcavity in Fig. 1 is expected to be much lower, while its damage threshold is expected to be much higher than that of the stand-alone GaAs layer. The role of the layered structure is threefold. First, it makes the optical limiter reflective, rather than absorptive. Second, it shields the vulnerable nonlinear layer (GaAs) from laser-induced damage, thereby greatly increasing the dynamic range of the limiter. Finally, our planar microcavity design provides much stronger transmitted light attenuation above the limiting threshold. The above features can be attractive for various applications, including sensor protection and mode locking for the generation of ultrashort laser pulses. The current design can only perform at short-wave IR, where GaAs displays negligible linear absorption and very strong nonlinear two-photon absorption. With a judicious choice of optical materials, the same principle can certainly be replicated for other wavelength ranges.

ACKNOWLEDGMENT

The authors acknowledge support by the Air Force Office of Scientific Research (AFOSR) through Program Officer Dr. Arje Nachman, award numbers LRIR12RY11COR and LRIR14RY14COR and MURI Grant No. FA9550-14-1-0037.


Waveguide photonic limiters based on topologically protected resonant modes

U. Kuhl,1 F. Mortessagne,1 E. Makri,1 I. Vitebskiy,1 and T. Kottos2

1Institut de Physique de Nice, Université Côte d’Azur, CNRS, F-06100 Nice, France
2Department of Physics, Wesleyan University, Middletown, Connecticut 06459, USA

We propose a concept of chiral photonic limiters utilizing topologically protected midgap defect states in a photonic waveguide. The chiral symmetry alleviates the effects of structural imperfections and guarantees a high level of resonant transmission for low intensity radiation. At high intensity, the light-induced absorption can suppress the localized modes, along with the resonant transmission. In this case the entire photonic structure becomes highly reflective within a broad frequency range, thus increasing dramatically the damage threshold of the limiter. Here, we demonstrate experimentally the loss-induced reflection principle of operation which is at the heart of reflective photonic limiters using a waveguide consisting of coupled dielectric microwave resonators.

The emerging field of topological photonics aims to realize photonic structures which are resilient to fabrication imperfections [1–10]. Usually, these structures, support topologically protected (TP) defect states within photonic band gaps. In this endeavor the manipulation of various symmetries has been proven extremely useful. An example case are resonator arrays with chiral symmetry [11] where a topological defect state appears to be insensitive to positional imperfections of the resonators [11,12]. In this Rapid Communication we connected the chiral symmetric array to leads, thus turning the TP defect mode to a quasilocalized resonant mode which was utilized for the realization of a topologically protected class of waveguide photonic limiters.

Limiters are protecting filters transmitting low-power (or energy) input signals while blocking the signals of excessively high power (or energy) [13–18]. Usually, a passive limiter absorbs the high-level radiation, which can cause its overheating. The input level above which the transmitted signal intensity does not grow with the input is the limiting threshold (LT). Another important characteristic is the limiter damage threshold (LDT), above which the limiter sustains irreversible damage. The domain between LT and LDT is the dynamic range (DR) of the limiter—the larger it is, the better. Unfortunately, material limitations impose severe restrictions on both thresholds. Importantly, these structures should be tolerant to deviation of the material and geometrical parameters from their ideal values.

Along these lines, the defect modes hosted by photonic band-gap [16,19–21] (or other resonant [22]) structures have been exploited as an alternative to achieve flexible, high efficiency photonic limiters. In most occasions, however, limiting action is achieved by a nonlinear frequency shift of the transparency window of the photonic structure. Such a shift is inherently small and, therefore, cannot provide broadband protection from high-power input. Other schemes, specifically in the microwave domain, exploit PIN diode (having spike leakage problems) [23], transistor-receiver (TR) tubes, or self-attenuating superconducting transmission lines that require high-power consumption [24]. To address these issues we have recently proposed the concept of reflective photonic limiters [25,26]. Such limiters reflect the high radiation, thereby protecting themselves—not just the receiving device—while they provide a strong resonant transmission for low incident radiation.

Here, we propose the use of chiral coupled resonator waveguides (C-CROWs) with alternating short and long distances from one another (see Fig. 1), as a fertile platform to implement structurally robust reflective waveguide limiters with a wide DR. In the presence of a phase slip defect [27,28], chiral symmetry provides topological protection to a midgap defect localized mode [11,12]. For low incident power (or energy) it can provide high transmittance shielded from (positional) fabrication imperfections. When (nonlinear) losses at the defect resonator (triggered from high-power, or energy, incident radiation) exceed a critical value, the resonant defect mode and the associated resonant transmission are dramatically suppressed, turning the C-CROW highly reflective (not absorptive) for a broad frequency range. As a result, the LDT increases with a consequent increase of the DR of the limiter. Using a microwave C-CROW arrangement we have tested experimentally the operational principle of this class of TP reflective photonic limiter by investigating the sensitivity and transport characteristics of the TP resonant defect mode in the presence of losses and imperfections.

The setup [see Fig. 1(a)] consists of $N = 21$ high index cylindrical resonators (radius $r = 4$ mm, height $h = 5$ mm, made of ceramics with refraction index $n \approx 6$) with an eigenfrequency around $\nu_0 = 6.655$ GHz and linewidth $\gamma = 1.4$ MHz [20]. The resonators are placed at alternating distances $d_1 = 12$ mm and $d_2 = 14$ mm corresponding to strong ($\tau_1 = 38$ MHz) and weak ($\tau_2 = 21$ MHz) evanescent couplings, respectively. A topological defect at the 11th resonator is introduced by repeating the spacing $d_2$ [11,12]. Close to the first resonator, we have placed a kink antenna that emits a signal exciting the first transverse electric (TE$_1$) resonant mode of the resonator. The structure is shielded from above with a metallic plate where a movable loop antenna (receiving antenna) is mounted and is coupled to the 13th resonator [29].

We assume that the defect resonator incorporates a nonlinear absorption mechanism, i.e., we assume that its losses are self-regulated depending on the strength of the incident radiation. One option to incorporate nonlinear losses is via an external element (fast diodes) [see Fig. 1(b)]. This option

DOR: 10.1103/PhysRevB.95.121409
FIG. 1. (a) The experimental setup: The resonators are separated by distances \( d_0 \) or \( d_0 + d_z \). A central defect is introduced by repeating the spacing \( d_z \). Various proposals for the implementation of nonlinear losses in the defect resonator. (b) A circuit with various modules (sensing antenna, diode, threshold DC voltage). (c) An epitaxial growth of a material that experiences a thermally induced insulator-to-metal phase transition. (d) Our measurements involve a defect resonator, which includes a manually modulated absorbing patch. (e) Measured transmittance \( T \), reflectance \( R \), and absorption \( A \) for two different patches. The linewidth \( \gamma \) (1.4 and 7.8 MHz) of the reflected signal mainly characterizes the losses due to the absorbing patches.

provides on-the-fly reconfigurability of the LT via an externally tuned DC voltage \( U_{DC} \). An alternative mechanism is associated with temperature driven insulator-to-metal phase transition materials, such as VO\(_2\) [30–33], which can be deposited on top of the defect resonator [see Fig. 1(c)].

In our experiment we are not concerned with the physical origin of the nonlinear losses at the defect resonator. Rather, we focus on demonstrating their effects on the transport properties of the photonic structure and how can be utilized for microwave limiters. Therefore, we have included losses \( \gamma \) by placing an absorbing patch on top of the resonator [see Fig. 1(d)]. This process results in a slight shift of the real part of the permittivity of the defect resonator, which we corrected by using resonators with a slightly higher eigenfrequency. The linewidth \( \gamma \) has been used in order to quantify the losses of the resonators.

In Fig. 1(e) we show the transmittance \( T \), reflectance \( R \), and absorption \( A \) as a function of the frequency \( \nu \) for two resonators with different losses. We observe that the transmittance of the standalone lossy resonator reduces as the losses increase, thus acting as a limiter. However, this reduction comes to the expense of increasing absorption, i.e., the standalone lossy resonator acts as a sacrificial limiter.

The photonic structure is described by a one-dimensional (1D) tight-binding Hamiltonian [34]

\[
H = \sum_n \omega_n |n| |n\rangle + \sum_n \varepsilon_n |n\rangle |n+1\rangle + |n+1\rangle |n\rangle ,
\]

where \( n = 1, 2, \ldots, 21 \) enumerates the resonators, \( \varepsilon_n = \nu = \nu_0 + i\gamma \) is the resonance frequency of the \( n \)th resonator, and \( \varepsilon_n = \varepsilon_{n+1} = \varepsilon_{n+2} = \varepsilon_n + t \) is the coupling between nearest resonators. The band structure consists of two minibands \( \nu_{01} - t_1 < \nu < \nu_{01} - t_2 < \nu < \nu_{01} - t_1 \) and \( \nu_{01} + t_1 - t_2 < \nu < \nu_{01} + t_2 \) separated by a finite gap of width \( 2t_1 - t_2 \). In the presence of the defect resonator at \( n_0 = 11 \), a TP defect mode at \( \nu_{D} = \nu_0 [11,12] \) is created. Its shape, in the limit of infinite many resonators, is [11]

\[
\psi_D^n \sim \frac{1}{\sqrt{2}} e^{\gamma n}, \quad n \text{ odd},
\]

\[
\psi_D^n \sim 0, \quad n \text{ even},
\]

where \( \psi_D^n \) is the amplitude of the defect mode at the \( n \)th resonator and \( \xi = 1/\ln(\nu_D/\nu_0) \) is the so-called localization length of the mode [11]. Hamiltonian Eq. (1) is invariant under a chiral symmetry, i.e., \( [H, C] = 0 \) where \( (\cdots) \) indicates an anticommutation and \( C = P_{\text{loss}} = P_{\text{abs}} C = 1 \) (\( P_{\text{loss}} \) is the projection operator in the even/odd sites). The staggering form of \( \psi_D^\nu \) is a consequence of the chiral symmetry [11,12].

We are modeling the transmitted (reflected) antenna, coupled to the \( n = 1 \) (\( n = 13 \)) resonator, by a 1D semi-infinite tight-binding lattice with coupling constant \( t_L = (t_1 + t_2)/2 \) and on-site energies \( \nu_L = \nu_0 \). The associated scattering matrix takes the form [35]

\[
S = -1 + \frac{2i \sin \kappa}{\nu - \nu_{\text{eff}}} W^T \frac{1}{\nu_D - \nu_0} W ,
\]

\[
H_{\text{eff}} = H_0 + \frac{i\nu}{\nu_D - \nu_0} W W^T ,
\]

where \( \hat{1} \) is the \( 2 \times 2 \) identity matrix, \( W_{\nu_0} = w_1 \delta_{\nu_0, \nu_{W1}} + w_2 \delta_{\nu_0, \nu_{W2}} \) is a \( \nu \times 2 \) matrix that describes the coupling between the array and the antennas, \( \nu = \nu_L + 2\nu_0 \cos k \) is the frequency of propagating waves at the antennas, and \( k \) is their associated wave vector.

When the system is coupled to the antennas, \( \psi_D^\nu \) becomes a quasilocalized resonant mode at frequency \( \nu_D = \nu_0 \), with a large but finite lifetime \( \tau \),

\[
\tau^{-1} \sim \left( \frac{\nu_0}{\nu_D} \right)^2 W W^T |\psi_D^\nu|^2 = |\psi_D^\nu|^2 + |\psi_D^\nu|^2
\]

where \( |\psi_D^\nu|^2 \) and \( |\psi_D^\nu|^2 \) are given by Eq. (2).

The measured transmittance \( T = \sum_{\nu} |w_{\nu}|^2 \), reflectance \( R = |S_{\nu,0}|^2 \), and absorption \( A = 1 - T - R \) versus frequencies \( \nu \) of the C-CROW (with global \( \gamma = 1.4 \) MHz) are shown in Figs. 2(a)–2(c) (solid lines). Measurements of the widths of the minibands and of the gap allow us to extract the couplings \( t_1 = 38 \) MHz, \( t_2 = 21 \) MHz. The presence of the defect resonator results in a transmission peak at \( \nu = \nu_D \) inside the band gap. A fitting of the height of this peak, for various \( \gamma_0 \) values, gives \( \nu_D = 10.915 \) MHz, \( w_T = 3.6875 \) MHz (see Fig. 3). The small peak in the absorption [solid line in Fig. 2(c)] is associated with the small ohmic component at all resonators. In Fig. 2(a) we also report (dashed lines) the measured transmittance for a defect with additional losses, i.e., \( \gamma_D = 7.8 \) MHz. We find that even a small increase in \( \gamma_D \) strongly suppresses the resonant transmission [see Fig. 2(a)].

In Fig. 2(b) we show \( R(\nu) \) of the C-CROW for \( \gamma_D = 1.4 \) MHz (solid line) and \( \gamma_D = 7.8 \) MHz (dashed line). We find that the suppression in \( T(\nu_D) \) is accompanied by an increase in \( R(\nu_D) \). Moreover, \( A(\nu_D) \) is decreasing as \( \gamma_D \) increases [see Fig. 2(c)]. In other words, our photonic structure becomes \textit{reflective} (not absorptive) as the losses of the defect resonator increase. This behavior is in distinct contrast to the case of a single (sacrificial) lossy resonator [see Fig. 1(e)] where the drop in transmittance is associated with an increase of
Shadowed areas indicate deviations in other resonators. Symbols (blue dashed-dotted lines) correspond to Numerics for the ideal C-CROW (red dashed lines) with Left: For the C-CROW with resonator losses $\gamma = 1.4$ MHz different values of $R$ values the absorption of the standalone resonator reaches large losses of an ideal C-CROW where all resonators have zero intrinsic absorption. These features are also observed in the simulations we observe relatively large (dashed-dotted lines) of a standalone lossy resonator where is contrasted with the measurements (diamonds) and numerics $\gamma_D$ has with lossless resonators ($\gamma = 0$) apart from the defect resonator which has $\gamma_D = 1.4$ MHz (solid lines) and $\gamma_D = 7.8$ MHz (dashed lines).

An overview of the measured (black circles) $T(\nu_D), A(\nu_D)$ and the corresponding numerical results (black solid lines) for the C-CROW of Fig. 1 versus $\gamma_D$ are reported at the left column of Fig. 3. We find that an increase of $\gamma_D$ leads to a decrease of $T(\nu_D)$ and $A(\nu_D)$ of the photonic structure. This behavior is contrasted with the measurements (diamonds) and numerics (dashed-dotted lines) of a standalone lossy resonator where we observe relatively large $T$ values $T \sim 10^{-3}$ as opposed to $T \sim 10^{-4}$ for the C-CROW, i.e., ultralow LT. For moderate $\gamma_D$ values the absorption of the standalone resonator reaches large values $A(\nu_D = 0.004$ GHz) $\approx 0.8$ corresponding to low LDT. In contrast, the C-CROW takes absorption values, which are at least one order of magnitude smaller (high LDT). On the right column of Fig. 3, we report the simulations for $T(\nu_D), A(\nu_D)$ for an ideal ($\gamma = 0$) C-CROW (dashed lines) versus the losses $\gamma_D$ of the defect resonator. These results are compared to the theoretical/experimental (dashed-dotted lines/diamonds) results for the standalone lossy resonator. Both cases show the same qualitative behavior. However, the C-CROW shows a two-order lower LT (i.e., a smaller $\gamma_D$ value for which the decay of transmittance occurs) as compared to a standalone resonator. At the same time the LDT of the C-CROW is at least two orders of magnitude higher than the one associated with the standalone resonator. The latter acquires a maximum value of absorption $A \approx 0.8$ at $\gamma_D = 0.01$ as opposed to $A \approx 0.01$ acquired by the C-CROW. The maximum absorption for the photonic structure occurs at much lower values of $\gamma_D \sim 10^{-4}$, which in the case of a nonlinear lossy mechanism corresponds to rather small, and therefore harmless, incident radiation.

The transport features of the TP resonant mode have been further investigated in the case of positional randomness corresponding to a box distribution for the coupling constants $\tilde{\nu}_{1,2} \in [\nu_{1,2} - 2$ MHz,$\nu_{1,2} + 2$ MHz]. The shadowed area in Fig. 3 indicates the variations in $T,A$. For $\gamma_D = 0$ (not shown) the resonant frequency $\nu_D \approx 6.655$ GHz remains protected and the resonant transmission is unaffected for both an ideal C-CROW $\gamma = 0$ and for resonators with losses $\gamma = 1.4$ MHz. Moreover, the experimental data in Fig. 3 incorporate an intrinsic disorder associated with the variation of the bare resonance frequencies, within a range of 1 MHz, and the precision of the resonator positioning, of the order of 0.2 mm (coupling uncertainty $\approx 500$ kHz). Nevertheless, the transport features remain largely unaffected (see Fig. 3).

The fragility of the resonant defect mode is further analyzed in Fig. 4. In Fig. 4(a) we report the simulated resonant defect fields for an ideal C-CROW (i.e., $\gamma = 0$) and for various $\gamma_D$ values. For $\gamma_D = 0$, a nice agreement between the numerics and Eq. (2) is observed, indicating that the coupling to the antennas does not affect the resonant mode profile. As $\gamma_D$ increases, a gradual deviation from the profile of Eq. (2) occurs and eventually a suppression of the defect mode is observed.
At $\gamma_0 = 20$ MHz the resonant localized mode is suppressed enough so that the field intensity in the vicinity of the defect lossy resonator is two orders smaller than the corresponding one for $\gamma_0 = 0$. Thus the lossy defect resonator is protected from damages induced by heat or electrical breakdown. This implies a huge increase in the DR of C-CROW. The comparison with the experimental data [see Fig. 4(b)], where $\gamma \approx 1.4$ MHz, indicates that the underlying mechanism which is responsible for the destruction of the resonant defect mode remains unaffected.

The destruction of the resonant defect mode can be understood intuitively as a result of a competition between two mechanisms that control the dwell time of photons in the resonant state. The first state is associated with the boundary losses due to the coupling of the photonic structure to the antennas. It results in a resonant linewidth $\Gamma_{\text{res}} = \gamma^{-1}$ [see Eq. (4)]. The other mechanism is associated with bulk losses and it leads to an additional broadening of the resonance linewidth. From first-order perturbation theory, $\Gamma_{\text{res}} \approx \gamma_{\text{D}} |\psi_n\rangle^2 + \gamma \sum_{n' \neq n} |\psi_{n'}\rangle^2 = (\gamma_0 - \gamma)/\xi + \gamma$. For small values of $\gamma_0$ such that $\Gamma_{\text{res}} < \Gamma_{\text{edge}}$, the dwell time is determined by $\Gamma_{\text{edge}}$ and it is essentially constant. Thus the absorption of the photons that populate the resonant state increases, as they are trapped for a relatively long time in the lossy C-CROW [see the peak of the black line in Fig. 2(a)]. When $\Gamma_{\text{res}} \approx \Gamma_{\text{edge}}$, the dwell time itself begins to diminish, and the resonant mode is spoiled. For even larger values of $\gamma_0$, the photons do not dwell at all in the resonant state and reflection from the whole structure becomes the dominant mechanism. As a result, the absorption decreases to zero. The above argumentation applies equally well for the standalone defect and for the photonic structure. However, in the latter case the condition for the destruction of the resonant mode $\Gamma_{\text{res}} = \Gamma_{\text{edge}}$ is achieved for exponentially smaller values of $\gamma_0$. It is exactly this effect that our proposal is harvesting in order to increase the damaging threshold (and the DR) of the photonic waveguide limiter.

We acknowledge partial support from AFOSR via MURI Grant No. FA9550-14-1-0037 (T.K.) and LRIR14RY14COR (E.V.). E.M. acknowledges partial support from Wesleyan University and from NSF EFMA-1641109. The stay of (T.K.) at LPMC-CNRS was supported by CNRS.


Reflective limiters based on self-induced violation of \(CT\) symmetry

Eleanna Makri, Roney Thomas, and Tsampikos Kottos

Department of Physics, Wesleyan University, Middletown, Connecticut 06459, USA

(Received 6 October 2017; published 30 April 2018)

Non-Hermitian bipartite photonic lattices with charge-conjugation \(CT\) symmetry can support resonant defect modes which are resilient to bipartite losses and structural imperfections. When, however, a (self-)induced violation of the \(CT\) symmetry occurs via tiny permittivity variations, the resonant mode is exposed to the bipartite losses and is destroyed. Consequently, the transmission peak is suppressed while the reflectance becomes (almost) unity. We propose the use of such photonic systems as power switches, limiters, and sensors.

DOI: 10.1103/PhysRevA.97.043864

I. INTRODUCTION

Symmetries and their violations constitute an important theme of investigation, both for their own fundamental interest [1] and for their potential technological use in managing wave transport [2,3]. For example, the violation of time-reversal \((T)\) symmetry is a necessary condition for the realization of isolators and circulators [4–6]. Similarly, chiral \((C)\) [7–9] and charge-conjugation \((CT)\) symmetries [7] have been proven important for the realization of defect modes which are topologically protected against disorder and which potentially enable robust unidirectional transport, mode selectivity, etc. [10–22]. Originally these topologically protected defect states attracted attention due to their possible realizations in condensed matter systems [23–25]. Recently, classical wave physics setups—like photonics, acoustics, and microwaves—have been proven fertile platforms for the implementation of topological defect modes [18,26–29].

In all of these cases the topological protection invokes a combination of judicious band-structure designs and symmetry implementations [10–21]. Among the well-studied setups are coupled resonator optical (or microwave or acoustic) waveguide (CROW) arrays [18,20–22,26,29]. Extensions to non-Hermitian CROWs have also been considered and were shown to support nontrivial topologically protected defect modes [30–32]. Nevertheless, very few studies address the transport properties of these defect states once the system is coupled to leads [20,21,33,34].

Here, we design a family of \(CT\)-symmetric non-Hermitian bipartite photonic CROW arrays with (self-)regulated transport characteristics. These arrays consist of resonators with the same resonant frequencies but different linewidths. In the presence of a topological defect [18], the associated \(CT\)-symmetric defect mode is strongly localized around the defect resonator and has nodal points at alternating resonators. This symmetry-induced staggered profile shields the defect mode from structural imperfections and from losses associated with the “nodal-point” resonators. We show that the symmetry protection persists also to the case of scattering setups where the associated resonant defect mode has a similar staggered form—thus minimizing the interaction with the lossy “nodal-point” resonators and enforcing a high resonant transmission peak. We refer to this phenomenon as symmetry-enforced transmittivity. When, however, the defect resonator is made of a material with a permittivity that is sensitive to either self-induced heating due to high fluorescence of the incoming electromagnetic radiation, or to high intensity field values, the resonant defect mode experiences a \(CT\)-symmetry violation. This self-induced explicit symmetry violation exposes the defect mode to the lossy “nodal-point” resonators, leading to its destruction together with the dramatic suppression of the associated resonant transmission. As a result, the entire structure becomes highly reflective at the resonant frequency. We propose to utilize the fragile nature of the resonant transport to \(CT\)-symmetry violations in order to realize a different family of photonic limiters and switches [35,36].

The structure of the paper is as follows. In Sec. II, we present the proposed CROW microwave photonic limiter and its symmetries. We also discuss the consequences of these symmetries in the structure of the defect mode and its resulting robustness against structural imperfections. In Sec. III, we analyze the scattering setup and demonstrate the hypersensitive nature of the defect resonant transmission against self-induced (explicit) symmetry violations. In Sec. IV we analyze an on-chip version of the photonic limiter and demonstrate its efficiency against previous proposals. Finally, in Sec. V we present our conclusions.

II. DESIGN AND MODELING OF \(CT\)-SYMMETRIC MICROWAVE CROWS

A design of our setup is shown in Fig. 1. It consists of a one-dimensional array of \(N\) resonators, which are arranged with alternating short \((d_1)\) and long \((d_2)\) distances from one another. We assume, without any loss of generality, that \(N = 21\). A central dimerization defect at \(n_0 = 11\), assumed to consist of a thermally (or intensity) modulated material, is introduced by repeating the spacing between the adjacent resonators on the left and right, respectively. The permittivity variation in the material making up the defect resonator \((\varepsilon_n = 11)\) is assumed to be self-induced (e.g., via heating by the incident radiation or via the local field intensity). Representatives of such materials include germanium-antimony-tellurium alloys [37], oxides of vanadium, etc. [38,39]. The resonant frequency \(\beta_{0_n}\) of the defect resonator matches the frequencies of the other resonators, \(\beta_n = \beta_0\). The two resonators on the left \((n = 10)\)
and right \((n = 12)\) of the central defect (see Fig. 1), involve large Ohmic losses. The losses are optimally minimized in a way that these resonators maintain the same resonant mode as the other cavities—condition that is necessary for CT symmetry—and at the same time overcome restrictions from the Kramers-Kronig relations. This can be achieved through deposition of thin layers of metals on top of the resonators or by using an absorbing paint like graphite powder. The losses due to the coating will be reflected in the resonant frequencies of these resonators which acquire an imaginary part, i.e., \(\delta \beta_n = \beta_n + i\gamma\). In our numerics, we have assumed that \(\beta_0 = 6.55\) GHz, \(\gamma = 50\) MHz, \(t_1 = 50\) MHz, and \(t_2 = 10\) MHz.

The array of Fig. 1 is described, in the resonant mode representation of the isolated resonators, by the following tight-binding Hamiltonian:

\[
H = \sum_{n} \beta_n |n\rangle \langle n| + \sum_{n} \iota_n |n\rangle \langle n+1| + |n+1\rangle \langle n|,
\]

where \(\iota_n\) is an imaginary number, \(\beta_n\) and \(\iota_n\) are the evanescent coupling strengths between the two nearby resonators \((d_1)\) and \(d_2\) distances, respectively. When \(\gamma = 0\), the Hamiltonian (1) is chiral symmetric, i.e., \([\mathcal{C}, H] = 0\) [40], where \([\cdots]\) indicates an anticommutation, and \(\mathcal{C} = P_{\text{even}} - P_{\text{odd}}\) is the chiral operator with \(P_{\text{even/odd}} = \sum_{n \in \text{even/odd}} P_n\) and \(P_n = |n\rangle \langle n|\) is the projection to a specific site \(n\). The eigencfrequencies \(\nu_n\) of Hamiltonian (1) are real and occupy two bands, \(\nu_0 - \nu_\iota = t_2 < \nu < \nu_0 - |t_1 - t_2|\) and \(\nu_0 + |t_1 - t_2| < \nu < \nu_0 + t_1 + t_2\), separated by a gap of width \(\Delta = 2(t_1 - t_2)\).

The central unpaired eigencfrequency \(\nu_0\) corresponds to a \(C\)-symmetric defect eigenmode \(\psi_0\) which is localized at the central resonator \(n_0 = 11\). At the infinite-link limit, the field amplitude \(\psi_0\) at the nth resonator takes the form

\[
\psi_0 \sim \begin{cases} 
1/\sqrt{2}, & n \text{ odd} \\
0, & n \text{ even}
\end{cases}
\]

where \(\xi = 1/\ln(t_1/t_2)\). Importantly, Eq. (2) indicates that this state is supported only by the odd \(n\) sublattice. Therefore, it is also an eigenstate of any diagonal operator \(D_{\text{local}}\), and the system is no longer chiral symmetric. We find that the Hamiltonian (1) becomes non-Hermitian and \([\mathcal{C}, H] \neq 0\); thus the system is no longer chiral symmetric. In order to verify the robustness of the defect state \([\i.e., \text{the position of the eigencfrequency } \nu_0 \text{ and the shape of the mode; see Eq. (2)}]\) against structural disorder, we introduced random variations of the coupling strengths \(t_1\) and \(t_2\), while preserving the dimer structure of the lattice; see Fig. 1(c). To this end, we have replaced each of the values of \(t_1\) in Eq. (1) with a random statistically independent coupling given by \(t_{1,n} \sim t_{1} + \xi W_n\), where \(W_n\) is a random number drawn from a uniform distribution in the interval \([-1,1]\). Finally, \(t = \xi t_1\). We note that a detailed experimental study of the robustness of the topologically protected defect mode has been performed in [21]. It turns out (see next section) that the resilience of the topologically protected mode carries over also in the case when the system is coupled to two leads. In this case, the defect mode becomes a topologically protected resonant mode, giving rise to a robust (against structural disorder) resonant transmission; see Fig. 1(d).

Let us now assume that the central resonator is made by a nonlinear material \((\text{say with a Kerr-like or thermal nonlinearity})\), thus making it more susceptible \((\text{with respect to the other resonators})\) to incident light radiation. In this case its permittivity, and consequently its resonant frequency \(\nu_0\), will be modified as \(\nu_{12} = \nu_0 \rightarrow \nu_0 + \delta\) whenever the power or fluence of the incident radiation is above some critical value. We find that the small detuning \(\delta\) will formally induce a
violation of C symmetry for $H(0)$ as well as a violation of CT symmetry for $H(\gamma)$. Furthermore, at some critical value of $\delta$ the staggered form Eq. (2) of the defect mode $\psi^d$ is destroyed, acquiring a nonzero field amplitude at the lossy resonators at $n = 10$ and $n = 12$; see Fig. 2(a). At the same time the associated eigenfrequency $\nu^d$ acquires an imaginary part—a signature of a low $Q$ factor due to the local losses at resonators $n = 10$ and $n = 12$. Using second-order perturbation theory with respect to $P_{\text{losses}}$ we estimate that for $\delta < 4\nu L/N$ ($4\nu L/N$ is the spacing between nearby levels for $\delta = 0$) the imaginary part of $\nu^d$ of the perturbed system $H(\gamma) + \delta P_{\text{losses}}$ scales as $\text{Im}[\nu^d] \propto \delta^2$.

### III. HYPERSENSITIVE TRANSPORT

Next, we couple the system of Eq. (1) with two antennas, at the first and last resonators. The antennas are modeled as one-dimensional semi-infinite periodic tight-binding lattices with coupling constants $t_2 = (t_1 + t_2)/2$ and on-site eigenfrequencies $\beta_1 = \beta_0$. These antennas support propagating waves with an eigenfrequency $\nu_2 = \nu_0 - 2\nu_0 \cos k$ where $k$ is the associated wave vector. The coupling between the antennas and the first and last resonator is assumed to be $t_2$.

Within the scattering framework, the defect mode becomes a resonant localized mode with small but finite linewidth. Its shape and transport properties are studied using the transfer matrix $M_n$:

$$
\begin{pmatrix}
\psi_{n+1} \\
\psi_n
\end{pmatrix}
= M_n
\begin{pmatrix}
\psi_{n+1} \\
\psi_n
\end{pmatrix},
M_n =
\begin{pmatrix}
\frac{\sin \beta_n}{\sin \beta_0} & \frac{-t_2}{\sin \beta_0} \\
\frac{\sin \beta_0}{\sin \beta_n} & 0
\end{pmatrix}
(3)
$$

Equation (3), together with appropriate boundary conditions, allows us to obtain the resonant mode profile at any resonator within the CROW. Without loss of generality we shall use the scattering boundary conditions $\psi_n = e^{i\theta_n}$ for $n \geq N$ and $\psi_n = e^{i\theta_n} + r e^{-i\theta_n}$ for $n \leq 1$ describing a left incident propagating wave with unit amplitude and reflection coefficient $r$.

The associated transmittance $T = |r|^2$ and reflectance $R = |r|^2$ are evaluated via iteration of Eq. (3). The scattering field intensities of the resonant defect mode for different values of the detuning $\delta$ are shown in Fig. 2(b). When $\delta = 0$, the scattering field profile resembles the staggered form Eq. (2) of the associated localized defect mode. Importantly, the position of the lossy resonators at $n = 10, 12$ coincides with the position of the (quasi-)nodal points of the resonant defect mode. Thus, the interaction of the field with these cavities is negligible and the structure demonstrates the phenomenon of “symmetry-enforced transmissivity”; i.e., we have a high resonant transmission peak at $\nu = \nu_0$; see Fig. 3(a). The spectral position of the resonant transmission peak is robust against positional disorder, as is demonstrated in Fig. 1(d).

When a small detuning $\delta$ is introduced, the CT symmetry is violated and the field amplitudes at the lossy resonators at sites 10 and 12 are different than zero; see Fig. 2(b). At the same time the resonant transmission peak decreases; see Fig. 3(a). Interestingly enough, also the absorbance shows the same decreasing trend; see Fig. 3(b) and discussion below.

For even larger values of $\delta$, the resonant localized mode is suppressed and for $\delta = \delta_{\text{crit}}$ it is eventually destroyed; see Fig. 2(b). One can estimate this critical detuning by realizing that the destruction of the resonant mode is associated with the competition between two physical mechanisms: the deterioration of the resonant $Q$ factor because of the radiative losses from the boundary which lead to broadening of the linewidth by $\Gamma_{\text{rad}} \propto \exp(\gamma/\nu_0)$, and the bulk (Ohmic) losses which are triggered by the interaction of the field with the lossy resonators at sites 10 and 12. The latter contributes to a linewidth $\Gamma_{\text{bulk}} \propto \delta^2$ (see previous discussion). Equating these two expressions we obtain $\delta_{\text{crit}} \propto \exp(-\gamma/\nu_0)$. In other words, even an exponentially small detuning results in the destruction of the resonant defect mode and a dramatic suppression of the associated resonant transmittance; see Fig. 4(a). The underlying physical mechanism associated with this abrupt change in the transport characteristics of the photonic structure relies on an underdamping-to-overdamping transition. In the former regime, the (small) radiative losses are the dominant factor.
mechanism that spoils the $Q$ factor of the structure, while in the latter case the $Q$ factor is dominated by the (strong) Ohmic losses. In this case, there is a strong impedance mismatch between the incoming wave and the resonant defect mode which, in turn, leads to the high reflection and consequently suppressed transmittance observed in our simulations.

In Figs. 4(b) and 4(c), we report the reflectance and absorbance at the associated resonant frequency, vs the detuning $\delta$. We find that, for $\delta > \delta_{\text{min}}$, the incoming photons do not couple at all with the resonant mode (strong impedance mismatch), but rather are reflected immediately. A quantitative understanding of this behavior requires the analysis of the absorbance $A(n)$ of the resonance mode. Using Eq. (3) we obtain

$$A(n) = 1 - R(n) = 1 - \sum_{n} |\psi_n|^2,$$

where $|\psi_n|^2$ is the $n$th component of the scattering field associated with a detuning $\delta$ and we have used the fact that the frequency $\nu(k)$ of the incident wave is real. Substituting in Eq. (4) the expressions of the field $\psi_n = e^{i\beta_n n} + r e^{-i\beta_n n}$ for $n \geq N$ and $\psi_n = e^{i\beta_n n}$, for $n \leq 1$, and taking into consideration that $\gamma_n = \gamma$ for $n = 10, 12$, and zero otherwise, we obtain

$$A \equiv 1 - T - R = 2|\nu_n|^2 |\psi_n|^2 / v_g,$$

where $\nu_n = \partial \nu / \partial k$ is the group velocity. From Eq. (5) one concludes that the absorbance depends on the (Ohmic) dissipation $\gamma$, the value(s) of the scattering field intensities at the position of the dissipative resonators, and is inversely proportional to the group velocity $\nu_n(k)$. In our case, $\gamma$ is constant. At the same time, $\nu_n(k)$ at the resonant mode can also be considered constant, to a good approximation (a small shift of the resonant position $\sim \delta$ is irrelevant for our discussion).

On the other hand, the change of the scattering field intensities $|\psi_{10}^0|^2$, $|\psi_{12}^0|^2$ can vary by orders of magnitude as $\delta$ increases; see Fig. 5(b). Specifically, for $\delta = 0$ we have $|\psi_{10}^0|^2, |\psi_{12}^0|^2 \approx 0$ and thus $A = 0$. For small detuning strengths $\delta < \delta_{\text{min}}$, the scattering field intensities $|\psi_{10}^0|^2, |\psi_{12}^0|^2$ increase [see $\delta = 2\%$ in Fig. 2(b)] and as a result the absorbance

$$A \approx \left| \frac{1 - \rho_{\text{CCW}}^0}{1 + \rho_{\text{CW}}^0} \right|.$$
Next, we evanescently couple the first and last ring with a Si bus waveguide and study the transmittance $T \equiv |S_{11}|^2 + |S_{10}|^2$, and reflectance $R = |S_{21}|^2 + |S_{20}|^2$ of an incident wave from port 1 (associated with the left bus waveguide). The scattering parameters $S_{11}, S_{10}$ describe transmission amplitudes from port 1 to ports 3 and 4 of the right waveguide, while $S_{21}, S_{20}$ describe transmission amplitudes from port 1 to port 2 and back to port 1 of the left waveguide (see top and bottom set of Fig. 6). Since there are intrinsic radiative losses we evaluate the Ohmic absorption (due to the metallic rings) directly via the expression $A = \frac{\sigma}{\omega} \int d^2\vec{r} |E(\vec{r})|^2 e^{i\phi(\vec{r})}$ [43]. The scattering parameters and the steady-state scattering field $E(\vec{r})$ associated with an incident monochromatic wave at frequency $\nu$ are calculated using the Maxwell’s equations coupled with the heat transport equations that dictate the steady-state temperature $\theta(\vec{r})$ within the CROW array:

$$\nabla^2 \vec{E} + \mu_0 \varepsilon(\vec{r}, \theta)e^{i\phi(\vec{r})} \vec{E} = 0, \quad \nabla \cdot [\varepsilon(\vec{r}) \nabla \theta(\vec{r})] = Q,$$

(7)

where $\varepsilon(\vec{r}, \theta) = \varepsilon(\vec{r}) + \varepsilon(\vec{r})e^{i\phi(\vec{r})}$ is the permittivity of the CROW array at position $z$ and steady-state temperature $\theta$ and $e^{i\phi(\vec{r})} = \sigma(\vec{r})/\omega$. The portion of the incident radiation which is absorbed by the defect resonator leads to a gradual heating of this resonator. This temperature increase, in turn, leads to a variation of the permittivity as we discussed above. Therefore, one needs to solve simultaneously the Maxwell’s and heat-transfer equations in a self-consistent manner in order to achieve steady-state transmittance, reflectance, and absorbance of the CROW array. Furthermore, we have assumed a fixed ambient temperature (293 K) at the boundaries surrounding the SiO$_2$ cladding. The parameter $Q = 0.5 \times |\text{Re} J \cdot \vec{E}|$, where $J = \sigma \vec{E}$, describes the electromagnetic energy deposited at the lossy metal-coated rings adjacent to the defect ring resonator which leads to an increase in temperature $\theta$. Finally, $\varepsilon(\vec{r})$ denotes the thermal conductivity of the rings making up the CROW array structure.

The upper (lower) panel of Fig. 6 shows the density plot of the scattering electric field intensity for incident signals with small (large) fluence. In the former case, the profile of the resonant defect mode respects the staggered form imposed by $CT$ symmetry. In contrast, in the latter case (lower panel of Fig. 6), the staggered profile is completely destroyed, thus leaving the defect mode exposed to the metallic resonators. In this case, the resonant transmissibility is completely suppressed.

In Figs. 7(a) and 7(b), we report $T$, $R$, and $A$ vs incident fluence for the CROW array (empty symbols) and SA ring-resonator structure (solid symbols). We observe that when the fluence of the incident light increases by an order of magnitude [i.e., from $10^4$ to $10^5$ W/cm$^2$]—see vertical orange...
lines in Fig. 7(a) the resonant transmission is also suppressed by an order. In Fig. 7(b), we also report the tiny relative permittivity variations (≈0.1%) which are associated with the increase of fluence of the incident light [see dashed vertical orange lines in Fig. 7(b)], due to the self-induced heating at the defect resonator caused by the incident radiation. Similarly, the Ohmic absorption $A$ decays as the fluence increases, thus protecting the CROW from self-damaging due to overheating. At the same time the reflectance $R$ increases as high as ≈0.55. Note that $R$ does not reach unity because there is a strong residual radiative absorption in the bus waveguide ($A$ ≈ 0.4).

For comparison purposes, we also show in the same figure the transmittance $T$ and the permittivity variation $\epsilon_r(\theta)$ vs incident fluences for the case of a stand-alone (SA) ring resonator (dashed lines) made by the same material (VO$_2$) as the defect resonator of the CROW arrangement. The resonator is now directly coupled to the bus waveguides. A similar SA resonator setup has been already investigated in Ref. [44] where it was shown experimentally that it can act as an on-chip limiter. The limiting action mechanism in this case relies on a resonant redshift—thus leaving the sensitive photonic elements exposed to damage in case of high-power broadband signal attacks. For extremely high fluencies the on-resonant transmittance is also suppressed due to an excessive heating which can lead to damage of the resonator [see Fig. 7(a)]. Conversely our design relies on complete suppression of the resonant mode at moderate fluences, thus protecting sensitive elements from any broadband (up to the size of the band gap) incident signal. In comparison, a complete resonant suppression in the case of the SA resonator requires a relative permittivity variation which is more than 1% (see the transmittance drop between the two black dashed lines in Fig. 7), which has to be compared with the 0.1% permittivity variation needed in the case of the CROW structure. Finally, we mark that our design demonstrates a limiting threshold (i.e., fluence value for which the transmittance drops to small values), which is smaller by an order of magnitude as compared to the SA ring-resonator structure; see Fig. 7(a). For completeness, we also compare the limiting performance of our photonic limiter with a CROW array consisting of the same number of VO$_2$-based resonators [dashed-star line in Fig. 7(a)]. The behavior of the latter is qualitatively similar to the one associated with the SA resonator. We find again that our CROW limiter has a lower limiting threshold (at least by an order) than the one utilized in Ref. [20] for suppressing high-power signals. In the latter case, for low incident field intensities or fluences, the system was chiral symmetric (not $C_T$ symmetric), and for high incident intensities or fluences one needed to utilize the presence of a strong nonlinear lossy mechanism in order to spoil the resonant $Q$ factor. Such strong nonlinear mechanisms are typically hard to realize in the microwave domain and require high incident field intensities or fluences in order to be activated. Here, instead, the structure is initially respecting a $C_T$ symmetry which guarantees the existence of high transmittivity for low incident field intensities or fluences via the phenomenon of symmetry-forced transmittivity. In the opposite limit of high incident field intensities or fluences, the abrupt drop of transmittance is triggered by the self-induced violation of $C_T$ symmetry which is achieved via (weak) nonlinear effects that change the value of the permittivity (for very small incident field powers) of the defect resonator by one to two percentage points—or even less.

V. CONCLUSION

We have investigated topologically protected defect modes and the transport properties of the associated resonant modes emerging in the frame of non-Hermitian bipartite CROW arrays. We show that an underlying $C_T$ symmetry enforces high resonant transmission and protects the resonant mode from positional disorder or local Ohmic losses that can potentially degrade the transport. When, however, a (self-)induced violation of $C_T$ symmetry occurs due to tiny variations of the permittivity of the defect, the resonant mode is destroyed and the transmission is completely suppressed. The fragile nature of resonant transport has been demonstrated for on-chip photonic and microwave CROW setups. Furthermore, it can be utilized in a variety of other frameworks including rf and acoustics for the realization of a different class of power limiters, switches, sensors, and modulators as well as for matter waves circuity.

Finally, we want to stress that the underlying physical mechanism invoked in this study is completely different from the one utilized in Ref. [20] for suppressing high-power signals. In the latter case, for low incident field intensities or fluences, the system was chiral symmetric (not $C_T$ symmetric), and for high incident intensities or fluences one needed to utilize the presence of a strong nonlinear lossy mechanism in order to spoil the resonant $Q$ factor. Such strong nonlinear mechanisms are typically hard to realize in the microwave domain and require high incident field intensities or fluences in order to be activated. Here, instead, the structure is initially respecting a $C_T$ symmetry which guarantees the existence of high transmittivity for low incident field intensities or fluences via the phenomenon of symmetry-forced transmittivity. In the opposite limit of high incident field intensities or fluences, the abrupt drop of transmittance is triggered by the self-induced violation of $C_T$ symmetry which is achieved via (weak) nonlinear effects that change the value of the permittivity (for very small incident field powers) of the defect resonator by one to two percentage points—or even less.

ACKNOWLEDGMENTS

We acknowledge partial support from Office of Naval Research (ONR) via Grant No. N00014-16-1-2803, from Air Force Office of Scientific Research (AFOSR) MURI Grant No. FA9550-14-1-0037, and from National Science Foundation (NSF) via Grant No. EFMA-1641109.

Bibliography


