Non-Spherical Particle Dynamics in Turbulence

by

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Abstract

The primary goal of this research is to better understand the dynamics of non-spherical particles in turbulence. This includes their preferential alignment with flow structures and their rotations in response to the velocity gradients of the flow or external forces, or both. We perform experimental measurements to study the dynamics of neutrally buoyant fibers and complex-shaped particles, and heavy, ramified particles as they sediment under the influence of gravity and turbulence.

In 3D homogeneous, isotropic turbulence, we measure the translational and rotational dynamics of small fibers while simultaneously resolving the fluid velocity field around the particles for the first time in experiments. To fully determine the dynamics of fibers in turbulence, it is required to specify a seven-dimensional joint probability density function of five scalars characterizing the velocity gradient tensor and two scalars describing the relative orientation of the fiber. We look at a lower-dimensional projection to simplify the problem and explore conditional averages. The preferential alignment of fibers with the velocity gradient tensor is observed and is in good agreement with direct numerical simulations.

The preferential alignment of elongated particles inspired us to design functionalized particles that show a preferential rotation in 3D homogeneous isotropic turbulence. We use 3D printing to fabricate so-called chiral dipoles, a rod with two helices of opposite handedness at either end. High aspect ratio chiral dipoles preferentially align with the extensional stretching field where the helical ends couple to the flow which results in a preferential rotation of the particle. These particles can be used to measure the rate at which fluid elements are stretched, one of the fundamental processes responsible for the energy cascade in turbulent flows.

The preferential alignment of non-spherical particles not only depends on particle shape, but also on the density difference between the particles and the fluid. To study the dynamics of heavy, non-spherical particles, we built a new apparatus that allows us to keep particles suspended and independently control the amount of turbulence they experience. We measure orientation distributions of ramified particles, quantify the dependence on turbulence intensity and look at preferential alignment. Moreover, we study the sedimentation and rotation rates and show that under certain conditions, a simple model is sufficient to capture most of the physics.
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Chapter 1

Introduction

The motion of non-spherical particles in turbulent flows is important to understand for many different areas of research, e.g. atmospheric sciences (Sabban and van Hout 2011; Heymsfield 1977), industrial processes including fiber suspensions (Lundell et al. 2011) and pharmaceutical processing (Erni et al. 2009), and natural phenomena like plankton in the oceans and marine snow (Pedley and Kessler 1992), and has a long history. Leal (1980) provides a review of the older literature, and a wide range of work has followed, for example: Koch and Shaqfeh (1989); Szeri and Leal (1993); Herzhaft et al. (1996); Olson and Kerekes (1998); Parsa et al. (2011); Andersson and Soldati (2013); Rosén et al. (2014); Voth and Soldati (2017).

The many applications have driven research in the particle-laden and multiphase flow community, whereas fundamental turbulence research mostly focused on the dynamics of the velocity gradient tensor through multi-point flow measurements. More recently, the potential of non-spherical particles as turbulence probes has been recognized and it has brought the two fields closer together. In many situations, non-spherical particles can provide a single particle measurement of the dynamics and the geometry of turbulent flow structures in complex environments where it is hard to make direct measurements. For example, fibers provide a single particle measurement that can be used to map out fluid stretching fields, since they preferentially align with the direction of extensional strain. Moreover, measurements of particles larger than the smallest length scale of the flow can give access to the velocity field and flow structures at the scale of the particle. This is especially valuable since it is computationally very expensive to
simulate large particles.

The first chapter is organized in the following way: First, we give a brief introduction to turbulence and explain the most important quantities that will be appearing throughout this thesis. Second, we focus on the motion of small, neutrally buoyant particles in isotropic turbulence, their preferential alignment and rotation rates and how the small scales of turbulence, which determine their motion, evolve. Last, we use the historical developments since the mid-19th century to introduce the different dynamics of heavy particles in quiescent fluid and in turbulence.

1.1 Introduction to Turbulence

The physics involved in many scientific and engineering problems like weather models, ocean currents and any pollutants carried by those currents, as well as channel flow and blood flow, rely on the Navier-Stokes equations\(^1\). They describe the forces acting on a fluid element in a viscous fluid and are used in the design of aircrafts and cars and, together with Maxwell’s equations, to study magnetohydrodynamics.

The equations, one for each dimension, can be derived from conservation of mass and momentum for a continuous media. For an incompressible fluid in the presence of gravity, the non-dimensional Navier-Stokes equations read

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + \frac{1}{Re} \nabla^2 \mathbf{u} + \frac{1}{Fr^2} \hat{g},
\]  

(1.1)

where \( \mathbf{u} \) is the velocity of the fluid, \( P \) is the pressure, \( \rho \) is the fluid density and \( g \) is gravity. The Reynolds number \( Re = UL/\nu \) appears as non-dimensional parameter, where \( U \) and \( L \) are the characteristic velocity and length of the flow and \( \nu \) is the kinematic fluid viscosity. It is the ratio of inertial forces to viscous forces, i.e. \( Re \ll 1 \) describes laminar flow, dominated by viscosity, whereas \( Re \gg 1 \) describes a flow that is unstable and will eventually become turbulent. The Reynolds number is used to characterize the behavior of fluid flows, where two flows with similar Reynolds number possess similar properties, e.g. mixing. Another non-dimensional

\(^1\)First derived by Claude-Louis Navier in 1821 with the formulation of the stress tensor, based on Euler’s perfect flow equations for frictionless fluids. Shortly after the work on viscous fluids by Simeon D. Poisson (1829), George G. Stokes (1845) re-derived Navier’s results with the formulation of the no-slip boundary condition.
number, the Froude number $Fr = \frac{U}{\sqrt{gL}}$, defines the ratio of fluid inertia to an external field, most commonly gravity.

For $Re = 0$, the Stokes flow limit, the Navier-Stokes equations are linear and many solutions are available. The difficulty in finding solutions to the Navier-Stokes equations when $Re \neq 0$ comes from their non-linearity, which becomes increasingly important with increasing Reynolds number.

At very high Reynolds numbers, the scales involved can be clearly separated. There are large scales, which strongly depend on the geometry and the forcing of the flow, and small scales, which only depend on the rate at which they receive energy from the large scales $\epsilon$ and the fluid viscosity $\nu$. It is the universal character of these small scales, their independence of geometry and forcing, that makes them so interesting. But how exactly does energy get to the small scales?

\begin{center}
Big whirls have little whirls, \\
That feed on their velocity; \\
And little whirls have lesser whirls, \\
And so on to viscosity – in the molecular sense.
\end{center}

This famous quote from Richardson (1922) illustrates the idea of an energy cascade. Kinetic energy enters the turbulence at the largest scales of motion and gets transported to smaller scales, where finally viscosity can dissipate the energy into heat. One of the mechanisms responsible for this cascade is called vortex stretching. This self-induced, self-amplifying process of turbulence (Tsinober 2001) will be discussed in greater detail later. It was Andrey N. Kolmogorov who quantified the energy cascade and identified the smallest scales of turbulence in 1941. He found their length scale, time scale and velocity to be:

\begin{align*}
\eta &\equiv \left(\frac{\nu^3}{\langle \epsilon \rangle}\right)^{1/4} \\
\tau_\eta &\equiv \left(\frac{\nu}{\langle \epsilon \rangle}\right)^{1/2} \\
u_\eta &\equiv \left(\frac{\nu\langle \epsilon \rangle}{\langle \epsilon \rangle}\right)^{1/4}
\end{align*}

This definition also provides a consistent definition of the velocity gradients of the small scales, $(u_\eta/\eta) = 1/\tau_\eta$. The importance of the velocity gradients are apparent when looking at the Navier-Stokes equations (Eq. 1.1). In the next section, we will show that the velocity gradients also determine the orientations and rotations of small, non-spherical particles. Since the small
scale structure of a turbulent flow plays a key role in the balance of turbulent kinetic energy, it is important to gain a better understanding of the velocity gradient field.

1.2 Neutrally Buoyant Particles in Isotropic Turbulence

When particles are small, on the order of the Kolmogorov length, and neutrally buoyant, they behave as Lagrangian tracers and move with the local fluid velocity. Advelted by the turbulent flow, they provide a compelling test case because of the nearly universal statistics of the small scales experienced by these particles in many turbulent flows at high Reynolds number.

The shape of particles determines their motion and their alignment with the local velocity gradients [Ni et al., 2015]. This makes them good probes for local vorticity [Marcus et al., 2014] and strain rate measurements [Kramel et al., 2016], and they can be used to gain information about the Lagrangian evolution of the velocity gradients. A tremendous amount of work has been done on fiber and disk motion, both experimentally [Parsheh et al., 2005; Parsa et al., 2012; Parsa and Voth, 2014; Marcus et al., 2014; Byron et al., 2015] and numerically [Zhang et al., 2001; Shin and Koch, 2005; Wilkinson et al., 2009; Pumir and Wilkinson, 2011].

1.2.1 Alignment of Non-Spherical Particles with the Velocity Gradient Tensor

The velocity gradients in three dimensions are described by a second-ranked tensor, the velocity gradient tensor $\mathbf{A} = \partial \mathbf{u} / \partial \mathbf{x}$. A natural decomposition of such a tensor has two components, defined by the symmetric and anti-symmetric part of $\mathbf{A}$. At this point, it is easier to switch to tensor notation:

$$\Omega_{ij} = \frac{1}{2}(A_{ij} - A_{ji}) \quad \text{(rotation rate tensor)} \quad (1.5)$$

$$S_{ij} = \frac{1}{2}(A_{ij} + A_{ji}) \quad \text{(strain rate tensor)} \quad (1.6)$$

where the trace of $\mathbf{S}$ is zero for an incompressible fluid (conservation of mass). The three independent elements of the rotation rate tensor form the vorticity vector, $\Omega_{ij} = 1/2 \, \varepsilon_{ijk} \omega_k$. The strain rate tensor $\mathbf{S}$ has three orthogonal eigenvectors $\hat{\mathbf{e}}_i$ and eigenvalues $\lambda_i$, with $\mathbf{Se}_i = \lambda_i \mathbf{e}_i$. 
By convention, the eigenvalues are ordered such that $\lambda_1 \geq \lambda_2 \geq \lambda_3$, and by conservation of mass, they add up to zero.

The eigenvector with the highest eigenvalue ($\lambda_1$, positive), $\hat{e}_1$, is pointing along the direction of greatest stretching. The eigenvector with the lowest eigenvalue ($\lambda_3$, negative), $\hat{e}_3$, is pointing along the direction of greatest compression. The intermediate eigenvector can either be extensional or compressional, but is on average slightly positive in 3D homogeneous isotropic turbulence [Tsinober, 2001].

A spherical fluid element that is stretched along the direction of $\hat{e}_1$ into an ellipsoid, will consequently be aligned with $\hat{e}_1$. Similarly, a rigid, non-spherical particle, e.g. a fiber, which is in a steady extensional strain field, will become aligned with $\hat{e}_1$. In turbulence, even a pseudovector like the vorticity aligns with the extensional eigenvectors of the strain rate tensor. Vortex stretching, one of the mechanisms responsible for the energy cascade, heavily relies on this alignment.

In a 3D turbulent flow, it has been observed that vorticity is on average better aligned with $\hat{e}_2$, the intermediate eigenvector [Ashurst et al. 1987; Xu et al. 2011], than with $\hat{e}_1$, and Ni et al. (2014) showed how Lagrangian stretching aligns the long axis of a fiber with the vorticity. This unexpected result comes from the fact that the alignment with the velocity gradients is an instantaneous measure of a dynamical process. One has to take into account the history of the fluid element or particle. The Cauchy-Green deformation tensor is then the natural quantity to look at. It describes the Lagrangian coherent structures in fluid flows and is closely related to the finite time Lyapunov exponents [Haller and Yuan 2000; Peacock and Haller 2013; Twardos et al. 2008].

A deformation tensor can be defined as $F_{ij} = \partial X_i / \partial x_j$, where $\mathbf{x}$ is the position of a fluid element at time $t_0$ and $\mathbf{X}$ the position of the fluid element at time $t = t_0 + \Delta t$. Since $F_{ij}$ is not necessarily symmetric and therefore may have not purely real eigenvalues, we calculate the left and right Cauchy-Green deformation tensors [Malvern 1969]:

\[
C^L_{ij} = F F^T = \frac{\partial X_i}{\partial x_k} \frac{\partial X_j}{\partial x_k} \quad (1.7)
\]

\[
C^R_{ij} = F^T F = \frac{\partial X_k}{\partial x_i} \frac{\partial X_k}{\partial x_j} \quad (1.8)
\]

Its eigenvalues encode the amount of stretching a fluid element has experienced over the time $\Delta t$. The eigenvectors and the process of fluid element stretching are illustrated in Fig. 1.1.
Figure 1.1: A fluid element at initial position \( \mathbf{x} \) at time \( t_0 \) is mapped to final position \( \mathbf{X} \) after time \( \Delta t \) by the flow. The circular fluid element is also deformed by the flow to an ellipse. The eigenvectors of the left (\( \hat{\mathbf{e}}_{1L} \) and \( \hat{\mathbf{e}}_{2L} \)) and right (\( \hat{\mathbf{e}}_{1R} \) and \( \hat{\mathbf{e}}_{2R} \)) Cauchy-Green tensor are shown.

It can be seen that a material line that was initially aligned with an eigenvector of the right Cauchy-Green deformation tensor, \( C_{ij}^R \), will end up aligned with one of the eigenvectors of the left Cauchy-Green deformation tensor, \( C_{ij}^L \). In that frame of reference, fibers and the vorticity vector will both show the strongest alignment with the direction of greatest stretching [Ni et al., 2014].

The preferential alignment was first shown experimentally by [Parsa et al., 2012], who observed the reduced tumbling rate of fibers when compared to the analytical prediction for randomly oriented fibers.

1.2.2 Rotation Rate of Non-Spherical Particles

In a turbulent flow, small particles rotate in response to the velocity gradients along their Lagrangian trajectory. Because these Lagrangian velocity gradients are controlled by the small scales, they are similar in many different turbulent flows and have been in the focus of extensive study [Meneveau, 2011]. We need to extend the investigation of the Lagrangian statistics of the velocity gradient tensor to include the particle orientations along their trajectories if we want to understand the dynamics of non-spherical particles in turbulence. This is a challenging problem, both because of the complexity of statistically quantifying the particle orientation with respect to the velocity gradient tensor, and due to the difficulty of measuring the dynamics of non-spherical particles simultaneously with the velocity gradient tensors.
The difficulty of accessing particle and fluid variables in turbulent flows, both in experiments and simulations, has hindered the progress in this field of research. Rod-shaped particles, or fibers, were the first to be studied. Shin and Koch (2005) provided an extensive numerical study of rotational diffusion and the tumbling rate of fibers in turbulence. They observed that the tumbling rate of fibers is much lower than that predicted for randomly oriented fibers. Pumir and Wilkinson (2011) showed from numerical simulations that this suppression of the tumbling rate is caused by fibers aligning with the vorticity. Parsa et al. (2012) extended numerical study of the tumbling rate across the full range of aspect ratios of spheroids (axisymmetric ellipsoids) and found that preferential alignment decreases the tumbling rate for almost all shapes. Chevillard and Meneveau (2013) studied the full parameter space of triaxial ellipsoids in numerical simulations and showed that the tumbling of fibers is a challenging test case for stochastic models of the velocity gradient tensor in turbulence. Gustavsson et al. (2014) used analytical and numerical methods to show that the differences in tumbling between fibers and disks can be understood using Lagrangian three-point correlations of the velocity gradient tensor. Parsa and Voth (2014) made experimental measurements of the rotations of fibers with lengths extending into the inertial range of turbulence and proposed that rotations of long fibers should show inertial range scaling.

The fact that turbulent strain aligns non-spherical particles raises the questions: what determines the rotation rate of these particles and how does the rotation rate depend on shape? Small, neutrally buoyant ellipsoids fall into the class of particles for which the Navier-Stokes equations (Eq. 1.1) can be solved analytically and the fluid forces can be determined. Jeffery (1922) was the first one to demonstrate that the tumbling rate in simple shear is given by

$$\dot{p}_i = \Omega_{ij}p_j + \frac{\kappa^2 - 1}{\kappa^2 + 1} (S_{ij}p_j - p_i p_k S_{kl}p_l)$$

(1.9)

where $\kappa$ is the aspect ratio of major to minor axis of an axisymmetric ellipsoid and $\hat{p}$ is a unit vector along the major axis. A full description of the tumbling rate of fibers in turbulence requires specifying a seven-dimensional joint probability density function (jPDF) of five scalars characterizing the velocity gradient tensor and two scalars describing the relative orientation of the fiber. If these seven parameters are known, then Jeffery’s equation specifies the tumbling rate and any statistic of fiber rotations can be obtained as a weighted average over the jPDF. Later, we will look for a lower-dimensional projection to simplify the problem and explore conditional averages of the mean-square rotation rate.
It is apparent that for spheres with \( \kappa = 1 \), the second term in the equation is zero. This makes sense because, by symmetry, they do not possess a unique orientation that could be affected by strain, and therefore vorticity is the only contributing term to their tumbling rate. We can take advantage of this fact and use spheres to measure the vorticity of a flow by measuring their rotations. Moreover, in 3D isotropic turbulence, \( \langle \Omega_{ij}\Omega_{ij} \rangle = \langle \epsilon \rangle / (2\nu) \), which means we can use spheres to measure the mean energy dissipation rate. There is just one problem: measuring the rotation of a sphere experimentally is very challenging. We developed a convenient way to address this issue which will be discussed in detail later.

1.2.3 Evolution of the Velocity Gradient Tensor

The orientations and rotational dynamics of small particles are determined by the velocity gradients, but what determines the geometry and the dynamics of the velocity gradients themselves? The evolution equation of the velocity gradient tensor can be obtained by taking the gradient of the Navier-Stokes equations (Eq. 1.1), which yields:

\[
\frac{D}{Dt} A_{ij} = -A_{ik} A_{kj} - \frac{\partial^2 P}{\partial x_i \partial x_j} + \nu \frac{\partial^2 A_{ij}}{\partial x_k \partial x_k}
\]  

(1.10)

where the material derivative \( \frac{D}{Dt} = \frac{\partial}{\partial t} + u_k \frac{\partial}{\partial x_k} \) has been used. It describes the rate of change of a quantity over time and through advection of the velocity field \( u \). \( H^p_{ij} \) and \( H^v_{ij} \) are the pressure Hessian and a viscous term, respectively.

Equation 1.10 can be decomposed into the following evolution equations for enstrophy \( \omega^2 = \Omega_{ij}\Omega_{ij} \) and strain \( S^2 = S_{ij}S_{ij} \):

\[
\frac{D}{Dt} \frac{\omega^2}{2} = \omega_i S_{ij}\omega_j + \nu \omega_i \nabla^2 \omega_i
\]  

(1.11)

\[
\frac{D}{Dt} \frac{S^2}{2} = -S_{ij}S_{jk}S_{ki} - \frac{1}{4} \omega_i S_{ij}\omega_j - S_{ij} \frac{\partial^2 P}{\partial x_i \partial x_j} + \nu S_{ij} \nabla^2 S_{ij}.
\]  

(1.12)

From Eq. 1.12 and 1.11 it becomes clear that together with \( \omega^2 \) and \( S^2 \), the third moments, enstrophy and strain production, \( \omega_i S_{ij}\omega_j \) and \( S_{ij}S_{jk}S_{ki} \), are among the key quantities of turbulence dynamics.

The velocity gradient tensor of an incompressible fluid can be fully determined with five invariant scalars. These could be the two independent eigenvalues of the strain rate tensor \( S \) and the
three components of the vorticity vector $\omega$ in the strain rate eigenframe. It has proven useful to define five different invariant scalars, especially when looking at the evolution equation of the velocity gradient tensor. Two of the most important invariants can be found from the characteristic equation of the velocity gradient tensor:

$$\Lambda_i^3 + PA_i^2 + QA_i + R = 0,$$

where $\Lambda_i$ are the eigenvalues of $A$ and $P$, $Q$ and $R$ are the first, second and third invariants, respectively. The first one is zero, due to incompressibility, and the other two are given by

$$Q = -\frac{1}{2} A_{ij} A_{ji},$$

$$R = -\frac{1}{3} A_{ij} A_{jk} A_{ki}.$$
$(P_Ω, Q_Ω$ and $R_Ω$), can be defined. Again, $P_S = P_Ω = 0$ and also $R_Ω = 0$, no self-amplification of vorticity. This leaves the five invariant scalars, $Q$, $R$, $Q_S$, $Q_Ω$ and $R_S$.

The physical interpretation of two invariants $Q$ and $R$ (Eq. 1.14 and Eq. 1.15) becomes clear when separating $A$ into its symmetric and anti-symmetric parts. Therefore, $Q = \frac{1}{4} (Ω_{ij}Ω_{ij} - 2S_{ij}S_{ij})$ quantifies the difference between enstrophy and strain, and $R = -\frac{1}{3} S_{ij}S_{jk}S_{ki} - \frac{1}{4} ω_iS_{ij}ω_j$ quantifies the difference between strain production and enstrophy production. Note the vortex stretching term causing enstrophy production.

The Restricted-Euler Equations [Vieillefosse 1982, Cantwell 1992] emerge from Eq. 1.10 when ignoring the non-local influence of the pressure Hessian $H^p$ and the viscous term $H^v$. Taking appropriate products with the velocity gradient tensor, we can obtain a simple set of coupled differential equations for $Q$ and $R$:

$$\frac{DR}{Dt} = \frac{2}{3} Q^2$$

$$\frac{DQ}{Dt} = -3R$$

(1.16)  (1.17)

Using this simple set of equations, many insights into the structures of turbulence can be obtained, e.g. for almost all initial conditions, the velocity gradient tensor evolves to a geometry with two positive principle rates of strain and the vorticity aligned with the intermediate positive strain rate (see Fig. 1.2). This suggests that this model may provide some useful insights into the mechanisms by which the geometry of small scale motions in turbulence can evolve. Moreover, the discriminant, $D = 27/4 R^2 + Q^3$, of $A$, is a scalar that can be used effectively to separate turbulent structures, e.g. $D < 0$ characterizes strongly dissipative events [Chacin and Cantwell 2000].

Investigations of turbulent flows by a number of groups (a compilation can be found in Tsinober 2001) have found a qualitatively similar “tear drop” shape (see Fig. 1.3) in joint PDF plots of $Q$ versus $R$. It is argued that this shape is a universal feature of the small scales in turbulence. The so-called Vieillefosse tail [Vieillefosse 1982], the most characteristic feature, illustrates the dominance of the strain production term over enstrophy production, in strain-dominated regions. Further analysis of the invariants have revealed that additional insight into turbulent motions can be obtained by studying the Lagrangian evolution of $A$ in the $Q$-$R$ plane. Martin et al. 1998 and Ooi et al. 1999 found a clockwise evolution of mean trajectories in this space and Chevillard et al. 2008 focused on understanding and modeling this evolution.
The scale dependence of the Lagrangian evolution of the velocity gradient tensor remains an open question, but non-spherical particles provide a powerful and promising tool to tackle this issue. When particles become larger than the Kolmogorov length, they filter the small scale dynamics of turbulence and respond primarily to structures of similar size. We will later show that the rotations of large particles, for example, can be used to measure the turbulent kinetic energy at the scale of the particle.

We are not just interested in the scale dependence of particle dynamics, but also what happens when particles become heavier than the fluid and inertial effects have to be considered. One consequence is sedimentation, which, in the case of non-spherical particles, depends strongly on particle orientation. Ice crystals in clouds are a good example, where inertial effects cause strong preferential alignment that can be observed by eye in the form of sun dogs.

### 1.3 Heavy Particles in Turbulence

Considering the vast parameter space and the associated complexity of the problem, it is no surprise that many studies focus on neutrally buoyant particles first and neglect the effects of particle inertia, or study heavy particles in quiescent fluid. For small particles, inertial effects can still be neglected, but they become increasingly important with increasing particle size (Voth and Soldati 2017). The particle inertia can drastically change the dynamics of these particles and the particle-fluid coupling, in quiescent fluid as well as in turbulence. To illustrate
the complexity of the problem in more detail, we can look at the historical development of the theoretical framework for the drag on particles in viscous flow. I will closely follow the description given by Eames and Klettner (2017), who have summarized this development nicely.

1.3.1 From Stokes Flow to Finite Reynolds Number in Quiescent Fluid

In 1851, George G. Stokes published his famous work on the effect of the internal friction of fluids on the motion of pendulums (Stokes, 1851). He was driven by the obsession for accurate timekeeping and therefore had to calculate the forces on a pendulum. He notes:

“I first tried a long cylinder, because the solution of the problem appeared likely to be simpler than in the case of a sphere. But after having proceeded a good way towards the result, I was stopped by a difficulty relating to the determination of the arbitrary constants [...]. Having failed in the case of a cylinder, I tried a sphere, and presently found that the corresponding differential equation admitted of integration in finite terms, so that the solution of the problem could be completely effected.”

The constant he mentions turned out to be the fluid viscosity, or as he called it the index of friction (the subject of fluid dynamics was barely established). The concept of a no-slip boundary condition also originated around this time. His theory resulted in the well-known expression for the drag on a sphere at low Reynolds number (Eq. 1.18):

\[ F_D = 6\pi \mu DU_\infty, \] (1.18)

where \( F_D \) is the drag force on a rigid sphere of diameter \( D \) in a uniform flow moving with speed \( U_\infty \). Here, the Reynolds number \( Re_D = DU_\infty/\nu \) is based on the sphere’s diameter and its velocity relative to the undisturbed fluid velocity at infinity. This result had great success as it has been used to determine two fundamental physical constants, the charge of an electron and Avogadro’s constant, and in theories which have won three Nobel prizes (Einstein in 1921, Millikan in 1923, and Perrin in 1926).

Sir Horace Lamb (1911) solved the problem of viscous flow past a cylinder 60 years after Stokes’s failure. Using the Oseen (1910) approximation in the momentum equations (Eq. 1.1) to account for the far field boundary conditions, the decay of the fluid disturbance is sufficiently fast that

\footnote{The Oseen approximation linearizes the inertia term in steady flow, \( u\nabla u \approx \partial u/\partial x \).}
a convergent solution can be found. With that, Lamb (1932) was able to derive the drag force on an infinite cylinder at $Re_D \ll 1$.

It took almost another 60 years after Lamb until Brenner (1964) obtained the flow past a solid body whose shape was a slightly deformed sphere at low Reynolds numbers Payne and Pell (1960) worked on the general case for lens-shaped bodies and Oberbeck (1876) worked on spheroids. Shortly after, Batchelor (1970) and then Khayat and Cox (1989) developed a theoretical framework based on matched-asymptotic expansions that captures the motion of slender bodies at arbitrary large Reynolds numbers (based on the body length $\ell$) as long as the Reynolds number based on the diameter of the particle remains small (high $Re_\ell$ possible as long as $\kappa \gg 1$).

This approach distinguishes between two regions, an inner and an outer region. Near the cylinder, where the cylinder appears infinitely long, it can be treated as a two dimensional problem with no-slip boundary condition at the particle, leaving the far field boundary condition open. Far away from the cylinder, the cylinder can be treated as infinitely thin – a line distribution of Stokeslets (point forces) – and the uniform-flow far field boundary condition can be met. Matched-asymptotic expansion forces the inner solution to match the outer solution in the intermediate region and therefore gives a valid solution everywhere, without having to deal with both boundary conditions simultaneously.

The big success of this analysis is the correct prediction of drag and lift force on a slender body moving at a relative velocity to the fluid. Moreover, it is the first time inertial torques can be quantified from a theoretical standpoint. Experimental observations of free falling, non-spherical particles have shown that they do not fall straight and do not have random orientation distributions, but the motion depends on the particle Reynolds number and shape Willmarth et al. (1964) Jayaweera and Mason (1965) Zikmunda and Vali (1972) Bragg et al. (1974) Field et al. (1997) Kajikawa (1992). The torque induced on thin cylinders, or prolate ellipsoids, in this Reynolds number regime causes the body to rotate into a stable orientation with its longest axis perpendicular to the direction of the relative velocity vector.

Following the historical developments, we arrive almost another 50 years later, to come up with a model that attempts to deal with more complex-shaped particles. We can use the theoretical framework developed for slender bodies Khayat and Cox (1989) and apply it to particles that are composites of thin cylinders. The so-called ramified particle model will be discussed in
(a) (b) (c) (d)

Figure 1.4: Steady-state of closed eddies formed behind a falling cylinder at $Re_D = 40$, (a) Broadside-on and (b) end-on. Shedding of vortices by a falling cylinder of $Re_D = 70$, (c) Broadside-on and (d) end-on. Figure from Jayaweera and Mason (1965).

greater detail later.

1.3.2 Sedimentation in Quiescent Fluid at High Reynolds Number

Direct numerical simulations (DNS) of the flow field around fibers and disk-like particles [Hashino et al., 2014] have revealed great insight into the vast parameter space, but due to the inherent complexity of the problem, many studies have been forced to ignore turbulence and focus on the slightly easier task of understanding sedimentation in quiescent fluid first.

When the particle Reynolds number based on the diameter $Re_D$ approaches 1 (or greater), analytical solutions and approximations are hard to come by. At $Re_D \approx 1$, simulations and experiments have shown that the boundary layer separates and a wake forms behind the particle. If $Re_D$ does not get too large, this becomes a steady-state (see Fig. 1.4 (a) and (b)), however, at around $Re_D \approx 100$ and greater, vortices start to shed of the particle. In 2D, this phenomenon is known as the Von-Karman vortex street [Mallock 1907, Von Kármán 1911, Von Karman and Rubach 1912].

It becomes clear why solutions are hard to find when looking at visualizations of the flow field, as shown in Fig. 1.4. Besides Richardson’s first finite-differences calculations with human

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3Note: Vincenc Strouhal, a Czech physicist who first investigated the singing of telegraph wires in the wind in 1878.
computers (~2000 operations per week), the steady increase in computational power since the
1960s provides a promising alternative to experiments to solve the problem of particle motion
at high Reynolds numbers.

Most DNS focus on the drag and lift forces on fibers and disks and curve fits to the data are
proposed, e.g. Vakil and Green (2009); Zastawny et al. (2012). There has also been great effort
in resolving the flow field around such particles (Hashino et al., 2014) to better understand
the dynamics of falling ice crystals. The results and curve fits show a very strong aspect ratio
dependence, which makes these models quantitatively only useful for particles with the same
parameters. Also, most simulations study the case of fixed particles, where the Reynolds number
does not depend on the particle orientation. There does not seem to be a consistent theory about
the forces on particles at higher Reynolds numbers. Some models predict greater lift coefficients
for fibers with angles greater than 45° with respect to the direction of the relative velocity
(Zastawny et al., 2012; Hoerner, 1965), whereas another model predicts a greater lift coefficient
for angles less than 45° (Vakil and Green, 2009). This highlights the importance of experimental
measurements, even in quiescent fluid.

1.3.3 Sedimentation in Turbulence

Many studies have investigated the case of heavy, spherical particles (Good et al., 2014) in
turbulence, experimentally (Nielsen, 1993; Aliseda et al., 2002; Yang and Shy, 2003, 2005);
and computationally in simple flows (Maxey, 1987; Davila and Hunt, 2001; Hill, 2005), as well
as in turbulence (Wang and Maxey, 1993; Yang and Lei, 1998). They have shown examples of
turbulence enhanced (sweeping) and reduced (loitering) settling speeds (Nielsen, 1993; Kawanisi
and Shiozaki, 2008) of small and large particles, respectively. It is unclear under what conditions
either mechanism exists in real turbulence.

Compared to spherical particles, where inertia only affects transport, non-spherical particles can
show very different orientation distributions and inertia can alter their preferential alignment
with the velocity gradients. DNS of small, heavy ellipsoidal particles (Mortensen et al., 2008,
Zhao et al., 2010; Marchioli et al., 2010; Challabotla et al., 2015) in turbulent channel flows have
shown that the preferential orientation changes non-trivially with increasing particle inertia,

\footnote{A linear drag law doesn’t capture sweeping/loitering, therefore a non-linear drag law is required, Wang and
Maxey (1993); Yang and Lei (1998); Ireland and Collins (2012).}
especially in the near-wall regions, which has important consequences for particle deposition. Most of these studies focus on small particles and ignore external forces. However, particles with a higher density than the fluid will sediment under the influence of gravity, and on top of turbulence and particle inertia comes now the situation where external forces have to be considered.

Only during the last decades has it been possible to study the influence of turbulence on sedimenting non-spherical particles (Newsom and Bruce, 1994; Krushkal and Gallily, 1988). This case is of special interest to the atmospheric research community, where prolate and oblate ellipsoids are used as archetypes for column- and plate-like ice crystals in clouds (Siewert et al., 2014). The in-cloud turbulence is often not able to destroy the strong alignment of these particles (Cho et al., 1981), which is in agreement with the orientation model of Klett (1995). As a result, their orientation statistics and sedimentation velocities can have important consequences for remote sensing applications like polarization LIDAR, which is a key component of climate research programs to characterize the properties of mixed-phase cloud systems, such as cirrus clouds (Noel et al., 2002; Westbrook et al., 2010; Hu, 2007). On a side note, the strong alignment of ice crystals in the atmosphere can also be observed by eye since it causes optical phenomena by scattering light, the origin of the Perry Arc (Westbrook, 2011).

In summary, the motion of heavy, non-spherical particles in turbulence sedimenting under the influence of gravity is an active field of research where current theoretical models need validation through experiments. One of the main contributions of this work is that we provide experimental techniques that shows how these measurements could be obtained.
Simultaneous Measurements of Fibers and Velocity Gradients in Turbulence

This Chapter represents the core of the paper “Measurements of the Coupling between the Tumbling of Rods and the Velocity Gradient Tensor in Turbulence”, published in the Journal of Fluid Mechanics [Ni et al. 2015]. This work was done in collaboration with Nicholas T. Ouellette from Yale University and Rui Ni, a joint post-doc between Wesleyan and Yale at that time.

In a turbulent flow, small particles rotate in response to the velocity gradients along their Lagrangian trajectory. Because these Lagrangian velocity gradients are controlled by the small scales, they are similar in many different turbulent flows. We need to extend the investigation of the Lagrangian statistics of the velocity gradient tensor to include the particle orientations along their trajectories if we want to understand the dynamics of non-spherical particles in turbulence.

We present simultaneous experimental measurements of the dynamics of fibers transported by a turbulent flow and the velocity gradient tensor of the flow surrounding them. A full description of the tumbling of fibers in turbulence requires specifying a seven-dimensional joint probability density function (jPDF) of five scalars characterizing the velocity gradient tensor and two scalars describing the relative orientation of the fiber. If these seven parameters are known, then Jeffery’s equation specifies the fiber tumbling rate and any statistic of fiber rotations can be
obtained as a weighted average over the jPDF. To look for a lower-dimensional projection to simplify the problem, we explore conditional averages of the mean-square tumbling rate. The conditional dependence of the mean-square tumbling rate on the magnitude of both the vorticity and the strain rate is strong, as expected, and similar. There is also a strong dependence on the orientation between the fiber and the vorticity, since a fiber aligned with the vorticity vector tumbles due to strain but not vorticity. When conditioned on the alignment of the fiber with the eigenvectors of the strain rate, the largest tumbling rate is obtained when the fiber is oriented at a certain angle to the eigenvector that corresponds to the smallest eigenvalue, because this particular orientation maximizes the contribution from both the vorticity and strain.

2.1 Experiments

The results presented in this Chapter are the first experimental measurements of fiber motion and the velocity gradients around them in three dimensional turbulence. Using multiple high speed cameras and small spherical tracer particles to visualize flow fields is a common technique, but to resolve the velocity gradients around small fibers, we require a very high spatial resolution of the velocity field. This poses a challenging task that can’t be achieved with common volume illumination particle tracking methods, because the high number of tracer particles required causes experimental difficulties as well as problems with the image analysis. To avoid these issues, we sacrifice temporal resolution for spatial resolution. Instead of illuminating the entire detection volume at once, we create a scanning laser illumination. This greatly reduces the number of tracer particles in each frame, while still being able to reconstruct the full detection volume from a complete scan.

2.1.1 Experimental Apparatus

We generated a turbulent flow in an octagonal Plexiglass tank measuring $1 \times 1 \times 1.5$ m$^3$ as shown in Fig. 2.1 (a). Two grids with a mesh size of 8 cm oscillate in phase with an amplitude of 12 cm and a grid frequency of 1 Hz. This creates nearly homogeneous isotropic turbulence in the center of the tank with Taylor Reynolds number $Re_\lambda = 140$. Details about the apparatus are also given in Blum et al. (2010). The Kolmogorov length scale of the flow is $\eta = 0.31$ mm and
Figure 2.1: (a) 3D rendering of the experimental apparatus showing the three cameras, the two grids and indicating the detection volume (not to scale). (b) Schematic of the scanning system (top view). The Nd:YAG laser beam passes two cylindrical lens groups and is steered by a piezo-electric driven mirror before entering the tank.

<table>
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<tr>
<th>Grid Freq. [Hz]</th>
<th>Re_λ</th>
<th>L [mm]</th>
<th>\bar{u} [mm/s]</th>
<th>\langle \epsilon \rangle [mm^2/s^3]</th>
<th>\nu [mm^2/s]</th>
<th>\eta [mm]</th>
<th>\tau_\eta [s]</th>
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<td>1</td>
<td>140</td>
<td>68</td>
<td>18.3</td>
<td>90</td>
<td>0.961±0.005</td>
<td>0.31</td>
<td>0.093</td>
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</tbody>
</table>

Table 2.1: Summary of experimental parameters. \( Re_\lambda = \sqrt{\frac{\bar{u} L}{\nu}} \), Taylor Reynolds number; \( L = \bar{u}^3 / \langle \epsilon \rangle \), energy input length scale; \( \bar{u} = \left( \langle u_i u_i \rangle / 3 \right)^{1/2} \), root-mean-square fluid velocity; \( \langle \epsilon \rangle \), mean energy dissipation rate; \( \nu \), kinematic viscosity; \( \eta = (\nu^3 / \langle \epsilon \rangle)^{1/4} \), Kolmogorov length scale; \( \tau_\eta = (\nu / \langle \epsilon \rangle)^{1/2} \), Kolmogorov time scale.

The temperature of the fluid was almost constant at \((21.8 \pm 0.2) \degree\) C. Table 2.1 summarizes the characteristic quantities of the experiments.

Three Photron Fastcam SA-5 cameras with a resolution of 1024 × 1024 pixels were used to image the particles in a small detection volume measuring approximately \( 3 \times 3 \times 4 \) cm\(^3\) in the center of the tank. To resolve the detection volume from more than half a meter away from the sidewalls of the tank, each camera was fitted with a Nikkor 200 mm macro lens and a Kenko 1.6 teleconverter. The cameras were mounted on a custom-built frame on an optical table uncoupled from the turbulence tank to minimize camera vibration. Two of the cameras (labelled as ‘top cameras’ in Fig. 2.1 (b)) were mounted in the same lateral plane with a 90\degree\ angular separation, and looked down into the detection volume at an angle of 16\degree. The third camera was aimed up
Fibers and tracer particle specifications: $L$, particle length; $D$, particle diameter; $\kappa = L/D$, aspect ratio; $\rho_p$, particle density. (Tracer particles are spherical).

<table>
<thead>
<tr>
<th>Fiber Specifications</th>
<th>Tracer Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>$L$ [mm]</td>
</tr>
<tr>
<td>23.3</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 2.2: Fiber and tracer particle specifications: $L$, particle length; $D$, particle diameter; $\kappa = L/D$, aspect ratio; $\rho_p$, particle density. (Tracer particles are spherical).

To measure the velocity gradient tensor simultaneously with fiber motion, we need two kinds of particles: the fibers themselves and small, spherical tracer particles. A key to this experiment is obtaining suitable particles. Fluorescent particles are convenient, as they will produce clean images, whereas some small residues from the aluminium top and bottom plates of the tank and the bearings, that might be carried into the detection volume, will not be visible. For tracer particles, we therefore use internally dyed polystyrene divinylbenzene (PS-DVB) particles with 30 µm diameter and density 1.05 g cm$^{-3}$, purchased from Thermo Scientific. The fibers (Nylon, density 1.10 – 1.13 g cm$^{-3}$) were purchased from DonJer Corp., with length and diameter of $L \approx 700$ µm and $D \approx 30$ µm, respectively, giving an aspect ratio of $\kappa = 23.3$. The fibers were dyed with Rhodamine-B to absorb green light from a frequency doubled Nd:YAG laser (wavelength $\lambda = 532$ nm) and emit red light at the same wavelength as the tracer particles (see Appendix 5.2.1 for emission spectrum). A typical image of these fluorescent particles, captured through a Schneider B+W MRC Orange 550 band-pass filter, is shown in Fig. 2.5. The shapes of the two types of particles are well-defined and distinct from each other, so that the particles can be easily separated using automated image analysis. Note that the brightness of the two kinds of particles is very similar, another important factor in the experiment: if the brightness of the particles were very different, it would be difficult to determine their positions accurately at the same time. Even though the tracer particles are much smaller than the fibers, the internal dye emits a much stronger fluorescent signal than the fibers, which we dyed ourselves.

In the subsequent analysis, we make the assumption that the fibers and tracers do not exhibit inertial effects caused by their finite size or density difference with the fluid. To characterize
the validity of this assumption, we consider the lengths and Stokes numbers of the particles. The length of a fiber is roughly $2.3\eta$. Studies of long neutrally buoyant fibers show that the tumbling rate of a particle of this length is very close to the short fiber limit (Shin and Koch, 2005; Parsa and Voth, 2014). The tracer particles have diameters of $0.1\eta$, which is clearly in the small sphere limit (Voth et al., 2002). The Stokes number $St = \tau_p/\tau_\eta$ is defined as the ratio between the time scale of the Stokes viscous drag $\tau_p = r^2/(3\beta \nu)$ and the Kolmogorov time scale $\tau_\eta$. Here, $r$ is the radius of the particle and $\beta = 3\rho_f/(2\rho_p + \rho_f)$ is a coefficient capturing the effect of a density difference between the fluid (with density $\rho_f$) and the particle (with density $\rho_p$). If the particle response time $\tau_p$ is much smaller than the smallest scale $\tau_\eta$, the inertial effect of the particle is negligible, and the particles can be safely treated as tracers. For our spherical particles, $St = 8.6 \times 10^{-4} \ll 1$. For a fiber, the response time is given by $\tau_p = 2\kappa \rho_p r^2 \log(\kappa + \sqrt{\kappa^2 - 1})/(9\nu \rho_f \sqrt{\kappa^2 - 1})$, where $r$ is again the radius (or semi-minor axis) (Shapiro and Goldenberg, 1993; Zhang et al., 2001). Using this expression, the Stokes number for the fibers is $St = 2.4 \times 10^{-3}$, which is also much less than unity. Thus, both the fibers and the spherical tracers are in the small particle limit and will follow the fluid motion accurately.

Once the particles are chosen, we need to determine the proper seeding density for the experiment. Generally speaking, we want a very high density of tracer particles and relatively low density of fibers. The tracer particle density is directly related to the spatial resolution at which we obtain velocity gradient measurements. The maximum density, however, is limited if we illuminate the entire measurement volume, because the images of individual particles will overlap with each other when the particle seeding density becomes high. Therefore, we use a scanning particle tracking system (Hoyer et al., 2005) for our measurements which will be described in the next section.

### 2.1.2 The Scanning System

The basic principle of this technique is to sacrifice temporal resolution for improved spatial resolution. By sub-dividing the detection into 10 sections, for example, and successively illuminating one section at a time, we can increase the particle seeding density by a factor of 10 (in the ideal case). Scanning the illumination across all sections enables us to reconstruct the entire detection volume, albeit at a cost of requiring a factor of 10 increase in camera frame rate and a decrease in total recording time.
Figure 2.2: (a) Reconstructed detection volume showing the trajectories that pass through at least five laser slabs in one typical movie with 5457 frames. Particle positions that belong to different laser slabs are shown by a different color. (b) The histogram of $y$ positions of the particles in each slab. Only particles from the center of the detection volume ($-5 \text{ mm} < x < 5 \text{ mm}$ and $-5 \text{ mm} < z < 5 \text{ mm}$) were used to ensure that the histogram represents the slab width and overlap.

A schematic of the scanning system is shown in Fig. 2.1 (b). The beam from a pulsed Nd:YAG laser with an average output power of 50 W was shaped independently in height and width by two sets of cylindrical lenses to create an illumination slab measuring $50 \text{ mm} \times 3 \text{ mm}$. To ensure a relatively uniform slab width over a height of 30 mm, we did not image the top and bottom 10 mm of the laser slab. The scanning motion of the illumination was created with a piezo-electric driven mirror (Model S-224, PI Inc.) with a diameter of 15 mm and a maximum deflection angle of 2.2 mrad with sub-mrad resolution. The small deflection angle is magnified through the optical system to give a 12 mm scanning range in the center of the flow chamber. For reasons we will explain later, each slab overlaps with the previous one by roughly 50%. Compared to previous designs, using a rotating prism to create the scanning motion, a piezo-electric driven mirror or alternatively an acousto-optic deflector have the potential to generate faster scanning rates, which are more suitable for turbulence at even higher Reynolds numbers.

The steering mirror was driven with an adjusted saw-tooth signal at a frequency of 500 Hz, rising linearly for 80% of each cycle and falling for the remaining 20%. This driving produced a nearly linear scanning motion of the laser beam through the measurement volume followed by a quick return to the initial position. The cameras recorded images at a frame rate of 5000 Hz,
so that each cycle of the mirror resulted in 10 images, with eight linearly positioned through the measurement volume. We only store these eight frames of each cycle on the cameras to conserve on-board memory, which could hold a total of 5456 images. Thus, approximately 1.4 s (14.7τη or 0.44 large eddy turn over times) of data is held in the cameras memory before being transferred to the computer hard drive. Our experimental protocol consisted of several steps. First, the two grids were driven at 3 Hz for 30 s to stir up the fluid and particles. The grids were then slowed down to 1 Hz, and the flow was given 1 min (approximately 17.5 large eddy turn over times) to stabilize. The cameras then recorded images until their memory was full (for approximately 1 s), and the system subsequently rested until the data transfer to the hard drive was complete. The timing of this system was automatically controlled with LabVIEW scripts. The results reported here come from a full day of measurements, giving roughly 300 data sets.

A single, unprocessed video file has a file size of 5.33 Gigabyte, and therefore, three cameras recording 300 data sets results in almost 5 Terabyte of raw data. Each camera was connected to a dedicated computer with an internal 2 TB hard drive for storage. After the data sets were analyzed, we compressed each video file to a final file size of roughly 100 Megabyte and stored all data sets on a network attached storage (NAS) system.

2.2 Data Analysis

Before describing our techniques for measuring fiber motion and the velocity gradient tensor in details, we show in Fig. 2.3 an example of a measured fiber trajectory along with vectors characterizing the local velocity gradients. The ribbon shows the full trajectory with a solid fiber plotted only once every 25 time steps. The color indicates the magnitude of fiber tumbling rate $|\dot{p}|$, which depends on the straining and vortical motion of the surrounding flow. The tumbling rate due to strain is given by Jeffery’s equation (Eq. 1.9) which tends to align the fiber with the most extensional direction of the local flow (red arrow), which is given by the eigenvector $\hat{e}_1$ of the strain rate tensor $S$ that corresponds to its highest eigenvalue $\lambda_1$. Local vorticity is characterized by the rotation rate tensor $\Omega$, which tends to rotate the fiber about the local vorticity direction $\omega$ (blue arrow) at a rate of $\dot{p}_i = \Omega_{ij}p_j$.

\[The location of the data and analysis codes will be specified in the Appendix.\]
For this fiber trajectory, the magnitude of the total tumbling rate $\dot{p}_i$ as well as its two component contributions $\dot{p}_i^\Omega$ and $\dot{p}_i^S$ are shown in Fig. 2.4 (a) and (b). The total tumbling rate can be computed in two ways: by differentiating the fiber orientation or by using measurements of the velocity gradient tensor and Jeffery’s equation $\dot{p}_i^J = \dot{p}_i^\Omega + \dot{p}_i^S$. As shown in Fig. 2.4 (a), the two measurements agree well with each other, indicating both that our measurement of the velocity gradient tensor is accurate and that the fiber is small enough that Jeffery’s equation holds. In Fig. 2.4 (c), we plot the cosine of the angle between $\dot{p}_i^\Omega$ and $\dot{p}_i^S$. When this quantity is negative, the contribution to the fiber tumbling due to strain works against that due to vorticity. But when it is positive, the two contributions work cooperatively and lead to high tumbling rates, as can be seen at 0.16 s and 0.56 s in Fig. 2.4 (corresponding to the red regions in Fig. 2.3).

### 2.2.1 Image Processing

To determine particle positions, the digital image from each camera is first segmented into clusters of bright pixels (blob detection), representing both tracer particles and fibers. Typically, the image of one spherical tracer particle contains 4-9 pixels, corresponding to 2-3 pixels in diameter. The number of pixels of a fiber depends on its relative orientation to the camera. The maximum size is about 320 pixels when its long axis is perpendicular to the optical path of the camera, in which case it is also highly anisotropic. But the image of a fiber becomes quasi-
Figure 2.4: The time series of the fiber tumbling rate for the trajectory shown in figure 1. (a) The magnitude of total tumbling rate from two different measurements: $\dot{p}$ determined by differentiating the rod orientation (curve) and $\dot{p}^J$ calculated from Jefferys equation using the velocity gradient measurements around the rod (circles). (b) Two contributions to tumbling rate from vorticity $|\dot{p}\Omega|$ and from strain $|\dot{p}^S|$. (c) The cosine of the angle between two vectors $\dot{p}\Omega$ and $\dot{p}^S$.

spherical with an area of roughly 16 pixels when it points directly toward the camera. Ideally, the images of fibers and tracer particles could be separated solely by their area. However, sometimes a tracer particle looks larger if it is out of focus. So we add another criterion for separating the images of spherical tracers and fibers: eccentricity, which is defined as the degree of geometrical deviation from a circle. It is determined by finding the best-fit ellipse to the pixel cluster and calculating the ratio of the distance between the foci and the major axis of the ellipse. We find that the eccentricity ranges from 0 to 0.6 for spherical particles and from 0.8 to 1 for fibers. In practice, a particle is considered a fiber if its eccentricity is greater than 0.9 and its area is larger than 30 pixels. These criteria separate almost all fibers from tracer particles. Sometimes a fiber that points directly toward one camera will be mistakenly identified as a tracer in that camera; but because we have other cameras viewing the same fiber from different directions, where it will be distinctly elongated, it can be correctly identified by combining the information from all
2.2.2 Particle Tracking

Once we know the particle type, its center is determined by averaging the positions of its pixels, weighted by their brightness values. This procedure is used for both types of particles. From the two-dimensional positions of the particles measured by each camera, the three-dimensional position can be found by stereomatching, and subsequently tracking over time from one volume scan to the next (Ouellette et al., 2006), although some modifications of common tracking algorithms are required.

In traditional particle tracking, the entire measurement volume is illuminated and is imaged at a constant rate of $1/\Delta t$, where $\Delta t$ is the time interval between two frames. Therefore, finding candidate particles to extend a given trajectory requires searching for potential matches only at a time $\Delta t$ in the future. But for the scanning system, as is sketched in Fig. 2.6, there are two relevant time intervals: $\Delta t = 1/500$ s, the time between two full scans of the volume; and $\delta t = 1/5000$ s, the time between the illumination of two neighboring slabs in the volume. Since a particle may or may not pass from one slab to another over an interval of $\delta t$, the time at which a particle corresponding to the continuation of a trajectory may be found is not obvious: it may be found at $\delta t$ in the future, for example, if the particle moved between two successive

Figure 2.5: (a) The full resolution and (b) cropped image captured by one of the three cameras. The circle in (b) is centered at the centroid of a fiber with 2 mm ($\sim 6\eta$) radius. The tracer particles that fall in such a sphere will be used to calculate the velocity gradient tensor.
slabs; at $\Delta t$ in the future if it remained in the same slab; at $\Delta t - \delta t$ in the future if it moved to a previous slab; or even potentially at other times. A similar scanning particle tracking system (Hoyer et al., 2005) performs the search for the continuation of a particle trajectory in slabs $n$, $n-1$ and $n+1$. To handle this ambiguity, we first track the particles found in each individual slab separately, and then subsequently merge the short segments that belong to different slabs but refer to the same particle. Within each slab, particles are tracked using a standard predictive tracking algorithm, with a recording time of roughly $d/\Delta t\bar{u} = 83$ frames, where $d = 3$ mm and $\bar{u} = 1.8$ cm s$^{-1}$ are the thickness of one laser slab and the root-mean-square velocity of the flow, respectively. In Fig. 2.2 (b), it is seen that each pair of neighboring slabs overlaps by $d_{ov} = 1.5$ mm. Thus, since the velocity of a particle is bounded, a particle that moves from one slab to the next will inevitably pass through the overlapping region and will be recorded twice during each volume scan. We refer to each doubly recorded position as a join between the two trajectory segments. In Fig. 2.6, where we schematically demonstrate our tracking method, we show 3 joins. In principle, two joins are enough to merge two trajectory segments into one longer track. Estimating the number of joins in the overlapping region using simple kinematics gives $d_{ov}/\Delta t\bar{u} = 42$. Thus, even a particle with a speed of $20\bar{u}$ would still have at least two joins.

**Figure 2.6:** Diagram of one particle passing from one slab $n$ (box in solid line) to the next one $n+1$ (box in dashed line). The positions of the same particle in slab $n$ and $n+1$ are represented by solid circles and crosses, respectively. The time between two consecutive volume scans is $\Delta t$, and the time between two slabs is $\delta t$. In the overlapping region, there are three pairs of joins, which can be used to connect two segments of trajectories in different slabs together.
in the overlapping region. We apply this merging procedure consecutively for all neighboring slabs, linking the segments in different slabs together into longer trajectories. In Fig. 2.2 (a), we show those trajectories that pass through at least five different slabs, which evenly cover the entire volume, to demonstrate qualitatively that our measurements are robust and that the overlapping regions are large enough for splicing.

The stereomatching and tracking procedures for fibers and tracers are almost the same. The primary difference is that we need to keep track of the orientation of a fiber in addition to its position. In two dimensions, we define the orientation as the angle between the major axis of the fiber and the horizontal image axis, which can be extracted from all three cameras. The three-dimensional orientation of a fiber can then be uniquely determined from the two-dimensional angles and the viewing directions of all cameras (Parsa et al., 2012). Errors in determining the orientation arise mainly from the finite aspect ratio of the fibers. In the experiments reported here, the aspect ratio of the fibers is \( \kappa = 23.3 \), roughly four times that of experiments by Parsa et al. (2012), resulting in smaller orientation uncertainties than previously reported.

The next step in processing the data is to smooth and differentiate the trajectories to obtain the velocities of the tracers and tumbling rates of the fibers. Common methods to accomplish this task include convolving the trajectory with appropriate kernels (Mordant et al., 2004) and polynomial fitting (Voth et al., 2002). In the scanning system, the convolution method is difficult to apply because the points along the trajectories are not evenly spaced in time, leading to difficulties in accurate numerical integration. We therefore use polynomial fitting. In addition to smoothing and differentiating, fitting also allows us to accurately interpolate the measured positions and velocities in time so that all data throughout the measurement volume are contemporaneous. That is, we can use the fits to extract the kinematics of the flow field not only at the measured spacetime positions of the particles, but rather at times \( t_n = 4\delta t + n\Delta t \) \((n = 0, 1, 2, \ldots)\). We acquire one full velocity field for each scan of the volume, measured at the time corresponding to the illumination of the center slab (halfway through the volume scan).

To accomplish this interpolation, for each \( n \), we fit a polynomial to the measured data points in the range \( [t_n - (\tau_f - 1)/2, t_n + (\tau_f - 1)/2] \) along each track, where \( \tau_f = 9\Delta t \) is the temporal length of the fit. This choice minimizes noise without unduly affecting the signal (Voth et al., 2002). From the polynomial fits, we extract smoothed positions, velocities, and accelerations of the tracers at \( t_n \), spread over the entire volume. We apply a similar process to the fibers
to extract their orientations and tumbling rates. We define the orientation of a fiber with a unit vector $\hat{p}$ along their major axis. Note: smoothing the orientations decreases the random error in the orientation measurements, but may change the magnitude of $\hat{p}$. Therefore, we must re-normalize the orientation vector for each fiber after smoothing.

To determine the maximum seeding density of the spherical tracer particles while still obtaining good measurements, we test the scanning system by slowly increasing the number of tracer particles. When the particle concentration is low, the ratio between the number of successfully stereomatched particles and the number of particles detected in each image is almost constant. We reach the maximum seeding density when this ratio begins to decrease. In our experiments, this point corresponds to roughly 500 stereomatched particles in each slab. After accounting for doubly imaged particles in the overlapping regions and trajectories shorter than $\tau_f$, for which we cannot measure velocities or accelerations, we can reliably measure about 2000 velocity vectors in each volume scan. This number varies somewhat over time due to sedimentation of particles to the sidewalls and bottom plate. We add particles over the course of the experiments to maintain a roughly constant particle concentration. The seeding density of fibers is kept low to avoid interactions between them. We have roughly 10 fibers in the measurement volume at any given time, so the non-dimensional concentration is roughly $nL^3 = 10^{-3}$, where $n$ is the number density and $L$ is the length of a fiber. This is far below the concentration at which fiber-fiber interactions become important.

After tracking, smoothing, and differentiation, we are left with trajectories of fiber orientations along with the velocities of many tracers surrounding them. Measuring the velocity gradient tensor around each fiber requires us to estimate the spatial gradient from multiple velocity vectors inside a small volume. To do this, we first locate all tracers within a $2 \text{ mm} \ (\sim 6\eta)$ radius of the center of a fiber. The radius is chosen to have a sufficient number of tracer particles surrounding the fibers to accurately estimate the velocity gradients. Given our seeding density, there are about 6-10 tracer particles within that radius, which is comparable to what has been used in previous experiments (Hoyer et al., 2005), where a radius of nearly $8\eta$ was found to be sufficient (viscous effects dominate and the velocity field is close to linear). To estimate the gradient, consider $N$ tracers at positions $\mathbf{x}^n(t)$ and with velocities $\mathbf{u}^n(t) \ (n = 1, 2, ..., N)$ that are sufficiently close to a fiber. Their relative position and velocity with respect to the center of mass of the tracer cloud are $\mathbf{x}^{\prime n}(t)=\mathbf{x}^n(t)-\sum_n \mathbf{x}^n(t)/N$ and $\mathbf{u}^{\prime n}(t)=\mathbf{u}^n(t)-\sum_n \mathbf{u}^n(t)/N$, respectively.
Determining the velocity gradient tensor $A_{ij}$ can then be formulated as a least-squares problem by finding the minimum value of the squared residuals $S = \sum_n \left[ u_i^n(t) - A_{ij}x_j^n(t) \right]^2$ (Pumir et al., 2013). In general, the tracer particles in the cloud surrounding the fiber are randomly distributed in space, but will sometimes lie in almost the same plane. Such cases will introduce a large error in the determination of the out-of-plane components of the velocity gradient. To exclude these cases, we use the inertia tensor $I = g / tr(g)$ with $g_{ij} = \sum_n x_i^n x_j^n$ to characterize the shape of the tracer cloud. It can be diagonalized in an orthogonal basis with three eigenvalues $I_1 \geq I_2 \geq I_3$. For a symmetric object, $I_1 = I_2 = I_3 = 1/3$, while for a coplanar particle cloud $I_3 \approx 0$. In our experiment, we require $I_3 > 0.1$ to rule out cases that will have large errors. Empirically, we find that our results are not sensitive to the choice of this threshold when it is in the range from 0.07 to 0.2. In addition to the overall shape of the tracer cloud, the distribution of the tracers inside the cloud may also affect the estimate of the velocity gradient: sometimes, the tracers will all be concentrated in one octant, for example, and the center of mass of the tracer cloud will be far away from the fiber center. This situation will lead to a biased estimate of the velocity gradient at the fiber center. To avoid this case, we require that the distance between the fiber center and center of the tracer cloud be no more than one third of the radius of gyration of the tracer cloud.

### 2.3 Direct Numerical Simulations

To compare with the experimental data, one data set from a direct numerical simulation (DNS) of homogeneous isotropic turbulence at Reynolds number $Re_\lambda = 180$ was used. There are a total of $N^3 = 512^3$ collocation points for the entire volume and $7 \times 10^4$ Lagrangian trajectories of the velocity gradient tensor were followed for $O(1)$ large eddy turnover times. The time step for integrating the Navier-Stokes equations and tracking Lagrangian points was $O(10^{-2}\tau_\eta)$. Along each trajectory, the orientation of a virtual, infinitesimal fiber with an aspect ratio of $\kappa = 20$ was computed by integrating Jeffery’s equation (Eq. 1.9) using a fourth-order Runge-Kutta method (Parsa et al., 2012). The details of this simulation are given by Benzi et al. (2009).
Chapter 2 - Simultaneous Measurements of Fibers and Velocity Gradients in Turbulence

2.4 Results

Details of the errors in our measurements and a comparison between the dissipation rate estimated from the measured velocity gradient tensor and the velocity structure functions are reported in Appendix 5.1.1 and 5.1.2.

2.4.1 Alignment between Vorticity and Strain Rate Tensor

To demonstrate the quality of our measurements, we briefly consider some of the well-known geometric properties of the velocity gradient tensor - in particular, the alignment between the vorticity \( \omega \) and the eigenvectors of the strain rate tensor \( S \). Reminder, \( S \) is a symmetric, second-rank tensor, and can be described by its three eigenvectors \( \hat{e}_i \) \( (i = 1, 2, 3) \), with corresponding eigenvalues \( \lambda_i \), where by definition \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \). Intuitively, one would expect that the vorticity \( \omega \) would tend to align with the most extensional eigenvector, \( \hat{e}_1 \). But, in general, \( \omega \) is preferentially aligned with the intermediate eigenvector \( \hat{e}_2 \) (Ashurst et al., 1987). We show the probability density functions (PDFs) of the cosine of the angle between \( \omega \) and the \( \hat{e}_i \) in Fig. 2.7 for the experiments and for the direct numerical simulation. We choose to use the cosine of the angle instead of the angle itself because two vectors that are randomly oriented with respect to each other in three-dimensional space will have a uniform distribution for the cosine of the angle between them. In Fig. 2.7, the overall trend of the PDFs is very similar for the experiments and the simulations: as expected, the vorticity is best aligned with \( \hat{e}_2 \). The trends, however, are more pronounced in the simulation due to experimental inaccuracies in the measurement of the velocity gradient tensor from a finite-sized tracer cloud.

2.4.2 Fiber Alignment

Figure 2.8 shows the alignment of fibers with respect to the vorticity \( \omega \) and the three eigenvectors \( \hat{e}_i \) of the strain rate tensor, for both our experiments and the simulations. In both cases, fibers are much more strongly aligned with \( \omega \) than they are with \( \hat{e}_i \). This is because \( \omega \) and each independently tend to align with the direction of strongest Lagrangian stretching, as defined by the maximum eigenvector of the left Cauchy-Green deformation tensor \( \hat{\rho} \) (Ni et al., 2014). The alignment of fibers with the strain rate eigenvectors are weaker because
fibers align with the Lagrangian stretching integrated over several Kolmogorov times, and this
direction is usually quite different from the instantaneous stretching direction defined by the
strain rate eigenvectors.

The alignment distributions from the simulations in Fig. 2.8 (b) are in excellent agreement with
previous simulations [Pumir and Wilkinson, 2011]. Our experiments agree quite well with
the simulations, except that the experiments show a slightly better alignment of fibers with \( \hat{e}_1 \) than
with \( \hat{e}_2 \), while the simulations show the opposite. Prior work on material lines [Wan, 2009]
has shown that the relative strength of their alignment with \( \hat{e}_1 \) and \( \hat{e}_2 \) is sensitive to Reynolds
number and perhaps to the particular driving of the flow. Since material lines rotate just like thin
fibers with high aspect ratio, the small difference between the experiments and the simulations
may result from differences in Reynolds number and the forcing mechanism.

### 2.4.3 Joint Probability Distribution Functions

In turbulence, the tumbling rate of fibers has been found to be much smaller than it would be
if the fibers were randomly oriented [Shin and Koch, 2005; Parsa et al., 2012]. This decrease
has been qualitatively understood to be a result of the alignment of fibers with the vorticity
vector. Given the alignment of fibers with the vorticity, fibers will preferentially spin around their
symmetry axis, and therefore the contribution of fluid rotation \( \Omega \) to the fiber tumbling rate \( \dot{p} \) will
be weak. To understand the fiber tumbling rate in more detail, we use Jeffery’s equation (Eq. 1.9) to estimate the parameter space and evaluate the complexity of the problem. Recall that the velocity gradient tensor $\mathbf{A}$ is a second-rank tensor with eight independent components, assuming incompressibility. For turbulent flows with isotropic small scales, the overall orientation in space does not matter so five scalars are sufficient to characterize $\mathbf{A}$ (Meneveau, 2011). As described in the introduction, there are many possible choices for these five scalars, such as two eigenvalues of the strain rate tensor $\mathbf{S}$ and the three components of the vorticity in the $\mathbf{S}$ eigenframe, or five scalars constructed from moments of the velocity gradient tensor, such as the well-known $R = -(A_{im}A_{mn}A_{mi})/3$ and $Q = -(A_{im}A_{mi})/2$ (Cantwell, 1992). Specifying the relative orientation of a fiber in the $\mathbf{S}$ eigenframe requires two additional independent angles. Thus, together with the five scalars required to determine $\mathbf{A}$, fully characterizing the fiber tumbling rate requires a seven-dimensional parameter space. Denoting the generalized coordinates in this space by $X(p, \mathbf{A})$ and the corresponding probability density function as $P(X(p, \mathbf{A}))$, the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2_8.png}
\caption{PDF of the cosine of the angle between the fiber orientation $\hat{p}$ and both vorticity $\hat{\omega}$ and eigenvectors of the strain-rate tensor $\hat{e}_i$: (a) experimental measurements, (b) simulation results.}
\end{figure}
mean-square fiber tumbling rate can be expressed as

$$\langle \dot{p}_i \dot{p}_i \rangle = \int dX(p, A) P(X(p, A)) \left[ \Omega_{ij}p_j + \frac{\kappa^2 - 1}{\kappa^2 + 1} (S_{ij}p_j - p_i p_k S_{kl}p_l) \right]$$

$$\times \left[ \Omega_{im}p_m + \frac{\kappa^2 - 1}{\kappa^2 + 1} (S_{im}p_m - p_i p_q S_{qn}p_n) \right]$$

$$= \int \left[ \Omega_{ij} \Omega_{mj} \Omega_{im}p_m + \frac{\kappa^2 - 1}{\kappa^2 + 1} \left( S_{ij} S_{im}p_m - p_i p_k S_{kl}p_l p_q S_{qn}p_n + T \right) \right]$$

$$\times dX(p, A) P(X(p, A)) \right) (2.1)$$

where $T$ represents six cross-terms between fluid strain and rotation. If we assume that fibers are randomly oriented and uncorrelated with the velocity gradient tensor, Eq. 2.1 simplifies considerably because some of the averages can be taken independently; for example, $\langle S_{ij} S_{im}p_m \rangle = \langle S_{ij} \rangle \langle p_m \rangle$. This assumption leads to

$$\langle \dot{p}_i \dot{p}_i \rangle = \frac{\langle \Omega_{ij} \Omega_{ij} \rangle}{3} + \frac{1}{5} \left( \frac{\kappa^2 - 1}{\kappa^2 + 1} \right)^2 \langle S_{ij} S_{ij} \rangle. \quad (2.2)$$

Given that, in isotropic turbulence, $\langle S_{ij} S_{ij} \rangle = \langle \Omega_{ij} \Omega_{ij} \rangle = \langle \epsilon \rangle / (2 \nu) = 1 / 2 \tau^2_\eta$, the normalized fiber tumbling rate would depend only on the fibers aspect ratio:

$$\langle \dot{p}_i \dot{p}_i \rangle \tau^2_\eta = \frac{1}{6} + \frac{1}{10} \left( \frac{\kappa^2 - 1}{\kappa^2 + 1} \right)^2. \quad (2.3)$$

However, in turbulence, fibers are not randomly oriented, but are coupled with the velocity gradient tensor. Characterizing their tumbling requires understanding of the full seven-dimensional PDF $P(X(p, A))$. Since the Lagrangian velocity gradients are similar in many different turbulent flows, this PDF should be approximately universal. Experimentally, we do not have enough samples to obtain this PDF at a reasonable bin size, and it would be difficult to present, even if we obtained it. A typical way to approach such a complicated PDF would be to try to find a suitable lower-dimensional projection to simplify the problem. We will show below that although the two most important dimensions are the magnitude of the strain and the enstrophy, the relative orientation of a fiber in turbulence can not be neglected when determining its tumbling rate.

In the rest of this section, we will use the conditional average of the fiber tumbling rate along different dimensions to characterize the importance of each dimension and to provide new experimental insights into the dynamics of fiber tumbling. Recall that, as shown in Fig. 2.4, we have two different ways of determining the fiber tumbling rate experimentally: a direct
Figure 2.9: mean-square tumbling rate of fibers conditioned on the alignment between fiber and vorticity \( \hat{\mathbf{p}} \cdot \hat{\omega} \).

The black dashed-dotted line represents the mean-square tumbling rate and the dashed line is from simulation results. Open symbols show the tumbling rate from differentiating the fiber trajectories of orientation, and closed symbols show the calculation from Jeffery’s equation applied to the measured velocity gradient tensor.

measurement, \( \dot{\hat{p}}_i \), and a measurement inferred from Jeffery’s equation, \( \dot{\hat{p}}_i^J \). To check for any potential systematic offset between these different measurements, we compute the mean-square tumbling rate for each, finding \( \langle \dot{\hat{p}}_i \dot{\hat{p}}_i \rangle = 0.10\tau_\eta^{-2} \) and \( \langle \dot{\hat{p}}_i^J \dot{\hat{p}}_i^J \rangle = 0.09\tau_\eta^{-2} \) for the experiments and \( \langle \dot{\hat{p}}_i \dot{\hat{p}}_i \rangle = \langle \dot{\hat{p}}_i^J \dot{\hat{p}}_i^J \rangle = 0.09\tau_\eta^{-2} \) for the simulations. Since \( \dot{\hat{p}}_i^J = \dot{\hat{p}}_i^S + \dot{\hat{p}}_i^\Omega \), the mean-square tumbling rate has three contributions, \( \langle \dot{\hat{p}}_i^S \dot{\hat{p}}_i^S \rangle, \langle \dot{\hat{p}}_i^\Omega \dot{\hat{p}}_i^\Omega \rangle \) and \( 2\langle \dot{\hat{p}}_i^S \dot{\hat{p}}_i^\Omega \rangle \), which are respectively equal to \( 0.053\tau_\eta^{-2} \), \( 0.069\tau_\eta^{-2} \) and \( -0.039\tau_\eta^{-2} \) for experiments and \( 0.062\tau_\eta^{-2}, 0.073\tau_\eta^{-2} \) and \( -0.045\tau_\eta^{-2} \) for simulations. To compare the experiments with the simulations in detail, we systematically shifted the values of \( \dot{\hat{p}}_i^J \) and the tumbling rates computed from the simulations so that their mean-square values are \( 0.10\tau_\eta^{-2} \). The shift was done by multiplying all data points in \( \dot{\hat{p}}_i^J \) and the simulation results by 1.11, which will only move the curves up without changing the trends.

In Fig. 2.9 we plot the mean-square fiber tumbling rate conditioned on \( \hat{\mathbf{p}} \cdot \hat{\omega} \) for \( \dot{\hat{p}}_i \), \( \dot{\hat{p}}_i^J \) and the simulations. All three curves nearly collapse, and show that the fiber tumbling rate monotonically decreases by more than 50% as its alignment with the vorticity increases. For \( \hat{\mathbf{p}} \cdot \hat{\omega} = 0 \), the fiber is perfectly perpendicular to the vorticity vector and its tumbling rate has a large contribution from the vorticity. If only the vorticity contributed, the mean-square tumbling rate should be \( \langle \dot{\hat{p}}_i \dot{\hat{p}}_i \rangle \approx \langle \Omega_{ij} \Omega_{ij} \rangle / 2 \). Since \( \langle \Omega_{ij} \Omega_{ij} \rangle = 1/2\tau_\eta^2 \), \( \langle \dot{\hat{p}}_i \dot{\hat{p}}_i \rangle \tau_\eta^2 / 2 \) at \( \hat{\mathbf{p}} \cdot \hat{\omega} = 0 \) should be 1/4,
Figure 2.10: mean-square tumbling rate of fibers conditioned on (a) normalized enstrophy and (b) normalized dissipation rate. The symbols are the same as in Fig. 2.9.

which is higher than the measured result of 0.14. This discrepancy indicates that the strain contribution for \( \mathbf{p} \cdot \hat{\mathbf{\omega}} = 0 \) is on average in the direction opposed to the fluid vorticity, just as it would be for a fiber in Jeffery orbits in a uniform shear flow.

In the other limit, when \( \mathbf{p} \cdot \hat{\mathbf{\omega}} = 1 \), the fiber is perfectly aligned with the vorticity, so its tumbling rate gains no contribution from it. The tumbling rate does not vanish, however, due to the coupling to the strain field. The contribution from fluid strain can be investigated by considering the alignment of \( \hat{\mathbf{p}} \) with the three orthogonal eigenvectors \( \hat{\mathbf{e}}_i \) of \( \mathbf{S} \). As shown in Fig. 2.11, both simulations and experiments suggest that the fibers tend to align parallel to \( \hat{\mathbf{e}}_1 \) and \( \hat{\mathbf{e}}_2 \) and perpendicular to \( \hat{\mathbf{e}}_3 \), indicating that fibers primarily lie in the plane spanned by \( \hat{\mathbf{e}}_1 \) and \( \hat{\mathbf{e}}_2 \).

Another way to assess the relative contributions to the fiber tumbling rate from fluid vorticity and strain is by conditioning the tumbling rate on the magnitude of \( \mathbf{\Omega} \) and \( \mathbf{S} \), which can be represented by the enstrophy \( \mathbf{\omega}^2 = \omega_i \omega_i \) and the dissipation rate \( \epsilon = 2\nu\langle S_{ij}S_{ij} \rangle \). Figure 2.10 shows the mean-square fiber tumbling rate conditioned on these two quantities. The tumbling rate monotonically increases by nearly two orders of magnitude with a similar dependence on both enstrophy and dissipation rate, indicating that the vorticity and the strain equally contribute to the tumbling rate of fibers on average. This two-decade change of the conditional fiber tumbling rate is greater than other dimensions, which also suggests that the enstrophy and the dissipation rate are the most important two dimensions.
Figure 2.11: mean-square tumbling rate of fibers conditioned on the alignment between the fiber and the eigenvectors of the strain-rate tensor: (a) $\hat{p} \cdot \hat{e}_1$; (b) $\hat{p} \cdot \hat{e}_2$; (c) $\hat{p} \cdot \hat{e}_3$. The symbols are the same as in Fig. 2.9.

In addition to the effect of the strain magnitude, we also investigated the effect of the orientation between the fiber and the eigenvectors of the strain rate tensor on the tumbling rate, as shown in Fig. 2.11. Recall that, in turbulence, the vorticity tends to be aligned with $\hat{e}_2$. It would then be expected that the conditional average of the tumbling rate on $\hat{p} \cdot \hat{e}_2$ (Fig. 2.11 (b)) would share some similarity with the conditional average on $\hat{p} \cdot \hat{\omega}$ (Fig. 2.9). In both cases, the tumbling rate monotonically decreases with increasing alignment, but the trend is much steeper for the case of $\hat{p} \cdot \hat{\omega}$, because the vorticity plays a bigger role in determining the fiber tumbling rate. The conditional average of the tumbling rate on $\hat{p} \cdot \hat{e}_1$ (Fig. 2.11 (a)) is more complicated to explain.

If the fiber were perfectly parallel or perpendicular to $\hat{e}_1$, the strain would act to stretch or compress the fiber rather than to rotate it, and the tumbling rate due to strain would be zero in both limits. But if a fiber were oriented at $45^\circ$ with respect to $\hat{e}_1$, the strain contribution to its tumbling rate would be maximized. So the squared tumbling rate should be small for both limits of alignment when $\hat{p} \cdot \hat{e}_1 = 0$ and $\hat{p} \cdot \hat{e}_1 = 1$, but should have a peak near $\hat{p} \cdot \hat{e}_1 = \cos(45^\circ) \approx 0.7$.

Our results in Fig. 2.11a are consistent with this expectation. The same argument can be made for the conditional average on $\hat{p} \cdot \hat{e}_3$, as shown in Fig. 2.11 (c). In this case, however, the peak is much higher and the data is skewed towards $\hat{p} \cdot \hat{e}_3 = 1$, as compared to the data conditioned on $\hat{p} \cdot \hat{e}_1$. Two factors are important here. First, fibers are rarely aligned with $\hat{e}_3$, as seen in Fig. 2.8. Events where this occurs are likely to be events with intense vorticity, so the tumbling rate will be high. Additionally, the vorticity is preferentially perpendicular to $\hat{e}_3$, so a fiber that is better aligned with $\hat{e}_3$ gains a larger contribution to its tumbling rate from vorticity. Both of these effects move the peak up and towards $\hat{p} \cdot \hat{e}_3 = 1$.

We have now described the dependence of fiber tumbling rate on five independent dimensions:
the magnitude of strain and vorticity, captured by the dissipation rate and the enstrophy, respectively, and three independent orientations of the fiber with respect to the velocity gradient tensor. All of them are important in determining the fiber tumbling rate, and none can be neglected. The additional two dimensions necessary for determining the problem concern the orientation of vorticity in the strain rate eigenframe, for example, \( \hat{\omega} \cdot \hat{e}_1 \) and \( \hat{\omega} \cdot \hat{e}_2 \). We find that the conditional mean-square tumbling rate has only a very weak dependence on these two dimensions. This may seem surprising since the relative orientation of vorticity and strain determine whether they reinforce or cancel each other as seen in Fig. 2.4. For prediction of the mean-square tumbling rate, the parameter space may be able to be simplified to five dimensions, but it seems that a complete picture of the tumbling of fibers in turbulence will require specifying the full seven-dimensional PDF of the fiber orientation and the velocity gradient tensor.

### 2.5 Conclusions

We have presented an experimental investigation of the tumbling rate of fibers in turbulence. To assess the relative importance of various factors on the tumbling rate and to thereby estimate the effective dimensionality of the problem, we also simultaneously measured the full velocity gradient tensor near the fibers. We obtained the gradient by implementing a scanning particle tracking system, allowing us to image a high concentration of tracer particles. The quality of our velocity gradient measurements is comparable to previous experiments, and we were able to measure the trajectories of anisotropic particles at the same time. We carefully explored the mean-square tumbling rate conditioned on several different variables, including \( \hat{p} \cdot \hat{\omega} \), \( \epsilon \), \( \omega^2 \), \( \hat{p} \cdot \hat{e}_1 \), \( \hat{p} \cdot \hat{e}_2 \) and \( \hat{p} \cdot \hat{e}_3 \). These variables were chosen to give a framework within which the relative contributions to the fiber tumbling rate are easily interpretable. We found that the mean-square tumbling rate was dependent on all five of these dimensions to some degree. As these variables can be separated into two classes, those that depend on the strain rate \( S \) and those that depend on the vorticity \( \Omega \), our results suggest that the fiber tumbling rate depends approximately equally on both. We provide experimental evidence that fibers are preferentially aligned with the vorticity, thus diminishing the potential contribution from \( \Omega \) and partially explaining why the measured fiber tumbling rates are smaller than they would for randomly oriented fibers. We also found that the mean-square tumbling rate monotonically decreases with increasing \( \hat{p} \cdot \hat{e}_2 \), where \( \hat{e}_2 \) is the intermediate strain rate eigenvector, just as it does with
The contribution from the local strain is small when the fiber is either aligned with or orthogonal to \( \hat{e}_1 \), but is largest when the fiber is oriented at roughly \( 45^\circ \) with respect to \( \hat{e}_1 \). The PDF of the fiber tumbling rate conditioned on \( \hat{p} \cdot \hat{e}_3 \) is similar to that conditioned on \( \hat{p} \cdot \hat{e}_1 \); the dependence is stronger, however, for \( \hat{p} \cdot \hat{e}_3 \) because the fiber experiences the full effect of both the strain and the vorticity simultaneously when it is aligned with \( \hat{e}_3 \). This result suggests that the intermittent tumbling rate of fibers may be linked back to the geometric information contained in the relative orientation of a fiber with the local velocity gradient. Finally, the new experimental technique we describe enables us to extract the flow motion near particles in a fully turbulent three-dimensional system, and has the potential to be applied to many other problems such as large particles where high particle Reynolds numbers make simulations much more difficult or the flow field generated by active anisotropic particles such as bacteria.
Chapter 3

Complex-Shaped Particles in Homogeneous Isotropic Turbulence

This Chapter summarizes the results from three published papers: *Measurements of the Solid-Body Rotation of Anisotropic Particles in 3D Turbulence*, *New Journal of Physics* (Marcus et al., 2014); *Methods for Measuring the Orientation and Rotation Rate of 3D-Printed Particles in Turbulence*, *Journal of Visualized Experiments* (Cole et al., 2016) and *Preferential Rotation of Chiral Dipoles in Isotropic Turbulence*, *Physical Review Letters* (Kramel et al., 2016), in collaboration with Saskia Tympel and Federico Toschi from the TU Eindhoven, NL.

**Disclaimer:** The experiments and results presented in Sec. 3.1 through Sec. 3.4 form the foundation of my doctoral work. Together with Guy G. Marcus (2013), we conducted the first experiments with 3D-printed particles in isotropic turbulence. He developed some of the orientation-finding algorithms which I continued using and modifying. The work presented in Sec. 3.4.3 was done with the help of Brendan Cole (2016). We tested various 3D printing technologies for particle fabrication and investigated the size dependence of the dynamics of non-spherical particles in isotropic turbulence.

The goal of the experiments in the first part of this chapter is to study the orientation distributions and rotational dynamics of particles in turbulence, with shapes far more complex than
fibers (see Fig. 3.1). The particles considered here are composites of individual fibers, which we call ramified particles. We use them as models for axisymmetric ellipsoids that enable us to study particle dynamics across the full range of aspect ratios. It can be shown that small ramified particles rotate like their equivalent ellipsoid (Marcus et al., 2014), e.g. a cross or triad rotates like a disk and a jack or tetrad rotates like a sphere. This specific design has several advantages over actual ellipsoids. We can accurately measure their full solid-body rotation rate, which is not readily accessible from experiments with actual ellipsoids. Other researchers have dealt with this problem by painting patterns on spheres (Mathai et al., 2016) to reconstruct their 3D orientation and measure rotation rates or embedded small mirrors inside the particles and measured the deflection of a laser beam for the same purpose (Frish and Webb, 1981). Measurements of their solid-body rotation rate can yield new insights into the dynamics of non-spherical particles in turbulence and the effects of particle size. Moreover, we can make use of the well developed slender-body theory, although it does neglect interactions between arms.

In the second part of this chapter, we introduce a new particle shape which shows preferential rotation in three dimensional homogeneous isotropic turbulence. We call these particles chiral dipoles and they consist of a rod with two helices of opposite handedness, one at each end. Measuring the rotations of these particles allows us to experimentally observe the mean stretching experienced by orientable elements in turbulent fluid flows.

3.1 Experiments I - Ramified Particles

All experiments in this chapter are conducted in the same octagonal tank as described in Sec. 2.1.1, where two oscillating grids generate homogenous isotropic turbulence (see Fig. 2.1 (a)). The measurements were performed at a grid frequency of 1 Hz and 3 Hz, resulting in Taylor Reynolds numbers between $Re_\lambda = 91$ and $Re_\lambda = 220$. Table 3.1 summarizes the characteristic turbulence quantities. We do not aim at resolving velocity gradients, but rather focus on the particle shape and size dependence of their orientations and rotations. The particle size and the number of particles available in these experiments is a major constraint and requires a different data acquisition approach compared to the scanning system of Sec. 2.1.2. In order to acquire enough particle trajectories to converge the statistics, we need a large detection volume and the

1 branched
We use a volume illumination created by two orthogonal laser beams, each reflected back upon itself, providing four directions of illumination to minimize self shadowing of particles. Depending on the particle size, we adjusted the size of the detection volume and use shorter or longer focal length lenses of 100 mm and 200 mm, respectively. We trade the three fast Photron cameras for four slower Mikrotron EoSens CL 1364 cameras with a similar resolution of $1280 \times 1024$ pixels at a maximum frame rate of 500 Hz. The additional fourth camera is located in the same lateral plane as the third camera, with a 90° angle between them, and looking up towards the detection volume at an angle of 21°. The cameras are equipped with either 200 mm macro lenses (Nikkor) for small particles, or 100 mm macro lenses (Sigma) for large particles. Having an additional camera is essential for reliably reconstructing the 3D orientation of complex-shaped particles.

The big advantage of these cameras is that we can use a custom real-time image compression system that enables us to record directly to hard drive (Wijesinghe, 2012). There is one com-

### Table 3.1: Summary of experimental parameters.

<table>
<thead>
<tr>
<th>Grid Freq. [Hz]</th>
<th>$Re_{\lambda}$</th>
<th>$L$ [mm]</th>
<th>$\bar{u}$ [mm/s]</th>
<th>$\langle \epsilon \rangle$ [mm$^2$/s$^3$]</th>
<th>$\nu$ [mm$^2$/s]</th>
<th>$\eta$ [mm]</th>
<th>$\tau_{\eta}$ [s]</th>
</tr>
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<tbody>
<tr>
<td>Crosses and Jacks (FDM)</td>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td>91</td>
<td>60</td>
<td>20.0</td>
<td>133</td>
<td>2.17</td>
<td>0.526</td>
<td>0.128</td>
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<tr>
<td>3</td>
<td>120</td>
<td>94</td>
<td>20.4</td>
<td>90</td>
<td>2.00</td>
<td>0.546</td>
<td>0.149</td>
</tr>
<tr>
<td>3</td>
<td>183</td>
<td>80</td>
<td>55.6</td>
<td>2150</td>
<td>2.00</td>
<td>0.247</td>
<td>0.030</td>
</tr>
<tr>
<td>Tetrads (FDM)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>94</td>
<td>20.4</td>
<td>90</td>
<td>2.00</td>
<td>0.546</td>
<td>0.149</td>
</tr>
<tr>
<td>3</td>
<td>183</td>
<td>80</td>
<td>55.6</td>
<td>2150</td>
<td>2.00</td>
<td>0.247</td>
<td>0.030</td>
</tr>
<tr>
<td>3</td>
<td>220</td>
<td>90</td>
<td>62.2</td>
<td>2600</td>
<td>1.72</td>
<td>0.46</td>
<td>0.13</td>
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<tr>
<td>Tetrads (STL)</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>94</td>
<td>20.4</td>
<td>90</td>
<td>2.00</td>
<td>0.546</td>
<td>0.149</td>
</tr>
<tr>
<td>3</td>
<td>220</td>
<td>90</td>
<td>62.2</td>
<td>2600</td>
<td>1.72</td>
<td>0.46</td>
<td>0.13</td>
</tr>
</tbody>
</table>

$Re_{\lambda} = \sqrt{\frac{\bar{u}L}{\nu}}$, Taylor Reynolds number; $L = \bar{u}^3/\langle \epsilon \rangle$, energy input length scale; $\bar{u} = ((u_i u_i)/3)^{1/2}$, root-mean-square fluid velocity; $\langle \epsilon \rangle$, mean energy dissipation rate; $\nu$, kinematic viscosity; $\eta = (\nu^3/\langle \epsilon \rangle)^{1/4}$, Kolmogorov length scale; $\tau_{\eta} = (\nu/\langle \epsilon \rangle)^{1/2}$, Kolmogorov time scale.
Figure 3.1: A collection of 3D-printed particles, showing a fiber, cross, jack, triad, small medium and large tetrad and a chiral dipole. The large tetrad was printed using a Stereolithography process, all other particles were printed using Fused Deposition Modeling.

pression system for each camera, equipped with a FPGA (field-programmable gate array) with 10 memory blocks (FIFO, first in first out) that can store approximately $1 \times 10^5$ pixels. Dark pixels below a set threshold are disregarded and only bright pixels are passed through to the frame grabber card (Epix PIXCI E4) in a dedicated computer. We are not limited to the internal buffer of cameras or random access memory (RAM) of computers anymore and can acquire continuous data for several days. A custom C++ program called IMPACT serves as interface for the compressed video stream. If the limit of $1 \times 10^5$ bright pixels is exceeded, IMPACT marks the frame with a FIFO overflow and the frame should not be used for further analysis. With an average particle number density of only $5 \times 10^{-3}$ cm$^{-3}$, there was a particle in view less than 20% of the time, which makes the image compression system crucial for these experiments.

3.2 Particle Fabrication - 3D Printing

We use 3D printing to fabricate anisotropic particles of different shapes as shown in Fig. 3.1. The shapes include fibers, crosses, jacks, triads and tetrads. The common feature of all these particles is that they consist of individual fibers, connected at a central point. The following

$^2$Chiral dipoles and triads will be discussed separately in Sec. 3.5 and Sec. 4.1.4, respectively
sections describe the fabrication process in detail (see also Appendix 5.2.1).

### 3.2.1 Crosses and Jacks

We fabricate crosses, two orthogonal fibers, and jacks, three mutually orthogonal fibers, in order to complete the measurements of the dynamics of axisymmetric ellipsoids across the entire range of aspect ratios. To print at both high resolution and in high quantity, we use a Connex 500 with the VeroClear print material to make 10,000 of each particle shape, using a process called Fused Deposition Modeling (FDM). Arm lengths are $\ell = 1.5$ mm, and therefore the longest particle dimension is $L = 3$ mm, or $6\eta$ (see Tab. 3.2). The diameter of any given arm of a cross or jack is 300 $\mu$m, the smallest we could achieve while maintaining the structural integrity of the particles. To print particles with such small, cylindrical arms, it is important to ensure that none of the arms are oriented along the build axes of the 3D printer. Arms lying along the vertical build axis show defects and often break off, while arms lying in the horizontal plane tend to flatten. Another difficulty in printing $O(10^4)$ particles is removing the support material. Connex printers use a different material for the support structure than for the particles. The support material can be partially dissolved using a strong base solution (e.g., NaOH) without affecting the particles themselves. We find that using an ultrasonic bath makes the removal process much more efficient, and the particles can be filtered out of the solution with almost no loss. (Note: When particles are exposed to NaOH for too long, they become soft and can deform, proper storage in density matched fluid is therefore recommended.)

<table>
<thead>
<tr>
<th>Particle (size)</th>
<th>$Re_\lambda$</th>
<th>$L/\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crosses &amp; Jacks ($L = 3$ mm)</td>
<td>91</td>
<td>6</td>
</tr>
<tr>
<td>Small Tetrads ($L = 9$ mm)</td>
<td>120</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>183</td>
<td>36</td>
</tr>
<tr>
<td>Medium Tetrads ($L = 18$ mm)</td>
<td>120</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>183</td>
<td>73</td>
</tr>
<tr>
<td>Large Tetrads ($L = 50$ mm)</td>
<td>120</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>220</td>
<td>237</td>
</tr>
</tbody>
</table>

*Table 3.2: Absolute and relative size of the particles used in the experiments.*
We choose the VeroClear print material, whose bulk density is quoted at 1.17 g cm\(^{-3}\). However, we find that the manufacturer’s quote differs significantly from the density of the fluid in which the particles are neutrally buoyant. After particles are immersed in the fluid for several hours, we find that different particles are neutrally buoyant at slightly different fluid densities ranging from 1.21 to 1.23 g cm\(^{-3}\). In order to have neutrally buoyant particles, the density of the fluid is matched to the population averaged particle density of 1.22 g cm\(^{-3}\) (by adding calcium chloride). The kinematic viscosity of the CaCl\(_2\) solution is \(\nu = 2.17 \text{ mm}^2 \text{ s}^{-1}\) as measured by a glass capillary viscometer. More work is needed to understand the mass density distribution inside 3D-printed objects, but our particles are sufficiently density matched that their rotations should accurately represent the neutrally buoyant case. The accelerations of large spheres are only weakly affected by density differences of 6\% from the fluid [Voth et al., 2002], so our particles, formed from thin fibers, should show much smaller effects from density mismatch. To make the particles fluorescent, they were placed in a high concentration Rhodamine-B solution at elevated temperature (60\(^\circ\)C - 80\(^\circ\)C) for several hours (see Appendix 5.2.1 for more details).

### 3.2.2 Tetrads

Tetrads, four fibers in tetrahedral symmetry, were printed at three different sizes: small (4.5 mm x 0.5 mm), medium (9 mm x 0.9 mm) and large (25 mm x 2 mm). Separate experiments at two turbulent length scales and three particle sizes give us measurements throughout the inertial range, starting with the small tetrads at 17\(\eta\) and up to 237\(\eta\) for the large tetrads (see Tab. 3.2). Compared to jacks, tetrads have two fewer arms, while maintaining the spherical symmetry, which reduces the chance of arms visually blocking each other.

Small and medium tetrads, were fabricated using the same FDM process and print material as described above. We are able to fabricate the large tetrads in house with a commercially available 3D printer (Formlabs, Form1+), using a process called stereolithography (STL). Due to this different process and print material, the large tetrads have a slightly lower density, on average 1.17 g cm\(^{-3}\), whereas the other particles have similar densities as crosses and jacks (1.21 g cm\(^{-3}\)). The viscosity of the fluid used for either density was measured to be \(\nu = 1.72 \text{ mm}^2 \text{ s}^{-1}\) and \(\nu = 2.00 \text{ mm}^2 \text{ s}^{-1}\), respectively.
3.3 Improved Orientation Finding Algorithm

Measuring the rotations of ramified particles requires a new orientation-finding algorithm. Compared to fibers, where the 3D orientation is determined by two Euler angles $\phi$, $\theta$, and can be calculated from the 2D angles as described in Sec. 2.2.2, the 3D orientation of ramified particles is determined by three Euler angles. An example of a jack, imaged by all four cameras, is shown in Fig. 3.2. When a particle is visible on all four cameras, we first find the three dimensional position of the particle using standard stereomatching methods (Ouellette et al., 2006). In order to determine the orientation of the particle from the same set of images, we developed a non-linear optimization routine. Any orientation is specified by a rotation matrix, $O$, which can be parametrized by three Euler angles $\phi$, $\theta$, $\psi$ (Goldstein et al., 2002). From the measurement of the particle’s position and an initial guess of its orientation, we project a computer-generated model particle onto the image plane of each camera using the camera calibration parameters (Tsai, 1987). The total difference in intensity between the model images and the experimental images provides a residual, the loss function, which is minimized by the non-linear optimization in Euler angle space.

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$^3$The third Euler angle is an arbitrary rotation around the symmetry axis of the fiber.
To minimize computational time, we only project the endpoints of each arm of the model onto the image planes and approximate the intensity distributions in two dimensions. The intensity of an arm is modeled by a Gaussian distribution along the width of each arm and a Fermi distribution along the length of the arm. For jacks and crosses, each arm in the model has an identical intensity distribution. The arms in the experimental images, however, are not of uniform intensity, as can be seen in Fig. 3.2. The observed intensity has a non-trivial dependence on the angles between the arms, the illumination, and the viewing direction (Parsa, 2013), which has not yet been included in our model. Nonetheless, we find that the simple model is adequate enough for our purposes.

The orientation-finding algorithm must be seeded with an initial guess for the orientation. Except for the first frame of a particle trajectory, we use the orientation from the previous frame as initial guess, since rotations between frames are $O(10^{-2})$ radians. The initial guess for the beginning of a trajectory is reliably generated using an optical tomographic reconstruction algorithm (Elsinga et al., 2006). We compare tomographic reconstruction as a method for finding particle orientation, but find it to be both less accurate and computationally more expensive than our own algorithm. Further analysis has shown that our orientation-finding algorithm is stable and reliable using a random initial orientation, which reduces the computational time even more.

The three Euler angles of a jack give 1 of the 24 orientations related by symmetry. To see this, consider 1 of the 6 arms of a jack and define it by the vector $\hat{p}' = \hat{z}$; there are 4 symmetrically identical orientations obtained by rotations of $\pi/2$ about the $z$ axis. There are 4 such orientations for each of the 6 arms, for a total for 24. A cross has 8 identical orientations, and a fiber has 2. We ensure that we have a consistent series of orientations along individual trajectories by comparing the orientation found in each frame with that from the previous frame, choosing the orientation of the particle that produces the smallest total rotation between frames.

Figure 3.3 (a) shows a representative track of a cross, which is $5.7\tau_\eta$ long. It demonstrates the effectiveness of our algorithm at determining the full range of orientations. At various points along the track, the rotation of the cross about a vector nearly coplanar with its two arms is clearly visible. In Fig. 3.3 (b) we also show an example trajectory of a jack that is $17.5\tau_\eta$.

Choosing an appropriate width of the Gaussian intensity distribution is crucial for this algorithm. We had good success with a standard deviation of $\sigma = \sqrt{2D}$. 

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[^4]: Choosing an appropriate width of the Gaussian intensity distribution is crucial for this algorithm. We had good success with a standard deviation of $\sigma = \sqrt{2D}$. 

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Chapter 3 - Complex-Shaped Particles in Homogeneous Isotropic Turbulence

Figure 3.3: (a) A reconstructed trajectory of a cross in three-dimensional turbulence. The two different color sheets trace out the path of the particle through space and time. The length of the particle track is 336 frames, or $5.7 \tau_n$. A cross is shown every 15 frames. (b) A reconstructed trajectory of a jack in three dimensional turbulence. The three different colors distinguish the arms of the jack and trace out their path as the particle rotates. The dark green line denotes the trajectory of the jacks center. The length of the particle track is 1025 frames, or $17.5 \tau_n$. A jack is shown every 50 frames. (Note: neither the crosses nor the jacks shown above are drawn to scale.)

The orientation of a ramified particle is again described by a unit vector $\hat{p}$, which is normal to the plane of the arms for crosses and along any of the arms for a jack and tetrad. Orientation measurements at each time step enable us to calculate the particle’s tumbling rate $\dot{p}$. For these particle shapes, we can also measure the full solid-body rotation rate vector $\Omega_s$. One method for doing this has been described in Zimmermann et al. (2011). We take a different approach using the tools we already developed for the least-squares optimization in Euler angle space. The problem can be framed as finding the initial orientation matrix $O(t_i)$ and the finite rotation matrix over a single time step $R$ that together give the particle orientation matrix as function of time

$$O(t) = R^{\frac{t-t_i}{\tau_f}}O(t_i),$$  \hspace{1cm} (3.1)

where $\tau_f$ is the period between frames and $t_i$ is the time of the initial frame. A non-linear least squares fit is used to find the six Euler angles defining the matrices $O(t_i)$ and $R$ that yield the best match to the measured orientation matrices\[^5\]. In accordance with Euler’s theorem (Goldstein et al., 2002), $R$ can be decomposed as a rotation by an angle $\Phi$ about the solid-body rotation

\[^5\]Assuming small rotations between consecutive time steps, the process of finding the rotation matrix can be framed as linear least squares problem, which significantly decreases computational time.
axis $\hat{\Omega}_s$ from which we obtain the magnitude of the solid-body rotation rate $\Omega_s = \Phi/\tau_f$.

The solid-body rotation rate $\Omega_s$ is related to the tumbling rate by $\dot{p} = \Omega_s \times \hat{p}$. The difference between the two quantities is that $\dot{p}$ does not depend on the vector component of $\Omega_s$ lying along $\hat{p}$. We use this relationship to determine the tumbling rate from measurements of the particle orientation and solid-body rotation rate.

In order to correct the mean-square tumbling rate for the contributions from orientation measurement errors, we need to measure $|\dot{p}|$ over a range of fit-lengths, $\tau_{fit}$. By extrapolating $|\dot{p}|$ for $\tau_{fit} \to 0$ for different fit-lengths, we can obtain an estimate of the error.

### 3.4 Results I

#### 3.4.1 Alignment

We can measure the preferential alignment of a particle directly by measuring the angle between the orientation of the particle and its solid-body rotation rate vector. In Fig. 3.4, we plot the PDF of the magnitude of the cosine of this angle, $|\hat{p} \cdot \hat{\Omega}_s|$, for both crosses and jacks, and
compare them with numerical simulations. The peak near $\hat{\mathbf{p}} \cdot \hat{\Omega}_s = 0$ for crosses in Fig. 3.4 (a) shows that disks preferentially align with $\hat{\mathbf{p}}$ perpendicular to $\hat{\Omega}_s$. Here, $\hat{\mathbf{p}}$ is normal to the arms of the cross, or the plane of the disk. Figure 3.4 (b) confirms the same story as Fig. 3.4 (a). It shows that an arm of a cross, $\hat{\mathbf{p}}'$, is preferentially aligned with the solid-body rotation rate vector. The height of the peak in Fig. 3.4 (b) is lower than that in Fig. 3.4 (a) because any vector in the plane of the disk is equally likely to align with $\hat{\Omega}_s$. This means that in a turbulent flow, disks preferentially rotate like a coin spun on its edge upon a table rather than a Frisbee rotating in flight. For jacks, Fig. 3.4 (c), there is no preferential alignment because they rotate like spheres.

Numerical results and experiment are in quite good agreement. The deviations near $\hat{\mathbf{p}} \cdot \hat{\Omega}_s = 0$ for crosses are likely the result of combined measurement error in the solid-body rotation rate vector and the orientation unit vector. Our ability to resolve the peak in the PDF is limited by the measurement error in $\hat{\mathbf{p}} \cdot \hat{\Omega}_s$, which would appear as horizontal error bars in Fig. 3.4 (a) and rounds off the peak. Recent numerical work has studied the preferential alignment of disks with the direction of fluid vorticity (Gustavsson et al., 2014). Their results are in qualitative agreement with our results on the alignment of disks with their own solid-body rotation rate vector.

The observed alignment in Fig. 3.4 seems natural if it is thought of as a result of the Lagrangian stretching of the fluid. It will align both the vorticity and the longest axis of a particle with the stretching direction, as defined by the maximum eigenvector of the left Cauchy-Green deformation tensor (Ni et al., 2014). This is also the direction vorticity is preferentially aligned with. For disks, where $\hat{\mathbf{p}}$ is perpendicular to the long axis, it follows that $\hat{\mathbf{p}}$ is also preferentially aligned perpendicular to vorticity (Reminder: fibers have $\hat{\mathbf{p}}$ preferentially aligned parallel with vorticity). Once crosses are correlated with the turbulence, their solid-body rotation rate will be dominated by vorticity and $\hat{\mathbf{p}}$ will be preferentially aligned perpendicular to the solid-body rotation rate vector.

### 3.4.2 Rotation Rates

The preferential alignment of crosses with $\hat{\mathbf{p}}$ perpendicular to their solid-body rotation rate vector creates a higher tumbling rate when compared to fibers, since $\hat{\mathbf{p}} = \Omega_s \times \hat{\mathbf{p}}$. This is
consistent with the data plotted in Fig. 3.5 where we show the mean-square tumbling rate $\langle \ddot{p}_i \dot{p}_i \rangle$ normalized by the Kolmogorov timescale as function of aspect ratio $\kappa$.

Previous work by Parsa et al. (2012) showed that the mean-square tumbling rate for axisymmetric, prolate ellipsoids is strongly affected by the alignment with the velocity gradients. They found agreement between simulations and experimental measurements of fibers, but particles with a wider range of aspect ratios could not be measured experimentally with the tools available at that time. We have now measured the mean-square tumbling rate of crosses and jacks using the techniques described in Sec. 3.4. Our measurements show crosses tumble at a considerably higher rate than jacks or fibers, but still less than predicted for randomly oriented particles. There is good agreement with the simulations across the full range of aspect ratios.

Figure 3.5 shows the mean-square tumbling rate as function of aspect ratio for randomly oriented axisymmetric ellipsoids. While the tumbling rate of jacks is unmodified from the randomly oriented case, both fibers and crosses show lower tumbling rates due to the effects of alignment by the velocity gradients. Shin and Koch were the first to notice that the tumbling rate of fibers that are correlated with a turbulent flow is reduced in comparison to that of randomly oriented fibers (Shin and Koch, 2005). More recent numerical work showed that the effects of
alignment persist across the full range of aspect ratios (Parsa et al., 2012). The leading-order effects are contained in the Lagrangian three-point correlations of the velocity gradients, which imply higher tumbling rates for disks than for fibers (Gustavsson et al., 2014). As shown in Sec. 2.4.2 and a number of other studies on the dynamics of fibers (Pumir and Wilkinson, 2011; Chevillard and Meneveau, 2013; Wilkinson and Kennard, 2012; Ni et al., 2014), the alignment of fibers with the vorticity can be used to explain the lower tumbling rate of fibers, because the component of the vorticity along the fiber axis does not contribute to its tumbling rate. Numerical simulations of disks show that they align with their symmetry axis perpendicular to the vorticity which explains a higher tumbling rate for disks than for fibers (Gustavsson et al., 2014).

Although disks tumble faster than fibers, they still tumble slightly slower than if they were randomly oriented, as seen in Fig. 3.5. Since disks have a much smaller difference from the randomly oriented case, one might conclude that disks are less strongly aligned with the velocity gradients than fibers. However, this is largely a result of the way \( \hat{p} \) is defined. We measure the normalized mean-square tumbling rate of a unit vector along one of the arms of a cross as \( 0.12 \pm 0.02 \), which is the same as measurements of fibers within experimental error. This explicitly indicates that crosses (disks) are also strongly aligned by turbulence, and it is only the definition of \( \hat{p} \) being perpendicular to the arms that increases the mean-square tumbling rate. It is not easy to directly compare the degree of alignment of fibers and disks because disks have an entire plane that can align with the stretching direction, whereas fibers only have a director. The picture that emerges from our data and previous work is that anisotropic particles align with the Lagrangian stretching direction, and this suppresses their tumbling rate for all aspect ratios, except nearly spherical, oblate ellipsoids. The amount of suppression is strongly dependent on the definition of the tumbling rate.

For experiments in turbulence, it is always challenging to measure the mean energy dissipation rate, which appears in the normalization of the vertical axis in Fig. 3.5 via \( \tau_\eta = (\nu/\langle \epsilon \rangle)^{1/2} \). Our attempt to make independent measurements using non-fluorescent tracer particles did not succeed because of the reduced light scattering in density-matched CaCl\(_2\) solution. However, the measurements of the rotations of jacks provide a new way to determine the mean energy dissipation rate. Because jacks rotate like spheres, they give a direct measurement of the vorticity, and in isotropic turbulence, the vorticity is directly related to the mean energy dissipation
rate through $\langle \Omega_{ij} \Omega_{ij} \rangle = \langle S_{ij} S_{ij} \rangle = \langle \epsilon \rangle/(2\nu)$. This implies that for spheres $\langle \dot{p}_i \dot{p}_i \rangle \tau_0^2 = 1/6$ (see Eq. 2.3). We use this to determine the mean energy dissipation rate, which makes our jack data at $\kappa = 1$ match the simulations by definition, and the agreement of the cross data at $\kappa = 0.1$ with numerical simulations is an independent result of the measurements.

From the measured solid-body rotation rates of crosses and jacks, we obtain the probability density function (PDF) of the squared tumbling rate, which is shown in Fig. 3.6. Also shown is the PDF of spheres, obtained from direct numerical simulations (Parsa et al., 2012). All three PDFs agree within experimental uncertainties. Numerical work has shown that there should be a slightly higher probability density in the tail of the PDF for fibers and disks than for spheres (Parsa et al., 2012). Within our measurement error, we cannot distinguish between the two PDFs. This is consistent with the numerical data where the mean-square rotation rate differs by no more than a factor of two up to $\dot{p}_i \dot{p}_i / \langle \dot{p}_i \dot{p}_i \rangle = 40$. The error bars shown in Fig. 3.6 account for random error as well as the systematic error that results from the fit-length dependence of the tumbling rate measurements. An additional source of error that is not included in the error bars is the self-shadowing of particles as they pass through certain orientations dependent on the camera configuration. The most dramatic cases are when an entire arm is missing from each of the four cameras, such that a jack, for example, will look like a cross along part of its trajectory. The reduction in accuracy when determining these particular orientations occasionally leads to erroneously high measurements of the solid-body rotation rate,

Figure 3.6: The PDF of the mean-square tumbling rate for our experimental measurements of crosses (red squares) and jacks (blue circles) as well as direct numerical simulations of spheres (solid line).
3.4.3 Inertial Range Scaling

In this section, we present our first measurements of the rotation rates of inertial range particles, aimed at resolving the power-law scaling of the energy cascade. We show that the rotations of tetrads can be used to probe the established inertial-range Kolmogorov scaling of velocity structure functions and that they provide a new, single particle measurement of these structure functions at variable scales.

The energy cascade of homogeneous, isotropic turbulence can be characterized by the moments of the velocity differences. If two points are separated by a vector $\mathbf{r}$, the $n$-th velocity structure function is defined as $D_n(r) = \langle (\mathbf{u}(\mathbf{x}) - \mathbf{u}(\mathbf{x} + \mathbf{r}))^n \rangle = \langle (\Delta \mathbf{u}_r)^n \rangle$. The velocity difference can be split into a longitudinal and transverse component. The longitudinal component, $D_{L_n} = \langle (\Delta \mathbf{u}_r \cdot \hat{\mathbf{r}})^n \rangle$, is the $n$-th moment of the velocity difference along $\mathbf{r}$; the transverse component, $D_{N_n} = \langle (\Delta \mathbf{u}_r \cdot (\hat{\mathbf{x}} - \hat{\mathbf{r}}))^n \rangle$, is the $n$-th moment of the velocity difference perpendicular to $\mathbf{r}$. 

Figure 3.7: mean-square tumbling rate of tetrads and fibers as function of particle size normalized by the Kolmogorov time. Kolmogorov theory is fitted to fibers and tetrads. The dotted purple line indicates the refined similarity theory.

which pushes additional probability density towards the tail of the PDF. This effect is stronger for jacks than for crosses, which may be why the jack PDF is sometimes slightly higher in our measurements at large tumbling rates.
We define the magnitude of the transverse velocity structure function. In general, there are two orthogonal components.

The transverse and longitudinal components are proportional to one another (Eq. 6.28 in Pope (2001)), therefore knowledge of one allows a full description of the energy cascade at the scale $r = |r|$. Using dimensional analysis, as in traditional Kolmogorov (1941) theory, it is straightforward to show that

$$D_{N_n} \propto D_{L_n} = C_n \langle \epsilon \rangle r^{n/3}.$$  \hspace{1cm} (3.2)

Here, $C_n$ is an approximately universal constant (Sreenivasan, 1995) that must in general be found experimentally for each moment and $\langle \epsilon \rangle$ is the mean energy dissipation rate. When scale-dependent energy fluctuations are taken into account, a similar analysis (so-called refined similarity theory (Kolmogorov, 1962)) can be performed, and one finds that

$$D_{L_n} = C_n \langle \epsilon \rangle r^{\zeta_n},$$  \hspace{1cm} (3.3)

where $\zeta_n = \frac{1}{3} n (1 - \frac{1}{6} \mu (n - 3))$. The intermittency exponent $\mu$ has been found experimentally to be approximately equal to $0.25 \pm 0.05$ (Sreenivasan and Kailasnath, 1993).

The magnitude of the solid-body rotation rate $\Omega_s = |\Omega_s|$ of a tetrad of size $r$ is determined by the transverse velocity difference at its tips, and therefore

$$\langle \Omega_s^n \rangle = \frac{2 D_{N_n}}{r^n} \propto \frac{r^{\zeta_n}}{r^n} = r^{\zeta_n - n},$$  \hspace{1cm} (3.4)

showing the $n$-th moment of the rotation rate of a tetrad follows a power law just as the velocity structure functions do.

Figure 3.7 compares the mean-square tumbling rate of tetrads with fibers from previous experiments (Parsa and Voth, 2014). Within the inertial range ($r/\eta \geq 15$), there is strong qualitative agreement between the scaling of both types of particles with size. However, unlike fibers which show preferential alignment with the local velocity gradients, tetrads do not have a preferred orientation, resulting in higher tumbling rates. Tetrads are perfectly suited for these measurements: they rotate just like spheres while at the same time minimizing the volume fraction and surface interactions with the fluid. The qualitative agreement of the measured tumbling rates can be compared to the power law predicted by the Kolmogorov theory.

The solid lines in Fig. 3.7 are functions of the form $A(r/\eta)^{4/3}$ fitted to measurements of tetrads and fibers. We have also included, for comparison, the prediction of refined similarity theory, of
the form $A(r/\eta)^{\zeta_2 - 2}$, where $\zeta_2 \approx 0.7$. The difference in the constant $A$ is a result of the different particle geometry and the corresponding alignment effects. Parsa (2013) has shown that for randomly oriented fibers (this should hold for tetrads), this constant equals $108/35 \ C_2$, which together with $C_2 = 2$ equals to $A \approx 6.2$.

More work needs to be done on this topic, but these particles enable interesting measurements and have the potential to shed new insight into the scale dependence of turbulence.

3.5 Experiments II - Chiral Dipoles

An incompressible turbulent fluid flow produces exponential stretching of material line segments. Batchelor (1952) conjectured that this must occur, and subsequent work has confirmed his conjecture, determining that their average exponential growth rate is $\zeta = \langle \ell_i S_{ij} \ell_j \rangle \approx 0.12 \tau^{-1}_\eta$, where $S_{ij}$ is the strain rate tensor, $\tau_\eta$ the Kolmogorov time, and $\ell_i$ is the orientation unit vector of the material line (Girimaji and Pope, 1990a; Goto and Kida, 2007; Byron et al., 2015). One might wonder how an incompressible flow can stretch material lines on average since every fluid element must combine extension with contraction to maintain constant volume. The answer lies in the Lagrangian advection of material lines, which causes them to preferentially orient along the extensional directions of the velocity gradient tensor.

The rate of separation of two material points is the longitudinal velocity difference, $D_L = \Delta u_r \cdot \hat{r}$. Randomly sampled points have $\langle \Delta u_r \cdot \hat{r} \rangle = 0$ due to incompressibility. To obtain insights into the dynamics of turbulence from longitudinal velocity differences at random orientations, one needs to consider higher moments. For example, the third moment in the inertial range is related to the mean energy dissipation rate by Kolmogorov’s 4/5 law: $D_{LLL} = \langle (\Delta u_r \cdot \hat{r})^3 \rangle = -\frac{4}{5} \langle \epsilon \rangle r$. However, two points advected by the flow develop a preferential orientation. In this oriented Lagrangian reference frame, the mean velocity difference is positive; in particular, for small $r$ the mean velocity difference is $\langle \Delta u_r \cdot \hat{r} \rangle / r = \zeta$.

Studying the stretching of material elements has led to many insights into the dynamics of turbulence. Since Richardson, many studies have explored two particle dispersion, focusing on the rate of separation of two particles that are initially close together (Salazar and Collins, 2009). The ‘advected delta-vec’ system (Li and Meneveau, 2005, 2006), in which velocity differences are sampled between two points advected in the flow but constrained to maintain fixed distance, has
illuminated the development of intermittency in turbulent flows. The positive mean stretching rate of vorticity in turbulence has been shown to result from vorticity becoming aligned with the extensional directions of the velocity gradient tensor (Ashurst et al., 1987; Tsinober et al., 1997; Ooi et al., 1999).

We introduce a new particle shape which shows preferential rotation in three dimensional homogeneous isotropic turbulence. We call these particles chiral dipoles and they consist of a rod with two helices of opposite handedness, one at each end. High aspect ratio chiral dipoles preferentially align with their long axis along the extensional eigenvectors of the strain rate tensor, and the helical ends respond to the extensional strain rate with a mean spinning rate that is non-zero. Measuring the rotations of these particles with multiple high speed cameras allows us to experimentally observe the mean stretching experienced by orientable elements in turbulent fluid flows. We use Stokesian Dynamics simulations of chiral dipoles in pure strain flow to quantify the dependence of spinning rate on particle shape. Based on their response to pure strain, we build a model for the spinning rate of small chiral dipoles using velocity gradients along Lagrangian trajectories from high resolution direct numerical simulations.

The spinning rate statistics determined with this model show surprisingly good agreement with the experimental measurements of much larger chiral dipoles, whose rotations were tracked in a turbulent flow between oscillating grids (see Sec. 3.1). The grid frequency was varied between 1 Hz and 3 Hz, which resulted in a Taylor Reynolds number $Re_\lambda = 120$ and 183, respectively. Spherical tracer particles with a diameter of 150 µm were used to measure the root-mean-square fluid velocity and to calculate the third order longitudinal structure functions from which we determine the mean energy dissipation rate. The characteristic turbulence parameters are summarized in Table 3.3.

<table>
<thead>
<tr>
<th>Grid Freq. [Hz]</th>
<th>$Re_\lambda$</th>
<th>$\mathcal{L}$ [mm]</th>
<th>$\bar{u}$ [mm/s]</th>
<th>$\langle \epsilon \rangle$ [mm$^2$/s$^3$]</th>
<th>$\nu$ [mm$^2$/s]</th>
<th>$\eta$ [mm]</th>
<th>$\tau_\eta$ [s]</th>
</tr>
</thead>
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</tr>
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<td>80</td>
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<td>2150</td>
<td>2.00</td>
<td>0.247</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Table 3.3: Summary of experimental parameters. $Re_\lambda = \sqrt{\frac{15 u \mathcal{L}}{\nu}}$, Taylor Reynolds number; $\mathcal{L} = \langle u_i^2 \rangle^{1/2}$, energy input length scale; $\bar{u} = \langle u_i u_i \rangle / 3$, root-mean-square fluid velocity; $\langle \epsilon \rangle$, mean energy dissipation rate; $\nu$, kinematic viscosity; $\eta = \langle \nu^3 / \langle \epsilon \rangle \rangle^{1/4}$, Kolmogorov length scale; $\tau_\eta = \langle \nu / \langle \epsilon \rangle \rangle^{1/2}$, Kolmogorov time scale.
3.5.1 Chiral Dipoles

An image of a chiral dipole is shown in Fig. 3.8(a). We call these particles chiral dipoles because of their similarity to electrical dipoles. The total chirality of the particle is zero; however, the two ends with opposite chirality are separated by a fixed distance. The chiral dipole vector, $\hat{d}$, points from the right-handed end to the left-handed end. A similar particle design was mentioned by Purcell (1997), predicting that such a particle would sink through quiescent fluid without spinning.

The particle shape can be specified by the pitch of the helices and the aspect ratio, $\kappa = L/D$, where $L$ is the length of the particle and $D$ is the diameter of the helices, as shown in Fig. 3.9. The pitch is defined as the length along the helix axis for a complete turn divided by $D$. The particle should have a high aspect ratio, $\kappa \gg 1$ to ensure good alignment with the extensional eigenvectors of the strain rate tensor (Parsa et al., 2012).
We use 3D printing to fabricate 2000 chiral dipoles with aspect ratio $\kappa = 20$, pitch 2 and a largest dimension of $L = 20$ mm. The particle size $L$ corresponds to $35\eta$ and $72\eta$, depending on the Reynolds number, which places them in the inertial range of our turbulent flow. The dimensions were chosen because the 3D printer is not able to mass produce structurally stable particles with smallest dimension less than $s = 0.8$ mm (see Fig. 3.9), and we also need $D \gg s$ in order to allow optical reconstruction of the particle’s 3D orientation.

With a slight modification of the orientation-finding approach described in Sec. 3.3 we can reliably reconstruct a chiral dipoles 3D orientation. Opposed to ramified particles, where each arm is connected at a central point, we generate a discretized model of a chiral dipole by connecting straight segments end-to-end, which is shown in Fig. 3.8 (b). To demonstrate the success of this method, we plot the three Euler angles along a particle trajectory in Fig. 3.8 (c). The algorithm works well for complex-shaped particles like chiral dipoles, is stable with respect to small variations in the particle shape and it can be easily modified to measure other particles made of slender filaments. The example trajectory in Fig. 3.8 (c) also indicates the typical amount of measurement noise. To estimate the accuracy of the orientation-finding algorithm, we fit a second-order polynomial curve to segments of trajectories and calculate the root-mean-square difference between measured Euler angles and fit. This yields an approximate accuracy of 0.02 radians, averaged over all three Euler angles.

3.5.2 Stokesian Dynamics Simulations

We use Stokesian Dynamics simulations (Brady and Bossis, 1988) to quantify the rotational motion of chiral dipoles in a pure strain flow in order to help choose the optimal shape before the fabrication process. A complex-shaped particle like a chiral dipole can be simulated with individual spheres, each fixed in their relative position as shown in Fig. 3.9. The composite particle is allowed to rotate freely while it is subjected to constant velocity gradients. When placed in a pure strain flow, a high aspect ratio chiral dipole tumbles until $\hat{d}$ points along the extensional strain direction, as shown in Fig. 3.10 (a). The strain flow then couples to the chiral

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6Particles were printed with the VeroClear material (dyed with Rhodamine-B), therefore the fluid density had to be increased to $\rho_f = 1.20$ g cm\(^{-3}\) to match the population averaged particle density and ensure neutrally buoyant particles.

7The biggest uncertainty comes from the orientation measurements along $\hat{d}$, for ramified particles, the algorithm achieves an accuracy less than 0.01 radians.
Figure 3.9: Model of a chiral dipole made of 40 spheres as used in the Stokesian Dynamics simulations. The dipole vector $\hat{d}$ points from right-handed helix to left-handed helix.

dipole shape to produce a solid-body rotation rate in the direction of the chiral dipole vector, $\Omega_s = \Omega_d \hat{d}$, where $\Omega_d$ is called the spinning rate.

Figure 3.10 (b) and (c) show the mean spinning rate $\Omega_d$ from these simulations in a two-dimensional pure strain flow with strain rate eigenvalues $\lambda_1 = -\lambda_3$ and $\lambda_2 = 0$. After an initial orientation phase of 5 to 10 times the characteristic strain rate time scale ($\lambda_1^{-1}$), the particle aligns with the extensional eigenvector of the strain rate tensor and begins to spin about its long axis at a rate $\Omega_d$, with the mean value calculated in this aligned state. Figure 3.10 (b) shows that increasing the aspect ratio with constant pitch increases the mean spinning rate of a chiral dipole in a pure strain flow. Figure 3.10 (c) shows the mean spinning rate as function of pitch with constant aspect ratio and suggests that there is an optimal pitch near 3.5. This is consistent with the optimal pitch near $\pi$ found for efficient propulsion by bacterial flagella (Spagnolie and Lauga, 2011). So a particle with pitch near 3.5 and very high aspect ratio should yield the strongest coupling of spinning rate to strain rate.

3.5.3 Direct Numerical Simulations

In addition to the experimental measurements, we use direct numerical simulations (DNS) of homogeneous isotropic turbulence at a Reynolds number of $R_\lambda = 400$ to calculate the motion of a chiral dipole along its trajectory. The simulation volume includes a total of $N^3 = 2048^3$ collocation points and $O(10^7)$ measurements of velocity gradients along Lagrangian particle trajectories for a few large eddy turnover times (Benzi et al., 2009). The characteristic quantities of the simulations are summarized in Tab. 3.4. We can use the velocity gradients from the DNS to integrate Jeffery’s equation (Eq. 1.9) and obtain the orientation valid for a particle in the dissipation range that has been aligned by the flow. Together with the known response to pure
Chapter 3 - Complex-Shaped Particles in Homogeneous Isotropic Turbulence

Figure 3.10: (a) Response of a chiral dipole to a pure strain flow as indicated by the arrows. (b) Mean spinning rate $<\Omega_d>$ as function of the overall aspect ratio, $\kappa = L/D$ (o: pitch = 2). (c) Mean spinning rate as function of helix pitch (o: $\kappa = 10$, □: $\kappa = 16$). The mean spinning rate is measured in the Stokesian Dynamics simulations and is normalized by the largest eigenvalue of the strain rate tensor $\lambda_1$.

strain, we build a model to estimate the spinning rate $\Omega_d$ of small chiral dipoles in isotropic turbulence.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$Re_\lambda$</th>
<th>$\mathcal{L}$</th>
<th>$\bar{u}$</th>
<th>$\langle \epsilon \rangle$</th>
<th>$\eta$</th>
<th>$\tau_\eta$</th>
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<td>0.0028</td>
<td>0.0225</td>
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</table>

Table 3.4: Parameters of the Direct Numerical Simulations. $Re_\lambda = \sqrt{\frac{\mathcal{L} \bar{u}^2}{\nu}}$, Taylor Reynolds number; $\mathcal{L} = \bar{u}^3/\langle \epsilon \rangle$, energy input length scale; $\bar{u} = (\langle u_i u_i \rangle / 3)^{1/2}$, root-mean-square fluid velocity; $\langle \epsilon \rangle$, mean energy dissipation rate; $\eta = (\nu^3/\langle \epsilon \rangle)^{1/4}$, Kolmogorov length scale; $\tau_\eta = (\nu/\langle \epsilon \rangle)^{1/2}$, Kolmogorov time scale; $\nu = 5 \times 10^{-4}$, kinematic viscosity.
3.6 Results II

High aspect ratio chiral dipoles can be approximated by elongated ellipsoids and their tumbling rate can therefore be described by Jeffery’s equation

\[ \dot{d}_i = \Omega_{ij} d_j + \frac{k^2 - 1}{k^2 + 1} (S_{ij} d_j - d_i d_k S_{kl} d_l). \]  

(3.5)

However, Eq. 3.5 is only an approximation for chiral dipoles. Marcos et al. (2009) showed that chiral particles in shear flow experience a translational motion along velocity gradients. Two opposite chiral centers that are spatially separated should show no cross-stream translational motion, but this same mechanism should produce a small torque. For our high aspect ratio chiral dipoles, this torque is negligible compared to the torques captured by Jeffery’s equation.

In addition to tumbling, a chiral dipole spins around \( \hat{d} \) with half of the fluid vorticity \( \omega \) along that direction and since chiral dipoles also couple to the fluid strain, their total spinning rate is given by

\[ \Omega_d = \frac{\omega_i}{2} d_i + \beta d_i S_{ij} d_j \]

\[ = A_S + \beta A_L \]  

(3.6)

In a turbulent flow, the particle becomes aligned by the strain and experiences \( \langle \Omega_d \rangle > 0 \), due to the persistence of strain along \( \hat{d} \). Preferential rotation requires isotropy to be broken, and even though the flow is isotropic on average, instantaneous flow structures are strongly anisotropic due to the stretching they have experienced. Carefully designed particles can couple to this local anisotropy to allow preferential rotation.

The constant \( \beta \) is strongly dependent on the particle shape and describes the strength of the coupling of the spinning rate to the strain rate. An approximate value for \( \beta \) for our particle shape was obtained from the Stokesian Dynamics simulations, where \( \beta = 0.39 \). This value of \( \beta \) is only valid for small particles since it assumes Stokes flow around the particle.

We adopt the compact notation developed for the analysis of the ‘advected delta-vee’ system by Li and Meneveau (2005, 2006) to define the longitudinal and transverse velocity gradients with respect to the particle. The longitudinal component is \( A_L = d_i S_{ij} d_j \) and the transverse component is the magnitude of the tumbling rate \( A_N = (\dot{d}_i \dot{d}_i)^{1/2} \). We can complete the picture if we include the spinning due to the fluid vorticity \( A_S = \frac{1}{2} \omega_i d_i \).
Figure 3.11: (a) PDF of the spinning rate $\Omega_d$ normalized by the standard deviation for both Reynolds numbers (blue $\circ$, $R_\lambda = 120$ and red $\circ$, $R_\lambda = 183$) and simulations (green solid line). The gray dashed lines show the experimental data mirrored around 0. (b) PDF of the individual contributions from strain, $A_L = d_i S_{ij} d_j$ (red dashed-dotted line) and vorticity, $A_S = \frac{1}{2} \omega \cdot \hat{d}$ (blue dashed line) to the spinning rate $\Omega_d$ (solid green line). The standard deviation of the simulations is $\langle \Omega_d^2 \rangle^{1/2} = 0.85 \tau_n^{-1}$.

3.6.1 Material Line Stretching Rate

The instantaneous spinning rate $\Omega_d$ (Eq. 3.6) depends on both the material line stretching rate $A_L$, and the vorticity component along the particle axis $A_S$. Because a chiral dipole is equally likely to be parallel or anti-parallel to the vorticity vector, the mean spinning due to vorticity is zero, and so the mean value of $A_L$ can be measured directly from $\langle A_L \rangle = \langle \Omega_d \rangle / \beta$.

Figure 3.11 (a) shows the probability density function (PDF) of the spinning rate from both experimental measurements and the simulations. The PDFs collapse surprisingly well given the fact that the experiments were performed with particles in the inertial range and the simulations are for particles in the dissipation range. There is a clear asymmetry around zero in both the experiments and the simulations. The higher probability of positive spinning rate demonstrates the preferential rotation of chiral dipoles advected in isotropic turbulence. In the simulations, we can separate the contributions from strain and vorticity as shown in Fig. 3.11 (b). Since the mean contribution from vorticity is zero, the contribution from the strain is responsible for the non-zero mean spinning rate. In addition to the mean, the PDFs show a strong positive skewness, $S = (\langle \Omega_d - \langle \Omega_d \rangle \rangle^3)/\langle \Omega_d^2 \rangle^{3/2}$. For the experiments, $S = 1.1$ at $R_\lambda = 120$ and $S = 1.0$ at $R_\lambda = 183$. For the simulations, $S = 0.27$. The skewness reflects both the skewness of the longitudinal velocity differences in the 4/5 law and the complex dynamics of preferential
alignment of slender bodies with vorticity and strain in turbulent flows. The discrepancy between the skewness of the experiments and the simulations is not fully understood, but is likely to be a result of the very different particle sizes.

The shape of the experimentally measured PDFs in Fig. 3.11(a) depends on the fit-length, which is the number of time steps used when extracting the solid-body rotation rate from the orientation measurements. Shorter fit-lengths include more noise from the orientation measurements, leading to larger tails, whereas longer fit-lengths filter out events of large rotational acceleration. Both experimental curves in Fig. 3.11 have been measured with a fit-length of $0.5\tau_\eta$.

The results from the DNS show a mean spinning rate of $\langle \Omega_d \rangle = 0.047\tau_\eta^{-1}$. This is in agreement with the previously measured material line stretching rate $\zeta = 0.12\tau_\eta^{-1}$ since $\langle \Omega_d \rangle = \beta \zeta$. The experimentally measured mean spinning rate is $\langle \Omega_d \rangle = (0.170 \pm 0.005)\tau_L^{-1}$ and $\langle \Omega_d \rangle = (0.26 \pm 0.01)\tau_L^{-1}$ for $R_\lambda = 120$ and $R_\lambda = 183$, respectively. The eddy turnover time at the scale of the particle, $\tau_L = \sqrt{1/15} L/u_L$, was chosen as normalization so that $\tau_L \rightarrow \tau_\eta$ for $L < \eta$. Here, $u_L = \langle (\Delta u_L \cdot \hat{r})^2 \rangle^{1/2}$ is the magnitude of the longitudinal velocity difference at separation $L$.

A simple scaling law with the mean spinning rate scaling with the eddy turnover time at scale $L$ does not hold. The higher than expected spinning rate of the larger chiral dipoles may be explained by two factors. First, the preferential alignment between the particle orientation and the extensional eigenvectors of the coarse grained strain rate tensor depends on particle size. Lüthi et al. (2007) measured the coarse grained velocity gradient tensor and showed that the preferential alignment of vorticity moves toward the maximum extensional eigenvector as the coarse graining length scale increases. Second, the coupling constant $\beta$ may depend on the particle Reynolds number, so that chiral dipoles spin more efficiently in a turbulent environment than in the Stokes flow limit. Future work using numerical simulations of particles with lengths in the inertial range and experiments using particles with lengths at the Kolmogorov scale could clarify how the crossover from dissipation to inertial range scales affects the rotations of chiral dipoles.

3.6.2 Vortex Stretching

Stretching of material lines is one of the fundamental processes in the energy cascade. Traditionally, vortex stretching has been emphasized with more recent work highlighting that strain-strain
interactions are equally if not more important. The ability to follow elongated particles through the flow and observe the preferential stretching they experience suggests new ways to quantify the dynamic processes of the energy cascade. Figure 3.12 shows the mean trajectories of fluid elements in the space of enstrophy, $\omega^2$, and the material line stretching rate, $A_L$. There is a clear cyclical pattern with a fixed point at large enstrophy and a positive value of the material line stretching rate. A qualitatively similar cycle has been observed for the vortex stretching process by Ooi et al. (1999) reflecting the similar physics involved in vortex stretching and material line stretching.

### 3.7 Conclusions

We developed a method for measuring the time-resolved Lagrangian orientation and solid-body rotation rate of anisotropic particles in turbulent flows. By measuring the rotation of 3D-printed crosses and jacks we are able to extend previous measurements of rods to cover the full range of aspect ratios of axisymmetric ellipsoids. Moreover, we provide a way to directly probe Lagrangian vorticity with a single particle measurement, which has potential for application in a wide range of flows and Reynolds numbers. We find that the mean-square tumbling rate $\langle \dot{\Omega}_i \dot{\Omega}_i \rangle$ agrees with DNS data of sub-Kolmogorov scale axisymmetric ellipsoids at points spanning the
full range of aspect ratios. Our measurements show that crosses are preferentially aligned in
turbulence with their orientation axis perpendicular to their solid-body rotation rate vector. To
the best of our knowledge, this is the first direct experimental measurement of the preferential
alignment of particles with their rotational motion in a turbulent flow. Our results support a
natural picture of alignment in turbulence where particles have their long axes aligned with the
Lagrangian stretching direction of the fluid flow.

The ability to fabricate particles with complex shapes and to measure their rotational motion
opens the door to the study of a wide variety of particle shapes beyond the axisymmetric
ellipsoids that have been the focus of most previous work. Chiral dipoles experience a preferential
rotation direction in isotropic turbulence. Elongated particles like chiral dipoles are aligned by
the fluid strain at the scale of the particle and therefore experience extensional strain on average.
The chiral ends couple to that extensional strain and respond with a corresponding preferential
rotation. These measurements highlight the importance of analyzing turbulent flows in an
oriented Lagrangian reference frame (Li and Meneveau, 2005, 2006). Future work is needed
to clarify the scale dependence of preferential alignment and rotation. Study of coarse grained
rotation and deformation have yielded substantial insights into the dynamics of turbulence (Lüthi
et al., 2007; Xu et al., 2011). Our current experiments and simulations show partial agreement
in the shape of the spinning rate PDF, but the experiments are limited to inertial range particle
sizes and the simulations are limited to dissipation range particle sizes. Tools to measure and
simulate particles across the full range of turbulent scales could provide a powerful new way to
analyze the dynamics of the turbulent cascade process.
Chapter 4

Sedimentation of Heavy, Non-Spherical Particles in Turbulence

The work presented in this Chapter was done in collaboration with Donald L. Koch, Udayshankar Menon and Anubhab Roy from Cornell University.

We focus on the dynamics of heavy, non-spherical particles sedimenting under the influence of gravity in a turbulent flow. The density difference between particle and fluid, and the associated effects of sedimentation and inertia, can lead to very different preferential alignment and rotations compared to neutrally buoyant particles.

An inertial ellipsoid, for example, experiences inertial torques which tend to align its long axis perpendicular to the direction of relative motion. Additionally, torques from the turbulent fluid strain are acting on the particle, favoring alignment with the local velocity gradients. Moreover, under the influence of gravity, an inertial particle will sediment and sample turbulence based on the sedimentation rate. Rapidly sedimenting particles will experience a “frozen turbulence field”, whereas slowly sedimenting particles will experience turbulence in a Lagrangian sense. The competition between turbulent torques and inertial torques plays a key role for the sedimentation statistics of non-spherical particles.

Dimensional analysis can simplify the large parameter space of this problem and yields five dimensionless groups. The aspect ratio $\kappa$, describing the particle’s shape and the relative density
ratio $\Delta \rho = (\rho_p - \rho_f)/\rho_f$ of particle and fluid, are already two dimensionless quantities. The Reynolds number of the turbulence $Re_\lambda$ and the non-dimensional particle size $L/\eta$, where $\eta$ is the Kolmogorov length, connecting the particle size $L$ with the turbulent length scale, are two more. Here, we define the last dimensionless quantity to be the settling factor $S_F$. It relates turbulent and gravitational accelerations, similar to the Froude number $Fr = \sqrt{Lg/\bar{u}}$, however, in this case the settling factor is defined as the ratio of particle rotations due to inertial torques (induced by gravity) and rotations due to the torques from turbulent strain.

In this chapter, we propose a new way of investigating non-spherical, inertial particles, which enables us to observe the transition from strongly aligned particles ($S_F \gg 1$) to almost randomly oriented particles ($S_F \ll 1$). We present experimental and computational results showing the orientation distributions, preferential alignment, and rotation and sedimentation rates of ramified particles, settling under gravity with various turbulence intensities. Using the well developed theory for fiber motion, we build a simple model to compare experimental measurements of ramified particles with computational results.

### 4.1 The Wesleyan Vertical Water Tunnel

We identified the non-dimensional parameters, $\Delta \rho$, $\kappa$, $Re_\lambda$, $L/\eta$, and $S_F$ that determine the sedimentation statistics of non-spherical particles in turbulence. With that in mind, we constructed a 14 ft. (4.2 m) tall, vertical water tunnel (see Fig. 4.1 (a)) that gives us control over each parameter and enables us to explore a large range of the parameter space. A through flow keeps heavy particles suspended and we can simultaneously control the amount of turbulence they experience. Keeping the particles suspended enables us to record long, individual trajectories using similar particle tracking methods as described in the previous chapter. The density ratio $\Delta \rho$ and the particle dimensions $L$ and $\kappa$ were chosen to yield particle sedimentation rates that could be supported by the through flow.

#### 4.1.1 Design and Construction

The design and construction of the water tunnel took about one year and we received great help from Dave and Bruce Strickland from the Wesleyan machine shop. Design Studies for a Closed-
Jet Water Tunnel, by Ripken (1951) was used as reference since it describes many essential parts of a typical water tunnel in detail, e.g. the contraction zones. Forcing the fluid through a contraction zone helps to create a flat flow profile, as indicated in Fig. 4.2 (a), which prevents particles from drifting to the channel walls. Moreover, we want to be able to study particle sedimentation under minimal turbulence, therefore being able to reach a flow with minimal velocity gradients in the through flow is crucial.

Figure 4.2 (b) illustrates Bernoulli’s equation (assuming negligible energy losses),

$$\frac{P_1}{w} + \frac{U_1^2}{2g} = \frac{P_2}{w} + \frac{U_2^2}{2g},$$  \hspace{1cm} (4.1)

and shows the uniformity of the velocity profile at point 2 in the contraction is considerably improved. Here, $P_i$ is the unit pressure, $U_i$ is the local fluid velocity on a particular streamline at points $i = 1, 2$ in the contraction, $w$ is the unit weight, and $g$ is the gravitational accelera-
Additionally, conservation of mass relates the cross sectional area $A$ and the mean velocities as follows

$$\langle U_2 \rangle = \frac{A_1}{A_2} \langle U_1 \rangle. \quad (4.2)$$

It can be shown that the turbulent velocity component along the principal axis of the contraction is reduced in magnitude approximately inversely proportional to the area ratio $A_1/A_2$, while the two lateral components increase proportional to $\sqrt{A_1/A_2}$. For that reason, suppression of the lateral turbulent motion should be a priority before the fluid enters the contraction zone. Therefore, the fluid first passes through a pressure plate (2% open area, 1 inch thickness) and a 20 cm tall honeycomb flow straightener (3/8 inch cell size, Plascore, Inc.) located in the bottom chamber of the water tunnel. An external supplier (Nor’Easter Yachts) built the contraction zone (area ratio $A_1/A_2 = 4$) from Fiberglass, and in order to keep the fabrication costs down, we chose a slightly simpler design (see Fig. 4.3 (a)) than suggested by Ripken (1951). Using these devices results in a mean flow profile that varies by less than 2% in the central region of the test section, as shown in Fig. 4.3 (b).

The exit conditions downstream of the test section are kept symmetric to the inlet conditions (except the honeycomb). The test section of the water tunnel is 1.5 m tall and has a cross-sectional area of $30 \times 30$ cm$^2$. The sides are 3/4 inch thick, clear plexiglass (Professional Plastics, Inc) and glued together using a two component glue (IPS Weld-On #40). The test section has two removable windows, one at the top end and one at the bottom end, for easy access.
Figure 4.3: (a) AutoCAD rendering of the contraction zone used in the water tunnel. The area ratio is 4:1, with a base area of $60 \times 60$ cm$^2$. (b) Mean fluid velocity $\langle U_z \rangle$ in the stream-wise direction as function of distance from the center, in (blue circles: $\hat{x}$; red squares: $\hat{y}$), normalized by the mean velocity at the channel center.

It is a recirculating system, driven by a 3 hp variable speed pump (Pentair, 011018 IntelliFlo). The pump can generate flow velocities of up to 10 cm/s in the test section, which is sufficient for our particles but sets an upper limit on the relative particle density. Initially, we planned on using a 10 hp magnetic drive pump (March, TE-10K-MD) and control the flow rate via valves. However, that setup generated too much heat and changed the fluid temperature by almost 10$^\circ$C from room temperature in 30 minutes. The through flow rate is monitored with a magnetic flow meter (Toshiba, LF654) with an accuracy of $\pm 0.2\%$ of the flow rate. Two cartridge filter systems (micron rating: 0.2 $\mu$m) in series are used to filter out any dirt or debris that might be in the apparatus.

One of the challenges of the experiments is keeping the particles suspended without clogging filters, valves or the pump. This is particularly difficult for fibers, which is the reason why they have been excluded from some of the experiments. Other particle shapes can be constrained to the test section using two coarse meshes (mesh size: 2.54 mm $\times$ 2.54 mm; Industrial Netting, Inc), one at the lower end (inlet) and one at the upper end (outlet) of the test section. Another challenge is corrosion of components, especially when dealing with liquids other than fresh water. Therefore, the entire apparatus was constructed so that only plastic components are in direct contact with the fluid (except the stainless steel electrodes of the flow meter).

The water tunnel is surrounded by a frame, constructed with 3$\times$3 inches, t-slotted, extruded
aluminum (Parco, Inc; also McMaster Carr). The frame stabilizes the water tunnel, allows a person to climb on to reach the upper sections and holds accessories. A spring system connects the water tunnel to the frame and transfers some of the tunnel weight to the frame.¹

4.1.2 The Jet-Array

The turbulence in the test section is generated and controlled with a jet-array, a 3D-printed grid (grid spacing $M = 6$ cm, 30% solidity) with internal channels and 40 individual nozzles as shown in Fig. 4.1(b) and (c). The jet-array was fabricated from Nylon using Selective Laser Sintering (SLS) by Xometry (see Fig. 4.4(a)). We tested different processes, such as Fused Deposition Modeling and Stereolithography, but found that either the anisotropy in the print caused small leaks or we were limited by the maximum possible print size. We also tested different nozzle designs, but had to make an educated guess for the final design since it could only be tested in an actual experiment in the water tunnel.

Each nozzle is controlled with a designated 12 V, 1/2 inch solenoid valve (Adafruit, Product ID: 997, Fig. 4.4(b)) and each solenoid valve is triggered with a solid-state relay ($5 \times 8$ Channel USB solid-state Relay Module, Numato Systems Pvt. Ltd.). We find that mechanical relays cause problems and interfere with the camera trigger signal. The jet-array is driven by the same pump as the through flow, the ratio can be adjusted with two external valves or the solenoid valves.

¹The springs should be tightened when the water tunnel is empty, to the point where they carry almost the entire weight of the apparatus.
Table 4.1: Experimental parameters: Through flow and total flow through the jet-array (measured with two magnetic flow meters); \( u'_z \), stream-wise rms fluctuating velocity; \( \langle U_f \rangle_z \), mean fluid velocity in the detection volume (measured with tracers). \( \langle U_p \rangle_z \), absolute mean particle velocity; \( u'_z / \langle U_f \rangle_z \), turbulence intensity in the direction of the mean fluid flow.

vals. To ensure equal pressure on all valves, we built four custom flow manifolds that hold ten valves each, one is shown in Fig. 4.4 (b). When a valve is opened, a jet of fluid is ejected into the flow from the corresponding nozzle on the jet-array. During the experiments, we keep the number of open valves equal on all four manifolds. The total flow rate through the jet-array is measured with a magnetic flow meter (Toshiba, LF654). Cavitation at the nozzles begins at a total flow rate of about 2.5 l/s with 8 open valves.

In the minimum turbulence configuration all solenoid valves are closed and the jet-array becomes a passive grid for the through flow. The stream-wise fluid velocity \( U_{f,z} \) has a small fluctuating component \( u'_z \), resulting in turbulence intensities \( u'_z / \langle U_f \rangle_z \) as little as 7\% (see Tab. 4.1), typical for passive grid configurations. We measured the anisotropy of the flow, \( u'_{(x,y,z)}/u'_z \), with about 20\% greater fluctuations in the direction of the through flow. Here, \( u'_{(x,y,z)} \) are the components...
of the root-mean-square (rms) fluctuating velocity.

The system can also be driven solely through the jet-array, similar to the random jet-array used by Variano and Cowen (2008), achieving much higher turbulence intensities. The intermediate turbulence regimes can be reached by either adjusting the number of jets, the duration each jet is firing or the jet velocity. For the experiments presented in the next sections, the turbulence intensity was controlled through the jet velocity, keeping the average number of jets (8 jets, 20% of the total number of jets) and the average duration of each jet (1 s ±0.25 s) constant. The 3 hp pump can produce an estimated jet velocity of up to 4 m/s at the nozzles, before cavitation becomes a problem. A more detailed characterization of the turbulence generated by the jet-array and the dependence on number of jets, duration and velocity can be found in Rees (2017).

Note: As far as we know, this is the first time 3D printing has been used to create a jet-array for generating and controlling turbulence. However, the idea of using active grids is not new (Alcaraz and Mathieu, 1965; Gad-El-Hak and Corrsin, 1974; Mydlarski and Warhaft, 1996; Ayyalasomayajula et al., 2006; Hearst, 2015). A mechanical grid, where brushless DC motors are driving shafts with vanes is an effective way to stir up the fluid and generate turbulence. The downside is the complexity of the system (both in construction and maintenance) and metal components being in contact with the fluid.

4.1.3 Controls and Data Acquisition

We use a similar data acquisition system for the experiments in the water tunnel as described before (3.1), where four multiple high-speed cameras (Mikrotron EoSens CL 1364) record particle trajectories. However, there are enough significant changes in the setup that it is worth mentioning some of the details here.

We switch from laser induced fluorescence to bright field imaging using LED backlights and black, opaque particles. This yields cleaner images since the particle signal does not depend on an even dye concentration or the laser intensity profile. We use two monochromatic pulsed, 30 cm × 30 cm LED lights (Smart Vision Lights, ODMOB-300x300-530) located on adjacent sides of the test section. The four cameras are located on opposite sides from the lights, so

\footnote{Since the cameras record monochromatic images anyway, this avoids chromatic aberrations}
that two cameras each obtain a uniformly lit background from one light. The cameras image a 12 x 12 x 10 cm$^3$ detection volume in the center region, 10\(M\) downstream of the jet-array. Two top cameras are located in the same lateral plane with a 90\(^\circ\) separation, looking down into the detection volume at an angle of 15\(^\circ\), and two bottom cameras are located beneath the top cameras, looking up toward the same volume at an angle of 10\(^\circ\). The cameras and LED backlights are triggered synchronously at 450 Hz, ensuring a single pulse illumination during each exposure.

The image compression systems (Sec. 3.1) have to be reprogrammed to work with a uniform bright background and dark pixels representing particles. The FPGAs can be set to ignore bright pixels above a certain threshold and only pass dark pixels on to the frame grabbers. Since particles that are in front or behind the detection volume will also appear on the camera images, special attention to the particle concentration has to be given to avoid FIFO buffer overflows.

Cameras, LEDs and the jet-array are controlled and synchronized from a dedicated computer running LabVIEW. We create a TTL signal (5 V for PIXCI E4 Rev.1; 3.3 V for PIXCI E4 Rev.5), which is converted to a low-voltage differential signal (LVDS), to trigger the cameras. The LED lights receive a DC voltage between 0 and 10 V to set the intensity, and a 5 V TTL signal for triggering. The lights can operate at a maximum duty cycle of 10\%, which means that at a maximum frequency of 5000 Hz, the maximum pulse duration is 200 \(\mu\text{s}\). At very low frequencies, the maximum pulse duration is limited to 125 ms.

The solid-state relay boards have a convenient USB interface and come with a LabVIEW driver. To avoid interference with other trigger signals, the relay boards are optically isolated via USB hub (HUB7i from Sealevel Systems, Inc). A string of hexadecimal numbers encodes the configuration of each relay, e.g. 4F (binary 0100 1111) will close relay number 2, 5, 6, 7 and 8, and open the corresponding valves. More details can be found in Rees (2017). The LabVIEW program is designed to either manually control each valve or to read in a .txt file containing strings of hexadecimal numbers. For the experiments, we created a .txt file with 6000 entries that will be repeated if the end of the list is reached. It randomly triggers 2 out of 10 valves on each manifold, keeping the total number of open valves constant, 8 on average. The duration for which any given valve is open is randomly picked from a normal distribution around 1 s (with standard deviation \(\sigma = 0.25\text{ s}\)). This approach was suggested by Variano and Cowen (2008).
the so-called sunbathing algorithm.

The flow rate of the through flow and the jet array can be adjusted using the built-in controller of the 3 hp variable speed pump. The maximum speed of the pump is 3450 rpm, which corresponds to a flow rate of roughly 10 cm/s in the test section (through flow only). We use two linear flow rate valves to change the flow rates of the through flow and the jet array, respectively. The flow rates are monitored with magnetic flow meters and can be logged with LabVIEW.

Note: The fluid in the experiments is degassed fresh water. When filling up the water tunnel, it is important to open all valves so that no air gets trapped inside the pipes or hoses. The water tunnel should be filled until the overflow container at the top of the water tunnel is filled. Running the pump at around 2000 rpm for a few minutes helps to get rid of any air in the system. This lowers the water level and air might get sucked into the system through the overflow container. Keeping the overflow container filled prevents that from happening.

4.1.4 Experiments

The first set of experiments investigates the sedimentation of fibers and triads in quiescent fluid. This will help to better understand the particle specific dynamics and to obtain a baseline measurement of their equilibrium orientations and sedimentation rates. Moreover, we measure the strength of the inertial torques by measuring the rotational response time of particles after they are perturbed from their equilibrium orientation. We develop a theoretical model for dissipation scale particles and show that many features of the sedimentation of complex-shaped particles in turbulent flows can be predicted using the results from these quiescent fluid experiments.

The second set of experiments focuses on the dynamics of triads sedimenting in turbulence. We investigate sedimentation as function of turbulence intensity and study how the orientation distributions, sedimentation rates and rotation rates depend on turbulence intensity and settling factor \( S_F \). The non-dimensional parameters for these experiments are summarized in Tab. 4.3. In addition to the experiments, we use direct numerical simulations to compare to our measurements.

The particles used in the experiments are 3D-printed ramified particles, using an opaque print material (VeroBlack, \( \rho_p = 1.14 \text{ g cm}^{-3} \)) and a FDM process. Three fibers of equal length \( \ell \)
Table 4.2: Turbulence parameters. $Re_\lambda = \sqrt{15 \bar{u}^3/L \nu}$, Taylor Reynolds number; $u'_{(x,y,z)}/u'_z$, components of the rms fluctuating velocity normalized by $u'_z$; $\bar{u} = \langle u'_i \rangle_i$, component-average rms fluctuating velocity; $L = \bar{u}^3/\langle \epsilon \rangle$, energy input length scale; $\langle \epsilon \rangle$, mean energy dissipation rate; $\eta = (\nu^3/\langle \epsilon \rangle)^{1/4}$, Kolmogorov length; $\tau_\eta = (\nu/\langle \epsilon \rangle)^{1/2}$, Kolmogorov time; $u_T^L = \sqrt{\langle (\Delta \mathbf{u} \cdot (\hat{1} - \hat{r} \hat{r}))^2 \rangle}$, $u_T^L = \sqrt{\langle (\Delta \mathbf{u} \cdot \hat{r})^2 \rangle}$, magnitude of the transverse, (longitudinal), rms velocity-difference at scale $L$ of the particle.

Table 4.3: Non-dimensional parameters of small (and large) triads. $\kappa = L/D$, aspect ratio; $\Delta \rho = (\rho_p - \rho_f)/\rho_f$, relative density ratio; $Re_\lambda$, Taylor Reynolds number; $L/\eta$, non-dimensional particle size, with length $L = 9$ mm ($L = 18$ mm); $S_F$, settling factor; $Re_D$, particle Reynolds number based on the diameter (additional non-dimensional parameter).
and radius \( r \) with aspect ratio \( \kappa = \ell/r = 20 \), oriented in planar symmetry and with a 120° angle between them, form a triad (see Fig. 4.8). A triad is used as a model particle for oblate spheroids (disks) with equivalent aspect ratio of \( \kappa = L/D \), where \( L = 2\ell \) and \( D = 2r \). The advantage of using ramified particles have already been described in great detail previously.

We fabricate 150 triads with smallest dimension \( r = 225 \pm 5 \, \mu m \) (referred to as small particles, but not in the dissipation range) and with \( r = 450 \pm 5 \, \mu m \) (large particles), both with \( \kappa = 20 \). The particle Reynolds number \( Re_D = \langle W \rangle D/\nu \) based on the fiber diameter ranges from \( Re_D \approx 10 \) for small triads to \( Re_D \approx 40 \) for large triads (see Tab. 4.3), where \( \langle W \rangle \) is the mean particle velocity. The fluid is fresh water at 24°C, with viscosity \( \nu = 0.9131 \times 10^{-6} \text{ m}^2 \text{ s}^{-1} \). Fibers used for the quiescent fluid experiments were manually cut from Nylon fishing line with a very similar density of \( \rho_p = 1.13 - 1.15 \text{ g cm}^{-3} \), but much smoother surface than the 3D-printed triads. The fiber radius and aspect ratio was close to the triads (although the small fibers had a diameter of \( D = 470 \, \mu m \)). Grey, polyethylene micro spheres with density 1.00 g/cm\(^3\) (Cospheric, LLC) with a diameter of 250-300 \( \mu m \) were used as tracer particles to measure the fluid velocity, calculate structure functions and extract the mean energy dissipation rate \( \langle \epsilon \rangle \) (Rees, 2017). For the experiments with small triads, the local fluid velocity at the particle position was determined using tracers within a sphere of radius \( 3L \) around the particle. The local mean fluid velocity \( \langle u_f \rangle \) did not depend very strongly on the size of that sphere for radii between \( L \) and \( 3L \), so a relatively large radius was chosen to minimize measurement noise. This was not successful for the experiments with the large triads due to an insufficient number of tracers. We define the relative particle velocity \( w = U_p - u_f \) when measured with respect to the local fluid velocity, and \( W = U_p - U_f \) when measured with respect to the mean fluid velocity. Here, \( U_p \) is the absolute velocity of a particle. For large particles, we can only determine \( W \).

The particles have to be suspended near the center of the test section in order to take continuous data and gather enough statistics. Depending on particle size, the through flow was adjusted to match the particle sedimentation rate in quiescent fluid. As we increase the turbulence intensity, triads start to rotate around their equilibrium sedimentation orientation which in return increases their sedimentation rate. The through flow was adjusted for each turbulence intensity to keep as many triads suspended as possible. For the highest turbulence intensities, triads were almost entirely suspended by strong jets from the jet-array, whereas for intermediate turbulence intensities, the through flow had to be increased to lift particles up into the detection
volume. The flow rates and mean velocities are summarized in Tab. 4.1.

4.2 Theory

We have seen that the dynamics of neutrally buoyant particles in turbulence solely depend on particle shape and the local velocity gradients, are well-captured by the theory for dissipation scale particles and that scaling arguments can be used to describe the dynamics of particles that extend into the inertial range. The theory for the forces and torques acting on heavy particles is far more complicated and analytical expressions currently only exists for high aspect ratio fibers at low particle Reynolds numbers. Moreover, heavy particles in turbulence sample the velocity gradients depending on their sedimentation rate. According to Taylor’s hypothesis, rapid settling particles will experience a “frozen turbulence” field, and sample random velocity gradients, whereas particles with a low sedimentation rate sample Lagrangian velocity gradients. It is not clear how the specific particle shape affects the forces and inertial torques on the particle and how the sampling of velocity gradients affects the orientation statistics.

In the next section, we develop a model that enables us to investigate the shape dependence of the forces and inertial torques for dissipation scale particles, and we show that the single non-dimensional parameter $S_F$ can be used to describe the orientation statistics. This parameter can also be defined for particles much larger than the dissipation scale, which enables us to compare the model with the experiments. The model assumes equilibrium particle dynamics, where the inertial torque is balanced by the torque from turbulent strain. We show that at high values of $S_F$, the particle dynamics become uncorrelated from the velocity gradients and that in either case, rapid or slow settling, the particle stays in a quasi-steady balance with the velocity gradients.

4.2.1 Sedimentation of Small Fibers

The sedimentation of small, slender fibers depends strongly on the fiber orientation $\hat{p}$. When the particle Reynolds number based on diameter $Re_D$ and half-length $Re_l$ are both small, they do not have a preferred settling orientation and their orientation is determined by their initial condition. With the inclusion of inertia, fibers experience an inertial torque that rotates the
Figure 4.5: Illustration of the coordinate system and the variables defining particle orientation $\hat{p}$, relative particle velocity $W$ and the angles $\theta$ and $\alpha$. (a) Fiber. (b) Triad.

particles to an equilibrium orientation where $\hat{p}$ is perpendicular to their relative velocity $W$. Khayat and Cox (1989) derived analytical expressions for the forces and torques on a particle, valid for arbitrary $Re_\ell$ as long as $Re_D \ll 1$ (Eq. 6.8 and 6.10 in Khayat and Cox (1989)), therefore requiring high aspect ratio (slender) fibers. These forces are balanced by any external force, usually gravity, and the balance of forces up to order $O(\ln(2\kappa)^{-1})$ reads

$$4\pi\mu L \ln(2\kappa) \left( \mathbb{1} - \frac{1}{2} \hat{p}\hat{p} \right) \cdot W - mg = 0,$$

where $\mathbb{1}$ is the identity matrix, $m = (\rho_p - \rho_f)\pi LD^2/4$ is the mass difference between a cylindrical fiber and the displaced fluid, $\mu$ is the dynamic fluid viscosity and $g$ is gravitational acceleration. A fiber in equilibrium will therefore move with relative velocity $W$. Equation (4.3) yields a well-known result for the transverse and longitudinal settling velocities

$$W_\parallel = 2W_\perp,$$

where $\parallel$ and $\perp$ means $\hat{p}$ is parallel to $W$ and $\hat{p}$ is perpendicular to $W$, respectively.

Analogous to the force balance, the rotation of small fibers is governed by a torque balance of inertial torque $G_{sed}$, rotational drag torque $G_{drag}$ and, in the case of turbulence, a torque from turbulent strain $G_{turb}$. The expressions for $G_{sed}$ is given by Eq. 6.12 in Khayat and Cox (1989), which becomes in the low Reynolds number limit (see their Eq. 6.22) and to order $\mathbb{1}$

The factor of 2 in $\ln(2\kappa)$ differs from Khayat and Cox (1989) and results in a first-order term that is more accurate at finite aspect ratios.
\frac{O(\ln(2\kappa)^{-2})}{24(\ln 2\kappa)^2}(W \cdot \hat{p})(W \times \hat{p}) (4.5)

The rotational drag torque \( G_{\text{drag}} \) from relative rotation can be quantified in the following way \cite{Batchelor, 1970}:

\begin{align*}
G_{\text{drag}} &= \frac{1}{L} \int_{-\ell}^{\ell} (r \times f_{\text{drag}}) \, dr \\
&= \frac{\pi \mu L^3}{3 \ln(2\kappa)} (\Omega_s \times \hat{p}) (4.6)
\end{align*}

where \( f_{\text{drag}} \) is the force per unit length which can be extracted from Eq. 4.3 when \( W \) is replaced with the tumbling rate \( \hat{p} = \Omega_s \times \hat{p} \), which is always perpendicular to \( \hat{p} \). Here, the solid-body rotation rate measures the relative rotation rate with respect to the fluid. On top of that, fibers sedimenting in turbulence, experience a torque from turbulent strain \( S \):

\begin{equation}
G_{\text{turb}} = \frac{\pi \mu L^3}{3 \ln(2\kappa)} (\hat{p} \times (S \cdot \hat{p})) (4.8)
\end{equation}

So for a symmetric fiber sedimenting in turbulence, the torque balance reads:

\begin{equation}
\begin{aligned}
\frac{5\pi \rho_f L^3}{24(\ln 2\kappa)^2}(W \cdot \hat{p})(W \times \hat{p}) + \frac{\pi \mu L^3}{3 \ln(2\kappa)} (\Omega_s \times \hat{p}) + \frac{\pi \mu L^3}{3 \ln(2\kappa)} (\hat{p} \times (S \cdot \hat{p})) &= 0.
\end{aligned}
\end{equation}

In quiescent fluid, the zero-torque tumbling rate can be found from the first two terms:

\begin{equation}
\dot{p}_{\text{sed}} = \frac{5Re_\ell}{8\ell \ln(2\kappa)} \hat{e}_1 W \cdot (1 - \hat{p} \hat{p}). (4.10)
\end{equation}

The typical time scale of rotations due to sedimentation at small \( Re_\ell \) can then be defined as the inverse of this tumbling rate, which depends on \( \theta \), the angle between \( \hat{p} \) and \( g \) (see Fig. 4.5 (a)):

\begin{equation}
\tau_{\text{sed}} = \frac{8\nu \ln(2\kappa)}{5|W|^2 \sin(2\theta)} (4.11)
\end{equation}

\begin{equation}
\tau_{\text{sed}, 45} = \frac{8\nu \ln(2\kappa)}{5|W|^2} (4.12)
\end{equation}

where we have used Eq. 4.3 to find \( W \) and \( \theta = 45^\circ \). For particles that are not in the low \( Re_\ell \) limit, \( \tau_{\text{sed}} \) can be empirically determined using the tumbling rate in quiescent fluid.

The typical timescale for rotations of small fibers due to turbulence \( \tau_{\text{turb}} \) is the Kolmogorov timescale

\begin{equation}
\tau_{\eta} = \frac{\eta}{u_\eta} = \sqrt{\frac{\nu}{\epsilon}}. (4.13)
\end{equation}
Figure 4.6: Mean-square particle orientation as function of settling factor $S_F$. The blue circles are simulation results at $Re_\lambda = 38$. The red lines indicate random orientations for which $\langle \cos^2(\theta) \rangle$ approaches $1/3$ (small $S_F$) and 0 for large $S_F$ (strong alignment perpendicular to gravity), proportional to $S_F^2$. There is a sharp transition at $S_F \approx 1$.

For particles larger than the Kolmogorov length scale, the appropriate turbulent time scale $\tau_{turb}$ is the eddy turn over time at the scale of the particle

$$\tau_L = \frac{L}{u_L} = \sqrt{\frac{4}{15} \frac{L}{u_L^T}}, \quad (4.14)$$

where $u_L^T = \sqrt{\langle (\Delta u \cdot (\hat{a} - \hat{r})^2 \rangle}$ is the magnitude of the transverse velocity difference at the scale of the particle and the factor of $\sqrt{4/15}$ is chosen such that the $\tau_L \to \tau_\eta$ in the limit $L < \eta$.

The orientation distributions of fibers can now be quantified by comparing the two time scales $\tau_{turb}$ and $\tau_{sed,45}$. We define the settling factor as the ratio

$$S_F = \frac{\tau_{turb}}{\tau_{sed,45}}. \quad (4.15)$$

For $S_F \gg 1$, the rotations due to turbulence are weak compared to rotations due to inertial torques and won’t significantly disturb the particles equilibrium orientation. The particles will be strongly aligned due to sedimentation. For $S_F \ll 1$, turbulence dominates the rotations and the particle orientations will be randomized accordingly. For small ($\ell \ll \eta$), slender fibers, the
analytical expression is given by

$$S_F = \frac{5}{8 \ln (2\kappa)} \left( \frac{|W|}{u_\eta} \right)^2. \quad (4.16)$$

In Appendix 5.3.2 we derive a model for the orientation variance $\langle \cos^2(\theta) \rangle$ of a fiber in the rapid settling limit. If we assume quasi-steady balance between inertial torques and turbulent torques (first and last term in Eq. 4.9), then the orientation variance follows a power law in $S_F$,

$$\langle \cos^2(\theta) \rangle = \frac{2}{15} \frac{1}{S_F^2}. \quad (4.17)$$

The orientation variance describes the amount of wiggling of a fiber around its equilibrium orientation $\theta = 90^\circ$. The power-law dependence is shown in Fig. 4.6 together with the results for fibers from direct numerical simulations.

It is interesting to see how inertial torques and turbulent torques are balanced in the rapid settling limit. One might think, that particles moving through an eddy of size $\eta$ at velocity $W$, do not have time to come to equilibrium if the sampling time is shorter than $\tau_{turb}$:

$$\tau_{samp} = \frac{\eta}{W} \ll \tau_{turb}. \quad (4.18)$$

Moreover, as Eq. 4.11 shows, the time scale of inertial torques $\tau_{sed}$ decreases quadratically with particle velocity $W$, and therefore becomes much shorter than the turbulent time scale $\tau_{turb}$ in the rapid settling limit. However, Eq. 4.11 also shows that $\tau_{sed}$ depends on the angle $\theta$, which fluctuates around the equilibrium orientation by $\langle \cos^2(\theta) \rangle^{1/2}$ for strongly aligned particles. Different eddies randomly kick particles around that equilibrium orientation, so the time it takes for eddies to rotate a particle to a quasi-steady angle is $\tau_{turb}\langle \cos^2(\theta) \rangle^{1/2}$. Therefore, the assumption of quasi-steady balance holds if

$$\frac{\tau_{turb}\langle \cos^2(\theta) \rangle^{1/2}}{\tau_{samp}} \ll 1. \quad (4.19)$$

Using Eq. 4.11 and 4.18 it follows that

$$\frac{\tau_{turb}\langle \cos^2(\theta) \rangle^{1/2}}{\tau_{samp}} \sim \frac{u_\eta}{W} \ln(2\kappa) \ll 1, \quad (4.20)$$

and we see that in the rapid settling limit, where $\eta/W \ll \tau_\eta$ or $u_\eta/W \ll 1$, particles are indeed in a quasi-steady balance. The faster a particle moves, the stronger the inertial torques it experiences and $\langle \cos^2(\theta) \rangle^{1/2}$ decreases quadratically with particle velocity $W$, whereas the sampling time $\tau_{samp}$ decreases linearly with $W$. For that reason, we obtain the counter-intuitive result that quasi-steady balance holds for particles described by Eq. 4.5 ($Re_D \ll 1$, $\ln(2\kappa)^{-2} \ll 1$), even in the rapid settling limit.
4.2.2 Corrections for Finite Particle Reynolds Number

The developed theory is valid for arbitrary Reynolds number as long as \( \text{Re}_D \ll 1 \), however, the full expressions derived by Khayat and Cox \((1989)\) include non-linear Reynolds number dependency coupled with particle orientation in a non-trivial way when including terms of order \( \mathcal{O}(\ln(\kappa)^{-2}) \). Solving the equations for force and torque balance is therefore computationally very expensive. Since the first-order term in aspect ratio nicely decouples drag and lift, we can add two constants, \( C_\perp \) and \( C_R \), that account for finite Reynolds number and aspect ratio effects:

\[
\frac{4\pi \mu L C_\perp}{\ln(2\kappa)} (1 - C_R \hat{p} \hat{p}) \cdot W - mg = 0 \quad (4.21)
\]

Here, \( C_\perp \) accounts for the overall change in drag on a particle sedimenting at non-zero Reynolds number and \( C_R \) accounts for the change of the drag ratio between a particle sedimenting with \( \theta = 0 \) and \( \theta = \pi / 2 \). In the low Reynolds number limit, \( C_\perp = 1 \) and \( C_R = 1 / 2 \) (see Eq. \(4.4\)). The expressions for \( C_\perp \) and \( C_R \) include the full analytical expressions given by Khayat and Cox \((1989)\) and the only approximation comes from interpolating the angle dependence at intermediate orientations. They are defined as:

\[
C_\perp = \frac{\ln(2\kappa)}{\ln(\kappa)} \left( 1 + \frac{\mathcal{F}_\perp}{\ln(\kappa)} \right) \quad (4.22)
\]

\[
C_R = \left( 1 - \frac{1}{2} \frac{(1 - \mathcal{F}_\perp / \ln(\kappa))}{(1 - \mathcal{F}_\parallel / \ln(\kappa))} \right) \quad (4.23)
\]

where \( \mathcal{F}_\parallel = \mathcal{F}_D(\text{Re}_\ell, \theta = 0) \) and \( \mathcal{F}_\perp = \mathcal{F}_D(\text{Re}_\ell, \theta = \pi / 2) \). The expressions for \( \mathcal{F}_D(\text{Re}_\ell, \theta) \) are given by Khayat and Cox \((1989)\) and in the Appendix \(5.3.1\). The two constants \( C_\perp \) and \( C_R \) are plotted as function of Reynolds number in Fig. \(4.7\) for different aspect ratios. The behavior of these functions at low Reynolds numbers shows that the theory is very sensitive to the high aspect ratio requirement and that the Stokes flow limit can only be recovered when \( \kappa \to \infty \).

Solving Eq. \((4.21)\) for the velocity of the fiber \( W \) yields the same expression as derived by Lopez and Guazzelli \((2017)\), who have approach this problem in terms of the mobility matrix

\[
W_i = M_{ij} F_j, \quad (4.24)
\]

where

\[
M_{ij} = M_\parallel p_i p_j + M_\perp (\delta_{ij} - p_i p_j). \quad (4.25)
\]
Figure 4.7: (a) \(C_\perp\) as function of Reynolds number \(Re_\ell\) for three different aspect ratios, \(\kappa = 20, 100\) and \(10^6\). The inset shows \(\kappa = 20\) for larger Reynolds number for comparison with experiments. (a) \(C_R\) as function of Reynolds number \(Re_\ell\) for the same three aspect ratios as in (a). The inset shows again \(\kappa = 20\) for larger Reynolds number for comparison with experiments.

The parallel and perpendicular mobility matrix \(M_\parallel\) and \(M_\perp\) refer to the mobility of a fiber when \(\hat{\mathbf{p}}\) is parallel or perpendicular to its velocity vector \(\mathbf{W}\), respectively, and

\[
M_\parallel = \frac{\ln(\kappa)}{4\pi\mu\ell}(1 - \mathcal{F}_\parallel / \ln(\kappa)) \\
M_\perp = \frac{\ln(\kappa)}{8\pi\mu\ell}(1 - \mathcal{F}_\perp / \ln(\kappa)).
\]

Equation 4.25 can be rewritten as

\[
M_{ij} = C_\perp M_\perp \left( \frac{1}{1 - C_R} p_i p_j + (\delta_{ij} - p_i p_j) \right),
\]

where we use the same constants as defined before.

For finite Reynolds numbers, experimental measurements of the sedimentation rate of horizontal fibers in quiescent fluid \(W_{\min} = |W|_{\theta = \pi/2}\) enable us to determine \(C_\perp\). This also includes the uncertainties in particle dimensions and density. With

\[
C_\perp = \frac{\ln(2\kappa)mg}{4\pi\mu L W_{\min}}.
\]

we measure \(C_\perp \approx 3.5\) and \(3.0\) for our small fibers and triads, and \(C_\perp \approx 7.8\) and \(7.6\) for our large fibers and triads, respectively. In order to determine \(C_R\), one has to measure the velocity of vertical fibers \(W_{\max} = |W|_{\theta = 0}\). It is well know, that for slender fibers in the Stokes flow limit, \(W_{\max} = 2W_{\min}\). Finite aspect ratio effects lower this ratio, but it increases again slowly when
considering finite Reynolds number effects. Our particles fall into the range where this ratio is approximately 2 again.

Note: One important difference in our work compared with the approach of Khayat and Cox (1989); Vakil and Green (2009) and Lopez and Guazzelli (2017) is that they use a particle Reynolds number that is independent of orientation. This is appropriate for a particle which is free to rotate, but fixed in a uniform flow, e.g. a typical experiment in a wind tunnel. For a sedimenting particle, however, $Re_{\ell}$ depends on orientation and can change by a factor of 2 and more.

### 4.2.3 Sedimentation of Ramified Particles

The motion of ramified particles can be modeled by treating the particle as a composite of several slender fibers (arms). If we neglect the interactions between the fibers, drag force and lift force of each arm of the ramified particle are now balanced by the external force. For $Re_{\ell} \ll 1$, the first-order term in aspect ratio of the drag and lift forces, when expressed in tensor form (see Eq. 4.3) can be summed over:

$$
\sum_{n=1}^{N} \left[ \frac{4\pi \mu \ell C_{\perp}}{\ln(2\kappa)} (\mathbb{1} - C_{D} \hat{p}'_{n} \hat{p}'_{n}) \cdot \mathbf{W}_{n} - \frac{\pi \Delta \rho D^{2} \ell}{4} g_{n} \right] = 0 \tag{4.30}
$$

Here, $N$ is the total number of arms of the ramified particle ($N = 3$ for triads) and $\ell$ is the arm length. The orientation of each arm is defined by $\hat{p}'$ and the orientation of the ramified particle is defined by $\hat{p}$, which is perpendicular to the plane of the arms of a triad. In the low Reynolds number limit, the ramified particle model predicts the same ratio of sedimentation velocities for triads as given for high aspect ratio disks (e.g. Clift et al. (2005)):

$$
W_{\text{max}} = 1.5W_{\text{min}} \tag{4.31}
$$

Since the definition of $\hat{p}$ has changed compared to fibers, the definition of $W_{\text{max}}$ and $W_{\text{min}}$ has also changed, so that now $W_{\text{max}} = |\mathbf{W}|_{\theta=\pi/2}$ and $W_{\text{min}} = |\mathbf{W}|_{\theta=0}$.

The model also predicts that fibers have twice the horizontal velocity of triads for all orientations and that independent of shape, the maximum horizontal velocity is reached when $\theta = 45^\circ$.

Analogous to Eq. 4.30, the torque balance becomes a sum over all arms of the ramified parti-
\[ \sum_{n=1}^{N} \frac{\pi \mu \ell^3}{3 \ln(2\kappa)} \left[ \frac{5}{8\nu \ln 2\kappa} (W_n \hat{\mathbf{p}}_n^\prime) (W_n \times \hat{\mathbf{p}}_n^\prime) - (1 - \hat{\mathbf{p}}_n^\prime \cdot \hat{\mathbf{p}}_n) \cdot \Omega_s + (\hat{\mathbf{p}}_n^\prime \times (\mathbf{S} \cdot \hat{\mathbf{p}}_n^\prime)) \right] = 0. \quad (4.32) \]

### 4.3 Direct Numerical Simulations

We are interested in simulating the particle orientation dynamics as it evolves through a Lagrangian velocity model. We expect the Lagrangian model to successfully capture the slow settling limit \( \frac{u_\eta}{W} \gg 1 \). It is important to point out that in the slow settling limit, where \( \frac{u_\eta}{W} \gg 1 \), particles sample Lagrangian velocity gradients and \( \tau_{\text{samp}} \gg \tau_\eta \). They are always in quasi-steady balance with turbulent eddies (see Eq. 4.19). A primary challenge in these kinds of simulations is generating an isotropic turbulent velocity gradient evolving in time. In this regard, a Lagrangian model for velocity gradient by Girimaji and Pope (1990b) was chosen. The stochastic model has been previously used to compare results obtained from direct numerical simulation of non-inertial particles in isotropic turbulence (Shin and Koch, 2005). In the model of Girimaji and Pope (1990b), the velocity gradient tensor, \( \mathbf{A} \), is modeled as a diffusive or Markovian process. The simpler stochastic models treating turbulence as Gaussian random velocity field are useful in simulation of particle coalescence and preferential concentration (Koch and Pope, 2002; Chun et al., 2005; Brunk et al., 1998). However, simulation of orientation dynamics requires a model that correlates the strain rate tensor and vorticity. The model of Girimaji and Pope (1990b) captures the non-linear terms in the evolution equation of the velocity gradient tensor (Eq. 1.10) exactly and captures the correlation between strain rate and vorticity, making it appropriate for the simulation of particle orientation dynamics.

### 4.4 Results - Quiescent Fluid

The experiments in quiescent fluid include two kinds of particles, fibers and triads, with two different sizes. Fibers were chosen to match the non-dimensional parameters of triads as close as possible. In quiescent fluid, both particles will assume a stable sedimentation orientation with their longest axis perpendicular to their sedimentation direction. In the lab reference frame, \( \hat{\mathbf{z}} \) is upwards and gravity \( \mathbf{g} \) is downwards, this means the stable orientation of fibers is \( p_z = 0 \) \( (\theta = 90^\circ) \) and \( p_z = 1 \) \( (\theta = 0) \) for triads.
The measurements of $W_{\text{min}}$ together with Eq. 4.29 determine $C_{\perp}$ of the particles in their stable orientation (or Eq. 4.24 to determine the mobility matrix elements). Moreover, the orientation distributions and variances of particles, sedimenting in quiescent fluid, contain valuable information about particle inhomogeneities and fabrication defects.

To gain insight into the sedimentation statistics in quiescent fluid, at orientations other than their equilibrium orientation, we disturb the particles by letting them hit a thin nylon string and recording the resulting trajectories. The sedimentation rate is measured for all orientations along the particles trajectory and does therefore include effects of the particles history. In other words, the sedimentation rate is measured during a transient phase whereas the ramified particle model assumes equilibrium sedimentation statistics.

### 4.4.1 Sedimentation Rates

In their equilibrium orientation, our small fibers and triads sediment at a mean rate of $W_{\text{min}} = 19.8 \text{ mm/s}$ and $23.2 \text{ mm/s}$, respectively, and our large fibers and triads at $W_{\text{min}} = 35.9 \text{ mm/s}$ and $36.8 \text{ mm/s}$, respectively.
Figure 4.9 (a) Relative velocity components of fibers (circles) and triads (triangles) in vertical and horizontal direction, $w_z$ and $w_h$, respectively, as function of particle orientation $|p_z|$. The symbols show experimental data in quiescent fluid, where small and large symbols indicate particle size. The dashed lines are the predictions for $w_z$ and $w_h$ based on a Stokes flow model. (b) Schematic of a fiber, with $\hat{p}$ along the fiber axis. (Inset: Fiber under the microscope shows smooth surface and end conditions). (c) Schematic of a triad, with $\hat{p}$ normal to the plane of the particle. (Inset: Triad under the microscope shows irregularities of the surface due to the fabrication process, FDM).

Figure 4.9 (a) shows the components of the relative particle velocity $W$ of fibers and triads as function of particle orientation. The top curves show the mean vertical component $W_z$ in the direction of gravity. Fibers of both sizes follow the predictions of the theoretical model very well when normalized by $W_{min}$, even though they are far outside the range of Reynolds numbers where this model is valid. Their sedimentation rate in the vertical orientation is about twice their sedimentation rate in the horizontal orientation, $\langle W_z|_{p_z=1} \rangle \approx 2 \langle W_z|_{p_z=0} \rangle$. Triads on the other hand are not quite reaching the sedimentation rates predicted by the ramified particle model, but $\langle W_z|_{p_z=0} \rangle \approx 1.4 \langle W_z|_{p_z=1} \rangle$. There are multiple reasons that could explain this lower ratio of sedimentation rates. For our particles, the low Reynolds number and high aspect ratio approximation is not valid ($Re_D > 10$ and $\kappa = 20$) and both effects are known to change that ratio (Khayat and Cox, 1989). As described before, one can model these effects in the ramified particle model by adjusting the constant $C_R$ (Eq. 4.21). However, if this was the dominant reason for a lower ratio of sedimentation rates, we would expect to see a similar effect for fibers. The most likely reason why we see a discrepancy between the model prediction
and the measurements of triads is that the model neglects the interactions between arms of a ramified particle.

In addition to vertical sedimentation, fibers and triads have a non-zero horizontal settling velocity depending on particle orientation. The bottom curves in Fig. 4.9 (a) show the mean horizontal component, $W_h = W \cdot \hat{h}$, where $\hat{h} = \hat{g} \times (\hat{p} \times \hat{g}) = (1 - gg) \cdot \hat{p}$, the projection of $\hat{p}$ in the plane perpendicular to $\hat{g}$ (horizontal). Based on the model, we expect this component to be maximized independently of shape when $\theta = 45^\circ$, or $p_z = 1/\sqrt{2}$. The experiments show that both fibers and triads reach the largest horizontal velocity at a different angle, when $p_z \approx 0.5$.

One has to keep in mind that $\hat{p}$ points along the symmetry axis of the fiber, but is perpendicular to the plane of the triad and therefore $p_z = 0.5$ for fibers means the long axis makes an angle of $30^\circ$ with respect to the horizontal, while for triads the long axis makes an angle of $60^\circ$ with respect to the horizontal, see Fig. 4.9 (b) and (c). The experiments show that fibers have roughly twice the horizontal velocity of triads for all orientations, which is in agreement with the ramified particle model.

Note: The shallowest trajectory occurs at a different angle than the maximum horizontal velocity.

### 4.4.2 Rotation Rates and Inertial Torques

The model for inertial torques, Eq. 4.5, is valid at low particle Reynolds numbers. Shin et al. (2006) have shown in simulations, that for $Re_\ell \sim 10$, the inertial torques stop increasing with Reynolds number and actually start decreasing again. We can therefore expect that the time scale $\tau_{sed,45}$, predicted by the low Reynolds number model (Eq. 4.12) is too short for the particles in the experiments. Figure 4.10 (a) shows the angle between gravity $\hat{g}$ and the particle orientation $\hat{p}$ as function of time, normalized by $\tau_{sed,45}$. The trajectories shown are averaged trajectories of many individual fibers and triads, with $t = 0$ chosen when each particle is at $\theta = 45^\circ$. The error bars are showing the standard deviation between individual trajectories, which is zero at $t = 0$ because we choose $\theta = 45^\circ$ to define time zero. We also show the simulation data from Shin et al. (2006) of fibers with lengths much smaller than the Kolmogorov length. For our particles, the low Reynolds number model clearly over-estimates the strength of the inertial torques and does not collapse the experimental curves nor the experiments with
the simulations. For that reason, we can not use the definition of $\tau_{sed,45}$ from the low Reynolds number model to calculate $S_F$ for our particles at $Re_\ell \gg 100$.

To gain information about the strength of the inertial torques when the particle Reynolds number is no longer small, we extract the time it takes a particle to come to alignment with its equilibrium orientation. We measure the the tumbling rate of the particles when $\theta = 45^\circ$ and use it to define an empirical time scale of the inertial torques as $T_{sed,45} = 1/|\dot{\theta}|_{\theta=45^\circ}$. Here, we determine the tumbling rate when $\theta = 45^\circ$ by fitting a straight line to the measurements of $p_z$ over a range of 0.05$T_{sed,45}$. Figure 4.10 (b) uses this definition to collapse the curves. Interestingly, $T_{sed,45}$ is roughly the same for all particle shapes with $T_{sed,45} = (1.8 \pm 0.1)$ s for small fibers and $T_{sed,45} = (1.9 \pm 0.1)$ s for large fibers and $T_{sed,45} = (1.7 \pm 0.1)$ s for small triads and $T_{sed,45} = (1.9 \pm 0.1)$ s for large triads. One notable difference is that triads are often significantly overshooting the $\theta = 0$ point (not shown) and return at slightly different rates. Averaging this over different trajectories makes it appear that the equilibrium angle is at $\theta \sim 10^\circ$ in the shown time range. In the long time limit $\theta$ will approach $0^\circ$. We will use this time scale of the inertial torques, $T_{sed,45}$, to calculate an empirical value of the settling factor $S_F$ for the turbulence experiments. Since the definition of $S_F$ remains the same as in Sec. 4.2.1 we can directly

Figure 4.10: (a) Experimental measurements (open symbols) of the angle between $\hat{p}$ and gravity as function of time, normalized by the predicted time scale for inertial torques based on the Stokes model (circles = fibers, triangles = triads). As comparison, simulation results from Shin et al. (2006) of dissipation range fibers are also shown. (b) Same data as in (a), but normalized by the predicted time scale for inertial torques, based on measurements of their rotation rate at $\theta = 45^\circ$. 
compare our measurements of particles that are much larger than the Kolmogorov scale with simulations of particles that are much smaller than the Kolmogorov scale.

4.5 Results - Turbulence

The sedimentation of ramified particles under different turbulence intensities gives insight into the competition between inertial torque, which aligns particles with their stable sedimentation orientation and turbulence, which tends to randomize particle orientations. We present orientation distributions of triads as two dimensional PDFs of the angles $\theta$ and $\psi$ and as three dimensional spherical histograms. Moreover, we will show the transition from strongly aligned particles to randomly oriented particles as function of $S_F$. In addition to the orientation statistics, we will present the sedimentation and rotation rate statistics of triads. We also compare the experiments to simulations and theoretical predictions of the ramified particle model.

4.5.1 Orientation Distributions

The relative particle orientation can be defined using two angles, $\theta$ and $\psi$. We define these two angles as

\[
\cos(\theta) = |\hat{p} \cdot \hat{g}|
\]

\[
\cos(\psi) = \left| \left[ (1 - \hat{p} \hat{p}) \cdot \hat{g} \right] \left[ (1 - \hat{p} \hat{p}) \cdot \hat{g} \right] \right|
\]

where $\theta$ is the angle between $\hat{p}$ and gravity and $\psi$ the angle between an arm $\hat{p}'$ and the projection of gravity into the plane of the particle $(\hat{p} - \hat{p} \hat{p}) \cdot \hat{g}$. In isotropic turbulence, the third Euler angle is a rotation around $\hat{z}$ which is randomly distributed and does not encode any relevant statistics.

Figure 4.11 shows how the probability density function (PDF) of $p_z = \cos(\theta)$ changes with turbulence intensity. For low turbulence intensities, small and large triads are strongly aligned with the direction of gravity, within a few degrees, reflected in the sharp peak near $p_z = 1$. This alignment becomes weaker as the turbulence intensity increases. The orientation PDFs become more uniform. Even for high turbulence intensities, particle orientations are not fully randomized.

The corresponding Reynolds numbers and settling factors are summarized in Tab. 4.4.
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Figure 4.11: PDF of the particle orientation at different turbulence intensities from low to high (color map cold to hot). $|p_z| = 1$ means horizontal alignment, $|p_z| = 0$ vertical alignment and the PDF of randomly oriented particles is uniform at 1. (a) Experiments with small triads. (b) Experiments with large triads.

Triads also show interesting preferential alignment within the plane of the particle. Figure 4.12 shows the PDF of the angle $\psi$. It is surprising that for low turbulence intensities, small triads show a strong peak at $\psi = \pi/3$, meaning any one of the three arms is pointing slightly upward. We assume that particle defects or fabrication inhomogeneities cause this alignment, e.g. one arm might experience slightly larger drag. We do not see such behavior for large triads, where these effects would have a smaller impact. With increasing turbulence intensity, the particles get kicked out of their equilibrium orientation and we assume interactions between the arms cause one arm to preferentially align parallel to the direction of gravity, pointing downward. Large

<table>
<thead>
<tr>
<th>Small Triads</th>
<th>Large Triads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Legend</td>
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<td>$\triangle$</td>
<td>192</td>
</tr>
<tr>
<td>$\triangle$</td>
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</tbody>
</table>

Table 4.4: Summary of Taylor Reynolds numbers $Re_\lambda$ and corresponding settling factors defined empirically for triads, $S_F = \tau_L / T_{sed}$. 
Figure 4.12: PDF of the particle orientation within the plane spanned by the arms of the particle at different turbulence intensities from low to high (color map cold to hot). \( \psi = 0 \) means an arm of the particle is aligned parallel with the direction of gravity (120° symmetry), \( \psi = \pi/3 \) mean an arm of the particle is anti-parallel to the direction of gravity. (a) Experiments with small triads. (b) Experiments with large triads.

Triads show that this effect is strongest for intermediate turbulence intensities, where turbulent fluctuations are strong enough to kick a particle far enough out of its equilibrium orientation so that interactions become relevant, but do not fully randomize the particles orientation yet.

Figure 4.13 shows spherical histograms of particle orientations for small and large triads for different values of \( S_F \). The histogram depicts the components of a unit vector defined in spherical coordinates by setting the polar angle equal to \( \theta \), and the azimuthal angle equal to \( \psi \). The large probability at the poles (see Fig. 4.13 (a) small triads and Fig. 4.13 (d) large triads at the lowest turbulence intensity) indicates that \( \hat{\mathbf{p}} \) is strongly aligned with the direction of gravity. The symmetric, but not random probability distribution around the equator (120° symmetry) shows the preferential alignment of one arm parallel to the direction of gravity. This preferential alignment can only be seen when the particles are significantly kicked out of their equilibrium orientation, as mentioned before. In fact, for small triads at the lowest turbulence intensity, the probability distribution shows small peaks, indicating opposite alignment with one arm upward.
Figure 4.13: Spherical histograms of particle orientations. The histogram shows the PDF of a vector defined in spherical coordinates by the polar angle $\theta$ and the azimuthal angle $\psi$. (a)-(c) Small triads, with increasing turbulence intensity ($Re_\lambda = 29, 162, 194$). (d)-(f) Large triads ($Re_\lambda = 36, 153, 200$).
4.5.2 Orientation Variance

To characterize the preferential alignment and quantify the amount of wiggling around the equilibrium orientation, we calculate the orientation variance. Figure 4.14 shows $(p_z^2) = \langle \cos^2(\theta) \rangle$ as function of settling number. For $S_F \ll 1$, triads approach random orientations where $(\cos^2(\theta)) = 1/3$, but with increasing settling number, inertial torques start to dominate, and for $S_F \gg 1$, triads approach their stable sedimentation orientation where $(\cos^2(\theta)) = 1$. For $S_F \ll 1$, small triads show in general stronger alignment and a smaller variance. This might be due to the different sampling of velocity gradients due to the smaller sedimentation rate of small triads. Both experimental curves are below the simulations of triads much smaller than the Kolmogorov scale. Since particles larger than the Kolmogorov scale filter the velocity gradients at the scale of the particles, this might damp some of the wiggling motion, which would explain the stronger alignment at lower values of $S_F$ compared to the simulations.

For $S_F \gg 1$, fibers approach $(\cos^2(\theta)) = 0$ (see Fig. 4.6). For that reason, we calculate the
Small Triads | Large Triads
---|---
$S_F$ | $\langle \cos^2(\theta) \rangle$ | $\frac{1}{2}(1 - \langle \cos^2(\theta) \rangle)$ | $S_F$ | $\langle \cos^2(\theta) \rangle$ | $\frac{1}{2}(1 - \langle \cos^2(\theta) \rangle)$
1.95 | 0.970 | 0.015 | 1.30 | 0.982 | 0.009
0.66 | 0.933 | 0.034 | 1.17 | 0.981 | 0.010
0.26 | 0.757 | 0.122 | 0.44 | 0.853 | 0.074
0.18 | 0.606 | 0.197 | 0.25 | 0.559 | 0.221
0.15 | 0.569 | 0.216 | 0.22 | 0.534 | 0.233
0.14 | 0.554 | 0.223 | 0.15 | 0.431 | 0.285

Table 4.5: Summary of settling factors defined empirically for triads, $S_F = \tau_L / \tau_{sed,45}$ and corresponding orientation variances $\langle \cos^2(\theta) \rangle$ and $0.5(1 - \langle \cos^2(\theta) \rangle)$. 

The orientation variance of the $z$-component of an arm of the triad, $\langle p_z^2 \rangle = 0.5 \left(1 - \langle \cos^2(\theta) \rangle\right)$, which approaches the same limit as fibers and allows a direct comparison of the two particle shapes. Theoretically, fibers and triads approach the strongly aligned limit according to a power law proportional to $S_F^{-2}$. The experiments and simulations follow that scaling law relatively well.

Perfect alignment with the stable sedimentation orientation can only be reached without turbulent fluctuations and ideal particles. However, particle inhomogeneities caused by fabrication defects set a lower limit for reliable measurements of the equilibrium orientation, which explains the deviation from the predicted power-law scaling for the small triads at the highest value of $S_F$. This can be seen in Fig. 4.14, where the measurements in quiescent fluid are also indicated.

When comparing fibers and triads, we see that the specific shape does not seem to make a big difference in the orientation statistics, as long as the aspect ratio stays comparable. This suggests that inertial fibers and triads depend on $S_F$ in a very similar way. However, it is known that turbulence couples quite differently to fibers and disks, which can be seen in the increased tumbling rate of disks compared to fibers in isotropic turbulence without inertia (Parsa and Voth, 2014). The dependence of inertial torques on particle shape and how sampling of velocity gradients of sedimenting particles affects their orientation statistics requires more research.

The aspect ratio will be a critical parameter in determining the exponent of the power law of arbitrary ellipsoids. Spherical particles do not experience inertial torques and will have random
### Small Triads

<table>
<thead>
<tr>
<th>$u_T^L$ [mm/s]</th>
<th>$\langle w_h \rangle$, $\langle w_z \rangle$ [mm/s]</th>
<th>$\langle W_h \rangle$, $\langle W_z \rangle$ [mm/s]</th>
<th>$\tau_L$ [s]</th>
<th>$T_{sed,45}$ [s]</th>
<th>$\tau_{samp}$ [s]</th>
<th>$\tau_p$ [s]</th>
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<td>0.01, -22.65</td>
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<td>0.24</td>
<td>1.7</td>
<td>0.35</td>
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**Large Triads**

<table>
<thead>
<tr>
<th>$u_T^L$ [mm/s]</th>
<th>$\langle W_h \rangle$ [mm/s]</th>
<th>$\langle W_z \rangle$ [mm/s]</th>
<th>$\tau_L$ [s]</th>
<th>$T_{sed,45}$ [s]</th>
<th>$\tau_{samp}$ [s]</th>
<th>$\tau_p$ [s]</th>
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<td>0.29</td>
<td>1.9</td>
<td>0.52</td>
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**Table 4.6:** $u_T^L$, magnitude of the transverse velocity difference at scale $L$ of the particle; $\langle w_h \rangle$ and $\langle w_z \rangle$, horizontal and vertical particle velocity, respectively; $w = U_p - u_f$, local relative particle velocity measured using tracers within a sphere of radius $3L$; $\langle W_h \rangle$ and $\langle W_z \rangle$, horizontal and vertical particle velocity, respectively; $W = U_p - U_f$, relative particle velocity with respect to the mean flow, $u_f$ could not be resolved for large particles; $\tau_L = \sqrt{k/15} L/u_T^L$, eddy turn over time at the scale of the particle; $\tau_{sed,45}$, inverse of the particle tumbling rate at $45^\circ$ in quiescent fluid; $\tau_{samp} = L/\langle |w| \rangle$ ($\tau_{samp} = L/\langle |W| \rangle$ for large triads), sampling time scale; $\tau_p = \kappa \rho_p D^2 \ln(\kappa + \sqrt{\kappa^2 - 1})/18 \nu \rho_f \sqrt{\kappa^2 - 1}$, particle response time. (Shapiro and Goldenberg, 1993; Zhang et al., 2001).
orientations independent of settling number, but with increasing deviations from the spherical shape, particles will experience enhanced coupling to the fluid and there should be a smooth transition to the high aspect ratio limit where the power law follows $S_F^{-2}$.

We also check the quasi-steady balance condition (see Eq. 4.19), however, for triads the condition reads

$$\frac{\tau_L[0.5(1 - \langle \cos^2(\theta) \rangle)]^{1/2}}{\tau_{samp}} \ll 1,$$

(4.35)

since the time it takes to come to a quasi-steady angle now depends on the variance of an arm. Here, $\tau_L$ is defined as the eddy turn over time at the scale of the particle $L$ (see Eq. 4.14). The sampling time for particles larger than the Kolmogorov time becomes

$$\tau_{samp} = \frac{L}{\langle |W| \rangle}.$$

(4.36)

With that, we find that small triads are in the range of $\tau_L[0.5(1 - \langle \cos^2(\theta) \rangle)]^{1/2} = (0.4 \pm 0.1)\tau_{samp}$ and large triads are in the range $\tau_L[0.5(1 - \langle \cos^2(\theta) \rangle)]^{1/2} = (0.25 \pm 0.05)\tau_{samp}$. We expect the quasi-steady balance condition to be valid. For high turbulence intensities, we are no longer in the rapid settling limit, but the slow settling limit, where the sampling time becomes comparable or larger than the eddy turn over time at the scale of the particle. In that case, we also expect to be in a quasi-steady balance, because the particle has enough time to come to equilibrium with the velocity gradients within each eddy, independent of orientation. For $S_F \gg 1$, inertial torques are dominant so that the particle orientation becomes uncorrelated from the velocity gradients. In that case, the sampling of Lagrangian velocity gradients (slow settling) yields similar orientation statistics as the random sampling of velocity gradients of a “frozen turbulence” field (rapid settling).

It is worthwhile to discuss the cases, in which the quasi-steady balance condition can not be satisfied. As mentioned before, when the particle Reynolds number becomes large, the low Reynolds number model of inertial torques doesn’t scale linearly with Reynolds number anymore. Moreover, when inertial particles become large ($L \gtrsim \eta$), their inertial response time $\tau_p$ has to be considered Voth and Soldati (2017). We use the same definition as described in Sec. 2.1.1,

$$\tau_p = \frac{\kappa \rho_p D^2 \ln(\kappa + \sqrt{\kappa^2 - 1})}{18 \rho_f \sqrt{\kappa^2 - 1}},$$

(4.37)

and find $\tau_p = 0.05$ s and $\tau_p = 0.2$ s for small and large triads, respectively. This time scale is short and we can assume that deviations from the tracer particle limit are small. If that time
scale becomes long, however, our assumption that particle tumbling rates are determined by Jeffery’s equation is no longer valid.

4.5.3 Sedimentation Rates and Dispersal

We present the first experimental measurements of the orientation dependent sedimentation rate of non-spherical, inertial particles in turbulence. First, we show the results for small triads, and then the results for large triads. Reminder, the relative particle velocity \( \langle w \rangle = \langle U_p \rangle - \langle u_f \rangle \) was measured with respect to the local mean fluid velocity \( \langle u_f \rangle \), at the position of the particle, whereas the relative particle velocity \( \langle W \rangle = \langle U_p \rangle - \langle U_f \rangle \) was measured with respect to the mean fluid velocity \( \langle U_f \rangle \).

Small Triads – Figure 4.15 (a) shows the mean vertical component \( \langle w_z \rangle \) of the relative particle velocity, as function of particle orientation. For \( p_z = 1 \), triads sediment at the same rate as in quiescent fluid, withing the measurement uncertainty, and this is independent of turbulence intensity. For the lowest turbulence intensity, particles are strongly aligned and we can only acquire data for a small range near \( p_z = 1 \). With increasing turbulence intensity, we obtain measurements at all orientations and we can see that triads sediment roughly 1.5 times faster when \( p_z = 0 \). This is in good agreement with the predictions of the ramified particle model (Eq. 4.30 with \( C_\perp = 3.0 \) and \( C_R = 1/2 \)). In Fig 4.15 (b), the relative particle velocity was measured with respect to the mean fluid velocity. It shows the effects of preferential sampling, which decreases the mean sedimentation rates, especially at orientations away from equilibrium, \( p_z \to 0 \). Particles that are significantly disturbed from their equilibrium orientation are most likely in an intense turbulent jet and are therefore subjected to higher upward fluid velocities. This emphasizes the importance of measuring the local fluid velocity at the position of the particle.

Figure 4.15 (c) shows the mean horizontal component \( \langle w_h \rangle \) of the relative particle velocity, as function of particle orientation. The horizontal component in lab coordinates is zero due to isotropy, but it is non-zero in the reference frame of the particle, where \( h = (1 - gg) \cdot \hat{p} \) is the projection of \( \hat{p} \) in the horizontal plane. According to the ramified particle model, \( \langle w_h \rangle \) should reach a maximum at \( \theta = 45^\circ \) \( (p_z = 0.7) \), independent of particle shape. In the experiments, we observe that this maximum occurs at a much lower angle, or steeper orientation, for triads, at
around $60^\circ$ ($p_z \approx 0.5$). The same trend has been observed in the quiescent fluid experiments. The most likely reason is that the interpolation of the angle dependence in this simplified model (Eq. 4.30) does not capture the lift forces as well as the drag forces. Another possible explanation for this discrepancy is that the model does not include the history of the particle, but only predicts the instantaneous vertical and horizontal components of the velocity of a particle in equilibrium. This would be an indication that the rotational time scale is much faster than the translational time scale (advection of inertial particles). Figure 4.15 (d) shows that the horizontal component $\langle W_h \rangle$ of the two most turbulent cases is significantly larger than for the two intermediate turbulence cases at its maximum when $p_z \approx 0.5$. This is most likely caused by the increased turbulent advection of the particles and is reduced when considering the local fluid velocity $\langle w_h \rangle$.

Even though many curves are spread within the error bars, they all collapse surprisingly well for different turbulence intensities and the sedimentation rates are in good agreement with the ramified particle model. This is very valuable because if the sedimentation rate of a particle in quiescent fluid is known for horizontal and vertical particle orientation, $C_{\perp}$ and $C_R$ can be determined and the ramified particle model can be used to predict the sedimentation statistics for different turbulence intensities even when the particle Reynolds number is large.

Figures 4.15 (e) and (f) show the angle between the relative particle velocity and the direction of gravity. Higher values indicate a shallower trajectory of the particle. The ramified particle model predicts this angle reaches a maximum of $11.5^\circ$ when the triad is oriented at $\theta = 39.2^\circ$ ($p_z = 0.775$)\(^4\). Again, we can see that triads reach their shallowest trajectory at steeper orientations than predicted.

Figures 4.16 (a) and (b) allow a direct comparison of the components of the sedimentation velocity of triads in turbulence with the results from the experiments in quiescent fluid as shown in Fig. 4.9. We normalized the velocities by $W_{\text{min}}$, the velocity of the particle when $p_z = 1$ in quiescent fluid. This way of presenting the data clearly shows that $W_{\text{max}} = 1.5W_{\text{min}}$ for triads, when the relative particle velocity is measured with respect to the local mean fluid velocity.

The standard deviations for the components of the sedimentation velocities $w$ and $W$ are presented in Fig. 4.17. Figures 4.17 (a) and (b) show the standard deviation of the vertical components.

\(^4\)For fibers, the maximum angle of $19.5^\circ$ is predicted when they are oriented at $\theta = 54.8^\circ$ ($p_z = 0.577$)
component $w_z$ and $W_z$. The particles are advected by intense turbulence which results in a higher standard deviation, especially at high turbulence intensities. The standard deviation is significantly reduced when the local fluid velocity is used to calculate relative particle velocities. Figures 4.17 (c) and (d) show the standard deviations of $w_h$ and $W_h$, indicating similarly strong effects. In all cases, the standard deviation shows an increase as $p_z \to 0$. As with the sedimentation rates, particles that are significantly disturbed from their equilibrium orientation are most likely experiencing intense turbulence. These strong, intermittent events result in a larger deviation from the mean when compared to strongly aligned particles at $p_z = 1$. 
Figure 4.15: Components of the relative particle velocity and the angle between the direction of gravity and particle velocity, as function of particle orientation. Panels (a), (c) and (e) measure the quantities with respect to the local fluid velocity, whereas panels (b), (d) and (f) are with respect to the mean fluid velocity. The color map is from low turbulence intensity (blue) to high turbulence intensity (red).
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Figure 4.16: Normalized components of the relative particle velocity as function of particle orientation. (a) \( \langle w_z \rangle \) and \( \langle w_h \rangle \), measured with respect to the local mean fluid velocity. (b) \( \langle W_z \rangle \) and \( \langle W_h \rangle \), measured with respect to the mean fluid velocity. \( W_{\text{min}} \) is the measured sedimentation velocity in quiescent fluid when \( p_z = 1 \). The color map is the same as in Fig. 4.15

Figure 4.17: Standard deviation \( \sigma(X) = \sqrt{\langle X^2 \rangle - \langle X \rangle^2} \) of the components of the relative particle velocity. The color map is the same as in Fig. 4.15
Large Triads – We were not able to resolve the local mean fluid velocity around large particles, due to an insufficient number of tracer particles. Therefore, we can only present measurements of the relative particle velocity \( \langle W \rangle \), measured with respect to the mean flow. Figure 4.18 (a) shows the mean vertical component \( \langle W_z \rangle \) of the relative particle velocity, as function of particle orientation for all turbulence intensities. The most turbulent case shows a clear deviation from the other data sets, the sedimentation rate is systematically lower. This is an experimental artifact caused by the specific way the particles were suspended. For the highest turbulence intensity, enough particles were elevated into the detection volume when they were sampling strong jets of the jet array. It was therefore not necessary to have a strong through flow as compared to the next two, lower turbulence intensity, data sets for which the jets were strong enough to significantly weaken the preferential alignment but not strong enough to lift particles up into the detection volume. Measurements of the local fluid velocity would most likely eliminate the effects of preferential sampling of strong jets and collapse all curves. All other curves follow the ramified particle model (Eq. 4.30, with \( C_\perp = 7.6 \) and \( C_R = 1/2 \)) surprisingly well. We also see that \( W_{\text{max}} = 1.5W_{\text{min}} \) is true for large triads.

Overall, the trends are very similar to small triads, indicating that the particle size does not affect the sedimentation statistics (besides a general shift towards faster sedimentation rates for larger particles) at high particle Reynolds numbers. Figure 4.18 (b) shows the horizontal component \( W_h \), and similar to small triads, the maximum horizontal velocity is reached at a lower angle, steeper orientation, than predicted by the ramified particle model.

Analogous to small triads, we show the normalized components of the sedimentation velocity in Fig. 4.19 and the standard deviations are in Fig. 4.20.
Figure 4.18: Components of the relative particle velocity, $W_z$ and $W_h$, and the angle between the direction of gravity and particle velocity, as function of particle orientation. The color map is the same as in Fig. 4.15.
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Figure 4.19: Normalized components of the relative particle velocity, $W_z$ and $W_h$, as function of particle orientation. $W_{min}$ is the measured sedimentation velocity in quiescent fluid when $p_z = 1$. The color map is the same as in Fig. 4.11.

Figure 4.20: Standard deviation of the relative particle velocity. The color map is the same as in Fig. 4.11.
4.6 Conclusions

We present the first results from the vertical water tunnel, which enables us to investigate the sedimentation of heavy, non-spherical particles in the shape of fibers and triads. We use a novel approach, a 3D-printed jet-array, to generate and control the amount of turbulence these particles experience as they sediment. Tracking their 3D positions and orientations using multiple high-speed cameras enables us to quantify the orientation distributions, rotation rates and sedimentation statistics. Experiments in quiescent fluid are used to measure inertial torques and the associated time scale. Based on these measurements, we determine the settling factor $S_F$ for the turbulence experiments and show that this single non-dimensional parameter can be used to quantify the orientation statistics of non-spherical, inertial particles. When $S_F \ll 1$, particle rotations are dominated by the turbulent motion of the fluid and the particle orientations are nearly randomly distributed. With increasing $S_F$, inertial torques start to preferentially align particles with their equilibrium orientation, $\theta = 90^\circ$ for fibers and $\theta = 0^\circ$ for triads. When $S_F \gg 1$, the particles are strongly aligned with that equilibrium orientation, and the variance around that orientation $\langle \cos^2(\theta) \rangle$ (or $0.5(1 - \langle \cos^2(\theta) \rangle)$ for triads) decreases according to a power law with $S_F^{-2}$.

We developed a model, based on slender body theory, that captures most of the statistics of inertial, ramified particles. For particles much smaller than the Kolmogorov scale, the analytical expression for the inertial torques is used to determine $S_F$. We show that in the limit $S_F \gg 1$, particles are strongly aligned and in quasi-steady balance of inertial torques and torques from turbulent strain. The power law dependence of the orientation variance can be derived analytically for this limit. The assumption of quasi-steady balance holds in the rapid settling limit, as well as in the slow settling limit, which enables us to use results from direct numerical simulations of Lagrangian velocity gradients to compare to our experimental results. Surprisingly, there are only a few ways to get out of the quasi-steady balance state. One way involves the inertial particle response time $\tau_p$. For $\tau_p \gg 1$, the particle dynamics become slow compared to the turbulent time scale and the particle does not have time to come to quasi-steady balance within a turbulent eddy. Another way to be out of the quasi-steady balance is when the particle Reynolds number becomes large. For $Re_\ell \gg 1$, inertial torques stop increasing linearly with Reynolds number. Therefore, the assumption of strongly aligned particles for $S_F \gg 1$ does not necessarily hold and more research needs to be done to reveal the behavior of inertial torques...
for high particle Reynolds numbers.

The ramified particle model agrees surprisingly well with the experimental measurements of the sedimentation rates of triads under different turbulence intensities. The model was developed for dissipation scale particles, but can be extended to large particles with the use of two constants, $C_\perp$ and $C_R$. The two particle specific constants can be measured in quiescent fluid and be used to predict the sedimentation rates in turbulence. The experiments show that triads sediment roughly 1.5 times faster when $\theta = 90^\circ$. This is in good agreement with the ramified particle model, and moreover, this is also what we would expect for high aspect ratio disks. This suggests that ramified particles not only mimic their equivalent ellipsoid when they are neutrally buoyant, but also that they can be used as models for heavy ellipsoids (with the exception of the preferential orientation of arms within the plane of the particle, $\psi$).

The sedimentation statistics are sensitive to the relative particle orientation, and therefore measurements of the local fluid velocity is important to accurately determine sedimentation rates. The reduced sedimentation rate with respect to the mean flow, or the increased standard deviation, when $\theta = 90^\circ$, shows that particles are sampling strong intermittent events when their orientation is significantly disturbed from their equilibrium orientation, $\theta = 0^\circ$.

The presented results provide the first experimental measurements of heavy, ramified particles in turbulence and the foundation to gain access to a complicated problem with a large parameter space. We identify a few non-dimensional parameters that can be used to describe the orientation and sedimentation statistics of non-spherical particles in turbulence. Still, more research needs to be done to fully understand the correlation between particle size and inertial torques for high particle Reynolds numbers and how the sampling of velocity gradients affect the orientation distributions.
5.1 Appendix A: Scanning Laser Particle Tracking System

5.1.1 Quality of Experimental Measurements

Here, we examine the quality of our experimental measurement of the velocity gradient tensor in several ways. First, we test the incompressible flow condition that requires the trace of the velocity gradient tensor to vanish. Figure 5.1 shows the joint probability density function (jPDF) of one of the diagonal components of the velocity gradient tensor versus the sum of the other two. The shape of the contour lines for all panels looks very similar, indicating that both the flow and the error are nearly isotropic. For perfectly incompressible flow, the vertical axes should be equivalent to the horizontal axes, as given by the dashed line. Deviations from the dashed line are caused by experimental uncertainties in measuring the velocity gradient. These deviations can be quantified using the ratio between major and minor axes of the contour ellipses in the jPDF. The larger this ratio is, the more accurate the measurement of the velocity gradient tensor. For all three panels in Fig. 5.1, this ratio is roughly 3.3, which is larger than the value of 2.6 in the early experiments of Lüthi et al. (2005). More recent experiments using scanning particle tracking systems by Hoyer et al. (2005) and Krug et al. (2014) have measured
deviations from incompressibility that are comparable or slightly smaller than ours.

A more stringent and relevant test of our experimental accuracy is comparing the directly measured fiber tumbling rate $\dot{p}_i$ computed by differentiating the time-dependent fiber orientation with that calculated from Jeffery’s equation, $\dot{p}_i^J$, using the measured velocity gradient tensor. Figure 5.2 shows the jPDF of $\dot{p}_i^J$ and $\dot{p}_i^J$. The contours of the jPDF again are ellipsoidal and deviate from the linear dashed line, just as they did in Fig. 5.1. The ratio between the major and minor axes of the contours is also close to 3.3, indicating that the uncertainty in the measurements of the velocity gradient tensor is not amplified through Jeffery’s equation. Previously, it has been proposed that the incompressibility condition can be used as a weighting factor for smoothing trajectories (Lüthi et al., 2005). The apparent incompressibility is characterized by

$$\delta = \frac{\left| \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right|}{\left| \frac{\partial u}{\partial x} \right| + \left| \frac{\partial v}{\partial y} \right| + \left| \frac{\partial w}{\partial z} \right|}$$

(5.1)

The weighting factor is a function of this $\delta$; it is designed to be 1 for small $\delta$, and smoothly changes to zero for large $\delta$. Thus if we smooth all the components of the velocity gradient using this weight, it nearly guarantees that the filtered velocity gradient has a small apparent compressibility. Note, however, that incompressibility only relies on the three diagonal components in the velocity gradient tensor, which are independent of the other six components; thus, compressibility may not be directly related to the accuracy of the full velocity gradient.
measurement. To test this argument, we plot in Fig. 5.3 (a) the incompressibility test we used in Fig. 5.1 (a) but only using samples with \( \delta < 0.15 \). This sampling is equivalent to applying a weighting function that equals 1 for \( \delta < 0.15 \) and 0 for \( \delta > 0.15 \), similar to the one used before (L"uthi et al., 2005). After enforcing incompressibility, we would expect that the contours should lie closer to the linear dashed line. We also plot the jPDF of \( \dot{p}_J^i \) and \( \dot{p}_i \) in Fig. 5.3 (b) as we did in Fig. 5.1 (b), but again enforcing \( \delta < 0.15 \). In this case, the shape of the contours does not change much, suggesting that weighting by the measured compressibility does not significantly improve the overall accuracy of the full velocity gradient tensor measurements.

### 5.1.2 Energy Dissipation Rate Measurements

From the velocity gradient tensor, we can directly obtain the mean energy dissipation rate \( \langle \epsilon \rangle = 2 \nu \langle S_{ij} S_{ij} \rangle \), one of the most important parameters in turbulence. For most experiments in homogeneous, isotropic turbulence without direct access to the velocity gradient tensor, the dissipation rate is extracted from the scaling of the second- or third-order Eulerian velocity structure functions in the inertial range. Applying Kolmogorov’s theory of isotropic turbulence (Kolmogorov [1941]), the longitudinal \( D_{LL} \) and transverse \( D_{NN} \) components of the second-order structure function \( (n = 2) \) scale as 

\[
D_{LL} = \langle \epsilon \rangle r^2 / 15 \nu \quad \text{and} \quad D_{NN} = 2 \langle \epsilon \rangle r^2 / 15 \nu
\]

in the dissipative range \( r < \eta \). Figure 5.4 (a) shows both \( D_{LL} \) and \( D_{NN} \) compensated with \( r^2 / 15 \nu \) and \( 2r^2 / 15 \nu \).
respectively. There is a very short plateau near \( r \approx \eta \), and the two compensated structure functions collapse with each other well in that range. Thus, the value of that plateau tells us the dissipation rate, which in our experiment is \( \langle \epsilon \rangle = 9 \times 10^{-5} \text{ m}^2 \text{ s}^{-3} \). In addition, Fig. 5.4 (b) shows the third-order longitudinal structure function \( (D_{LLL}) \). The dissipation rate can be obtained from the Kolmogorov 4/5 law \( (D_{LLL} = -\frac{4}{5} \langle \epsilon \rangle r) \) in the inertial range \( \eta \ll r \ll L \), where \( L \) is the integral length scale. Here, because of the finite Reynolds number and the small measurement volume, there is a sharp cutoff near the infrared end of the inertial range. There is therefore only a narrow peak roughly from 50\( \eta \) to 60\( \eta \), but the estimate of \( \langle \epsilon \rangle \) from that peak is very close to the estimate from the dissipation-range scaling of the second-order structure functions.

The dashed line in both panels shows the estimate from the velocity gradient measurement from tracer particles within a 6\( \eta \) radius of the fiber. It is clear that this value is slightly smaller than the estimate from both velocity structure function methods. This is because the velocity gradient is coarse-grained over a small volume, which removes contributions from some of the smallest scales to the energy dissipation rate.

Figure 5.3: Comparison of the joint probability density function with previous figures after setting the condition \( \delta < 0.15 \): (a) same plot as Fig. 5.1; (b) same plot as Fig. 5.2.
Figure 5.4: The mean energy dissipation rate $\langle \epsilon \rangle$ calculated from different methods: (a) comparison between $\langle \epsilon \rangle$ obtained from the dissipative range of the longitudinal ($D_{LL}$, dot-dashed line) and transverse ($D_{NN}$; solid line) second-order structure function and the direct measurement of the velocity gradient tensor (black dashed line). (b) Dissipation rate obtained from compensated third-order structure function (solid line) and the direct measurement of the velocity gradient tensor (black dashed line).

5.2 Appendix B: Complex-Shaped Particles

5.2.1 Methodology of Particle Fabrication

In this section, we focus on some of the important details of the particle fabrication. A visual description can also be found in Cole et al. (2016). We start with the particle design in AutoCAD. The simplest particle shape and basic building block of a ramified particle is a single cylinder. For that, we create a \texttt{CIRCLE} with diameter $D$ and use the \texttt{EXTRUDE} command to create a cylinder of length $L$. Any number of cylinders can now be assembled (\texttt{3DROTATE}, \texttt{3DMOVE}) to form a ramified particle, e.g. three cylinders in a plane with 120$^\circ$ angle between them is what we call a triad.

Before exporting the model as 3D printable .STL file (\texttt{STLOUT}), a few things should be considered. Particles that oriented in the $x$-$y$ plane will be printed flat. This can cause arms with a circular cross section to be distorted into arms with elliptical cross section due to fabrication process of Fused Deposition Modeling printers. Similarly, if thin arms are oriented along the $z$-axis, the arm will show larger irregularities compared to arms along other directions. An even

\footnote{Bold text represents commands that can be used directly in AutoCAD with \texttt{DYNAMIC INPUT} set to 1.}
distribution of inhomogeneities across the entire particle should be the main goal, rather than trying to minimize the feature size on any single arm. We had the best success with triads, when they were printed at $\theta = 45^\circ$, and one arm either pointing upwards, $\psi = \pi/3$, or downwards, $\psi = 0$.

Since the cost of 3D printing is strongly influenced by the print time, it is advisable to stack particles as closely together as possible. The minimal distance between particles is set by the specific 3D printer and should be determined before stacking. Printing in the $x$-$y$ plane is faster than printing in the $z$-direction, therefore, particles should be stacked in the $x$-$y$ plane, as shown in Fig. 5.5.

We had good success with Fused Deposition Modeling. The big advantage over other 3D printing processes is that the support material (see Fig. 5.6) can be easily removed and does not leave residues on the particles. In order to keep the fabrication cost down, we remove the support material ourselves. It can be dissolved using a sonicator and a strong base solution, e.g. NaOH. If the particles are kept in the solution for too long (more than 60 minutes), they soften and might deform. It is therefore important to remove as much of the support structure by hand as possible. Different stages of this process are shown in Fig. 5.6.

After the support material is removed and the particles are cleaned, we dye the particles with Rhodamine-B\textsuperscript{2}. This fluorescent dye absorbs green light (e.g. from a frequency doubled Nd:YAG at 532 nm) and emits in the red wavelength range, near 600 nm. The spectrum, as measured

\textbf{Figure 5.5:} AutoCAD rendering of 200 triads, stacked closely together, viewed from different directions.

\textsuperscript{2}This step is obviously only necessary when using laser induced fluorescence.
with a spectrometer from Ocean Optics, is shown in Fig. 5.7 (a). The particles absorb the dye at room temperature, however, baking the particle at a slightly elevated temperature (∼70°C) results in more evenly distributed dye within the particle. The dye concentration is another important factor. Figure 5.7 (b) shows the result of two different dye concentrations and baking times. It might be counter intuitive, but the lighter particle shows a much stronger fluorescent signal.

Particles should be stored in the same fluid as the experiments. Since they absorb a small amount of that fluid, the particle density can change significantly. Moreover, the amount of fluid absorbed by the particles depends on particle size, which means the density of each particle type has to be measured separately before the experiments, the 3D print material specifications provided by the manufacturer are not reliable. Storing the particles in fluid also protects the particles and any excess dye can bleed out, preventing contamination of the experimental apparatus.

5.2.2 Details of the Improved Orientation-Finding Algorithm

We do most of the image processing in MATLAB and I will use it for the examples presented in this section. Most modern languages have similar libraries/packages and it shouldn’t be too difficult to translate the ideas.

The first step of any orientation-finding algorithm is image segmentation, where we identify the
Figure 5.7: (a) Comparison of the emission spectrum of Rhodamine-B dyed particles. Darker particles show less emission than light particles. (b) Image of two dyed tetrads, the darker one (left) was dyed for 3 hours at 70°C in a 0.6 g/l Rhodamine-B concentration. The lighter one (right) was dyed for 1 hour at 70°C in a 0.6 g/l Rhodamine-B concentration.

Pixel clusters on the 2D images of each camera that correspond to particles (also known as blob detection). The MATLAB function **regionprops** finds bright pixel clusters, calculates the area, mean and max brightness, 2D orientation and many other features. Non-spherical particles like fibers or ramified particles can be distinguished from spherical tracer particles using these features.

Example #1 - using regionprops:

```matlab
% SAMPLE CODE FOR IMAGE SEGMENTATION USING MATLABS REGIONPROPS COMMAND
function clusters = sk_find_particles(IMG)
    pix_info = regionprops(IMG,'Centroid','Area','MeanIntensity');
    center = cat(1,pix_info.WeightedCentroid);
    area = cat(1,pix_info.Area);
    meanbright = cat(1,pix_info.MeanIntensity);
    ind = find(area>CL_MIN_AREA & area<CL_MAX_AREA);
    if ~isempty(ind)
        clusters(:,1:2) = center(ind,:); % x,y center position of particle
        clusters(:,3) = meanbright(ind); % brightness
    end
end
```

This function takes an image as argument and uses the **regionprops** command to find the center, area and mean brightness of each cluster. Defining the constants CL_MIN_AREA and CL_MAX_AREA as the minimum and maximum area of a particle, respectively, enables us to
separate large particles from small tracers.

After finding the clusters on all images from each camera, and together with the camera calibration data\(^3\) we use a C++ stereomatching and tracking code (Ouellette et al., 2006) to calculate the 3D position of the particles. Since the 2D center of the pixel cluster found with regionprops is not necessarily the actual particle center, the tolerances on the stereomatching algorithm have to be relaxed. This gives a rough estimate of the 3D position and serves as initial guess for the orientation-finding algorithm described next. Note: The stereomatching and tracking code can be used for spherical tracer particles and non-spherical particles. The number of tracked tracer particles can be increased if we allow particles to be seen on only three out of four cameras. This is not recommended for non-spherical particles since the orientation-finding algorithm has not been tested with less than four cameras and might not yield reliable results.

Now we are ready to find the 3D orientation of the particle. We create a model of the particle and use the determined 3D position and camera calibration parameters to project the model onto the image plane of each camera. The model consists of individual straight line segments, e.g. a triad is defined by four points, the center and three end points of each arm.

Example #2 - creating the model:

```
1 % SAMPLE CODE TO GENERATE A MODEL OF A TRIAD
2 function model = sk_triad(armlength, armdiameter)
3     arm1 = [0,1,0].*armlength;  % armlength in mm
4     arm2 = [cos(7*pi/6),sin(7*pi/6),0].*armlength;
5     arm3 = [cos(-pi/6),sin(-pi/6),0].*armlength;
6     arms = [arm1;arm2;arm3];
7     model.arms = arms;
8     model.cntr = zeros(size(arms,1),3);
9     model.rad = armdiameter/2;  % armdiameter in mm
10    end
```

This function takes the arm length and arm diameter (in physical units) as arguments and defines the three end points of the arms of a triad around the particle’s center at \( x = y = z = 0 \). One arm points along the \( y \)-direction, the other two arms are in the same plane, but rotated relative to that orientation by \(+120^\circ\) and \(-120^\circ\), respectively. The model also stores the radius of the arms.

\(^3\)calibration process is not described here. See Wijesinghe (2012) for more details.
Figure 5.8: Illustration of the reduced Euler angle space and the residual landscape. Bad particle orientations (red box) correspond to large residual values. Correct orientations (green box) correspond to the global minimum.

More complicated particle shapes can be built in a similar fashion. We use the same algorithm for triads, tetrads and chiral dipoles. In the case of chiral dipoles, the individual line segments are not connected at a central point, but end-to-end, to form a discretized model of a chiral dipole. The computational time does not grow linearly with segments used and has to be tested to find a good balance between accuracy and speed.

To keep the computational time at a minimum, we only project the end points onto the image plane and calculate the intensity distribution in 2D, rather than creating a full 3D model and projecting all points onto the image plane. The difference between projected model and image data enables us to define a loss function. We calculate the residual as the squared difference between the intensity of the projected model and the data on all cameras. Since we don’t know the 3D orientation of the particle a priori, we use a random initial guess for the three Euler angles that define the particle’s orientation. These are most likely not the correct three Euler angles, which leads to a large residual value from the loss function. This is shown schematically in Fig. 5.8, where we plot the residual value landscape for two Euler angles (the third Euler angles is not shown for visualization purposes). A non-linear optimization routine (fminsearch
function in MATLAB) can now search in Euler angle space until it finds the minimum, at which point the correct particle orientation is identified. Complex-shaped particles like chiral dipoles have several local minima, which correspond to wrong particle orientations, e.g. the model finds the rough orientation of the particle but is flipped (chiral dipoles are not symmetric!). For that reason, we compare the residual value to a second optimization that starts with the flipped orientation. Up to this point, the 3D position is only a rough estimate. This means we also have to optimize the particle position. We find that three position variables and three Euler angles, for a total of six parameters, is often times too much to minimize at once, especially if the initial guess is far off. The question is if one should optimize the particle position first with a possibly wrong orientation, or optimize the orientation when the position of the projected model might be offset? Unfortunately, there is no simple solution to this problem and the answer is that it depends on the particle shape. Fortunately, it is only a problem for the first frame of each track, since we can use the position and orientation from the previous frame as initial guess for the next optimization (translation and rotation between consecutive frames is small).

5.3 Appendix C: Sedimentation of Heavy, Non-Spherical Particles in Turbulence

5.3.1 Inertia Effects on the Motion of Long, Slender Bodies

The calculations in this section lay out some of the steps involved in deriving the low Reynolds number expressions for the forces and torques on fibers and how the calculations from Khayat and Cox (1989) relate to the results from Lopez and Guazzelli (2017). I start with the equation of the total force on a body with straight center line and arbitrary orientation, (Eq. 6.4 in
Khayat and Cox (1989):

\[
\frac{F}{2\pi\mu W\ell} = \left(\frac{2}{\ln(\kappa')}\right) (\cos(\alpha)\hat{p} - 2\hat{e}_W - 2\pi\mu W\ell)
\]

where \( X = R_{\ell}(1 - \cos(\alpha)) \), \( Y = R_{\ell}(1 + \cos(\alpha)) \), \( \gamma = 0.577... \) is Euler’s constant, and

\[
E_1(x) = \int_x^\infty \frac{\exp(-y)}{y} \, dy.
\]

The particle Reynolds number \( R_{\ell} \) is based on the particle half-length \( \ell \). \( \hat{p} \) is a unit vector along the fiber axis (\( \beta \) in Khayat and Cox (1989)), \( \hat{e}_W \) is a unit vector in the direction of the velocity \( W \) and \( \alpha \) is the angle between \( \hat{p} \) and \( W \). Note, Khayat and Cox (1989) defined the aspect-ratio \( \kappa' = \frac{r}{\ell} \), whereas I will be using \( \kappa = \frac{\ell}{r} \).

In the case of circular cross-section and fore-aft symmetry, \( K(s) = 0 \) and \( \ln(K(s)) = 0 \) for \(-1 \leq s \leq 1 \) and \( \int_{-1}^{1} R_s(s) \, ds = 0 \), Eq. 5.2 simplifies and the total force on the body can be decomposed into a drag component \( D \) parallel to the relative velocity and a lift component \( L \), perpendicular to the relative velocity:

\[
\frac{D}{\mu W\ell} = 4\pi(2 - \cos^2(\alpha)) \left( \frac{1 + F_D(R_{\ell}, \alpha)}{\ln(\kappa)} \right)
\]

\[
\frac{L}{\mu W\ell} = -2\pi\sin(2\alpha) \left( \frac{1 + F_L(R_{\ell}, \alpha)}{\ln(\kappa)} \right)
\]

Khayat and Cox (1989) approximate these equations for high aspect ratio with:

\[
\frac{D}{\mu W\ell} = \frac{-4\pi(2 - \cos^2(\alpha))}{\ln(\kappa) + F_D(R_{\ell}, \alpha)}
\]

\[
\frac{L}{\mu W\ell} = \frac{-2\pi\sin(2\alpha)}{\ln(\kappa) + F_L(R_{\ell}, \alpha)}
\]
Here, $F_D(Re\ell, \alpha)$ and $F_L(Re\ell, \alpha)$ are:

$$
F_D(Re\ell, \alpha) = \frac{1}{2(2 - \cos^2(\alpha))} \left[ \frac{\cos^2(\alpha)}{2 Re\ell} (E_1(X) + \ln(X) + \gamma - X) + \frac{\cos^2(\alpha)}{2 Re\ell} (E_1(Y) + \ln(Y) + \gamma - Y) + 3 \cos^2(\alpha) - 2 \right] 
+ \frac{1}{2} \left[ E_1(X) + \frac{1 - \exp(-X)}{X} + \ln(X) + \gamma 
+ E_1(Y) + \frac{1 - \exp(-Y)}{Y} + \ln(Y) + \gamma \right] - \ln(4) \tag{5.7}
$$

$$
F_L(Re\ell, \alpha) = \frac{1}{\sin(2\alpha)} \left[ \frac{(2 - \cos(\alpha) + \cos^2(\alpha)) \sin(\alpha)}{2X} (E_1(X) + \ln(X) + \gamma - X) 
- \frac{(2 + \cos(\alpha) + \cos^2(\alpha)) \sin(\alpha)}{2Y} (E_1(Y) + \ln(Y) + \gamma - Y) \right] 
+ \frac{1}{2} \left[ E_1(X) + \frac{1 - \exp(-X)}{X} + \ln(X) + \gamma 
+ E_1(Y) + \frac{1 - \exp(-Y)}{Y} + \ln(Y) + \gamma - 2 \ln(4) \right] - \frac{3}{2} \tag{5.8}
$$

(Eq. 6.9 and Eq. 6.11 from Khayat and Cox (1989)). Eq. 5.3 & Eq. 5.4 yield the following expression for the velocity $W$:

$$
W(2 - \cos^2(\alpha)) = \frac{\ln(\kappa)}{4\pi \mu \ell} D \left( 1 - \frac{F_D(Re\ell, \alpha)}{\ln(\kappa)} \right) \tag{5.9}
$$

$$
W \sin(\alpha) \cos(\alpha) = \frac{\ln(\kappa)}{2\pi \mu \ell} L \left( 1 - \frac{F_L(Re\ell, \alpha)}{\ln(\kappa)} \right) \tag{5.10}
$$

For $\alpha = 0$ and $\alpha = \pi/2$, Eq. 5.9 yields the mobility coefficients for a translational stationary particle in uniform fluid flow.

$$
W_\parallel = \frac{\ln(\kappa)}{4\pi \mu \ell} M_{\parallel} \left( 1 - \frac{F_\parallel}{\ln(\kappa)} \right) \tag{5.11}
$$

$$
W_\perp = \frac{\ln(\kappa)}{8\pi \mu \ell} M_{\perp} \left( 1 - \frac{F_\perp}{\ln(\kappa)} \right) \tag{5.12}
$$

where $F_\parallel = F_D(Re\ell, \alpha = 0)$ and $F_\perp = F_D(Re\ell, \alpha = \pi/2)$. For the limiting case where the angle $\alpha \to 0$ or for the evaluation of the equation at low Reynolds numbers, this expansion is
useful:

\[
\lim_{x \to 0} E_1(x) = \lim_{x \to 0} \int_{x}^{\infty} \frac{\exp(-y)}{y} \, dy
\]

\[
= \lim_{x \to 0} \left[ \exp(-y) \ln(y) \bigg|_x^{\infty} + \int_x^{\infty} \exp(-y) \ln(y) \, dy \right]
\]

\[
= -\gamma - \ln(x) + x - \frac{x^2}{4}
\]

The expressions for \(F_\parallel\) and \(F_\perp\) can also be found in Lopez and Guazzelli (2017) (Eq. B.9):

\[
F_\parallel = \frac{1}{2} \left[ \frac{E_1(2Re_\ell) + \ln(2Re_\ell) + \gamma - \exp(-2Re_\ell) + 1}{2Re_\ell} \right.
\]

\[
+ \frac{E_1(2Re_\ell) + \ln(2Re_\ell) + \gamma}{2Re_\ell} \right] - \ln(4) + \frac{1}{2}
\]

\[
F_\perp = E_1(Re_\ell) + \ln(Re_\ell) - \frac{\exp(-Re_\ell) - 1}{Re_\ell} + \gamma - \ln(4) - \frac{1}{2}
\]

For \(Re_\ell \to 0\), Eq. 5.7 and Eq. 5.8 yield the same results as Khayat and Cox (1989):

\[
F_\parallel(Re_\ell \to 0) = \frac{Re_\ell}{4} - \ln(4) + \frac{3}{2}
\]

\[
F_\perp(Re_\ell \to 0) = \frac{Re_\ell}{2} - \ln(4) + \frac{1}{2}
\]

Balancing the forces on the particle yields the same set of non-linear equations for the velocity \(W\) and the angle \(\alpha\) as Eq. 5.9 and Eq. 5.10:

\[
\sum F_x = 0 = -mg + D\cos(\theta - \alpha) + L\sin(\theta - \alpha)
\]

\[
= f - W \left((2 - \cos^2(\alpha))(1 + F_D(Re_\ell, \alpha)/\ln(\kappa))\cos(\theta - \alpha)\right)
\]

\[
+ W \left((\sin(\alpha)\cos(\alpha))(1 + F_L(Re_\ell, \alpha)/\ln(\kappa))\sin(\theta - \alpha)\right)
\]

\[
\sum F_y = 0 = -D\sin(\theta - \alpha) + L\cos(\theta - \alpha) = 0
\]

\[
= - \left((2 - \cos^2(\alpha))(1 + F_D(Re_\ell, \alpha)/\ln(\kappa))\sin(\theta - \alpha)\right)
\]

\[
- \left((\sin(\alpha)\cos(\alpha))(1 + F_L(Re_\ell, \alpha)/\ln(\kappa))\cos(\theta - \alpha)\right)
\]

Solving Eq. 5.21 for \(F_L(Re_\ell, \alpha)\) or \(F_D(Re_\ell, \alpha)\) and substituting into Eq. 5.20 gives the desired result:

\[
W(2 - \cos^2(\alpha))(1 + F_D(Re_\ell, \alpha)/\ln(\kappa)) - f\cos(\theta - \alpha) = 0
\]

\[
-W(\sin(\alpha)\cos(\alpha))(1 + F_L(Re_\ell, \alpha)/\ln(\kappa)) + f\sin(\theta - \alpha) = 0
\]
Figure 5.9: Comparison of the different models. The velocity is averaged over all orientations (in 2D) and normalized by $f$. (blue dotted line: slender-body limit without inertial nor aspect ratio corrections; red dashed dotted line: aspect ratio corrections; yellow dashed line: low Reynolds number inertial corrections; green solid line: inertial corrections, but not the full non-linear system.)

with $f = mg \ln(\kappa)/4\pi\mu\ell$.

Similarly, the torque on a slender fiber is given by (Eq. 6.6 in Khayat and Cox (1989)),

$$
\frac{G}{2\pi\mu W\ell^2} = \left(\frac{1}{\ln(\kappa)}\right)^2 \left[\frac{\cos(\alpha)}{X} \left(2 + 2\frac{\exp(-X) - 1}{X} - E_1(X) - \ln(X) - \gamma\right)ight. \\
+ \frac{\cos(\alpha)}{Y} \left(2 + 2\frac{\exp(-Y) - 1}{Y} - E_1(Y) - \ln(Y) - \gamma\right) \\
\left. - \frac{2}{X} \left(1 + \frac{\exp(-X) - 1}{X}\right) + \frac{2}{Y} \left(1 + \frac{\exp(-Y) - 1}{Y}\right)\right] \hat{p} \times \hat{e}_W \\
+ O\left(\frac{1}{\ln(\kappa)}\right)^3 
$$

(5.24)

This yields the magnitude of the torque

$$
\frac{G}{\mu W\ell^2} = -2\pi \left(\frac{1}{\ln(\kappa)}\right)^2 \mathcal{F}(Re, \alpha) + O\left(\frac{1}{\ln(\kappa)}\right)^3 
$$

(5.25)
Figure 5.10: Illustration of the coordinate system and the variables defining particle orientation $\hat{p}$, relative particle velocity $W$ and the angles $\theta$ and $\alpha$. (a) Fiber. (b) Triad.

where

\[
\mathcal{F}_G(Re_\ell, \alpha) = \left[ \frac{\cos(\alpha)}{X} \left( 2 + 2\exp(-X) - \frac{1}{X} - E_1(X) - \ln(X) - \gamma \right) \right. \\
+ \frac{\cos(\alpha)}{Y} \left( 2 + 2\exp(-Y) - \frac{1}{Y} - E_1(Y) - \ln(Y) - \gamma \right) \\
- \frac{2}{X} \left( 1 + \frac{\exp(-X) - 1}{X} \right) + \frac{2}{Y} \left( 1 + \frac{\exp(-Y) - 1}{Y} \right) \left. \right] \sin(\alpha) \\
(5.26)
\]

This becomes in the limit $Re_\ell \to 0$:

\[
\mathcal{F}_G(Re_\ell, \alpha) = -\frac{5}{12} \sin(2\alpha) Re_\ell. \\
(5.27)
\]

If the particle is free to rotate, it also experiences a rotational drag. This drag torque $G_{\text{drag}}$ can be quantified in the following way:

\[
G_{\text{drag}} = \frac{1}{L} \int_{-\ell}^{\ell} (r \times F_{\text{drag}}) \, dr \\
= \frac{\pi \mu L^3}{3 \ln(2\kappa)} (\Omega \times \hat{p}) \\
(5.28)
\]

where $F_{\text{drag}}$ can be extracted from Eq. 5.2 when $W$ is replaced with the tumbling rate $\dot{\hat{p}} = \Omega \times \hat{p}$, which is always perpendicular to $\hat{p}$. Setting this drag torque equal to the inertial torque expression at low Reynolds numbers yields the tumbling rate of a fiber (same expression as in Lopez and Guazzelli (2017)): 
An alternative expression that is in the literature (Lopez and Guazzelli, 2017), and includes higher terms in the expansion

\[ |\dot{p}| = -\frac{3}{4} \frac{W}{\ell} \frac{F_G}{\ln(\kappa)} \]  

(5.30)

In the low Reynolds number limit (Eq. 5.27), this simplifies to

\[ |\dot{p}| = -\frac{5}{16} \frac{Re \ell}{\kappa^{\frac{1}{2}}} W \sin(2\alpha) \]  

(5.31)

which is the same as Eq. 4.10.

### 5.3.2 Rapid Settling of Small Fibers

We know that in absence of turbulence, the equilibrium orientation of sedimenting fibers is with their symmetry axis perpendicular to their direction of motion - \( \hat{p} \perp \hat{g} \). Inertial torques align particles with that equilibrium orientation. Turbulence tends to align particles with the velocity gradients, and therefore tends to randomize the particle orientation. We derive a model to characterize the relative strength of each effect in the rapid settling limit - when fibers are settling at a faster rate than the Kolmogorov shear rate - \( \eta/W \ll \tau_\eta \) or \( u_\eta/W \ll 1 \). We are considering sub-Kolmogorov fibers with \( \ell \ll \eta \).

The mean-square tumbling rate of fibers due to inertial torques can be calculated from Eq. 4.10

\[ \langle \dot{p}_3 \dot{p}_3 \rangle_{\text{sed}} = \left( \frac{5Re \ell W}{8\ell \ln(2\kappa)} \right)^2 \langle p_3^2 \rangle = \frac{1}{\tau^2_{\text{sed},45}} \langle p_3^2 \rangle. \]  

(5.32)

where we assume \( p_3 \) is along the direction of \( W \), without loss of generality. The components of the mean-square tumbling rate in the 1 and 2 direction are small compared to the 3 direction, since rapid settling fibers will be strongly aligned by inertial torques and therefore \( W \) in the 1 and 2 direction will be small.

In the rapid settling limit, the orientation of a fiber becomes uncorrelated from the velocity gradients and we can assume the mean-square tumbling rate due to turbulence is the same as that of randomly oriented fibers in isotropic turbulence, as given by Eq. 2.3

\[ \langle \dot{p}_i \dot{p}_i \rangle_{\text{turb}} = \frac{1}{\tau^2_{\eta}} \left[ 1 + \frac{1}{6} \left( \frac{\kappa^2 - 1}{\kappa^2 + 1} \right)^2 \right] = \frac{1}{\tau^2_{\text{turb}}}. \]  

(5.33)

For a strongly aligned particle, \( p_3 \ll 1 \), that samples a random, isotropic turbulence field, \( \langle \dot{p}_3 \dot{p}_3 \rangle = 2\langle \dot{p}_1 \dot{p}_1 \rangle = 2\langle \dot{p}_2 \dot{p}_2 \rangle \), because \( \dot{p} \) is perpendicular to \( \dot{p} \). Therefore,

\[ \langle \dot{p}_3 \dot{p}_3 \rangle_{\text{turb}} = \frac{1}{2} \langle \dot{p}_i \dot{p}_i \rangle_{\text{turb}}. \]  

(5.34)
If we assume a quasi-steady balance, where the mean-square tumbling rate due to sedimentation is equal to the mean-square tumbling rate due to turbulence (in the 3 direction), we can use Eq. 5.32 and Eq. 5.33 with Eq. 5.34 to solve for the orientation variance

\[ \langle p_3^2 \rangle = \left( \frac{8 \ell \ln (2\kappa)}{5 Re \ell W} \right)^2 \frac{1}{\eta^2} \left[ \frac{1}{12} + \frac{1}{20} \left( \frac{\kappa^2 - 1}{\kappa^2 + 1} \right)^2 \right]. \]

This yields in the high aspect ratio limit

\[ \langle p_3^2 \rangle = \frac{2}{15} \frac{1}{S_F^2}. \]

Anubhab Roy, our collaborator from the Department of Applied Mechanics at the Indian Institute of Technology Madras (formerly in Donald L. Koch’s group at Cornell University), independently derived the same result. The following is his derivation.

The total tumbling rate of a fiber settling in turbulence is given by the tumbling rate due to inertial torques, given by Eq. 4.10, and the tumbling rate due to turbulence, given by Jeffery’s equations (Eq. 1.9).

\[ \dot{p}_i = \frac{5 Re \ell p_3 W}{8 \ell \ln (2\kappa)} (p_i p_3 - \delta_{i3}) + \Omega_{ij} p_j + \frac{\kappa^2 - 1}{\kappa^2 + 1} (S_{ij} p_j - p_i p_k S_{kl} p_l) \]

Since the body will remain horizontal on average, the rapid settling limit gives \( p_3 \ll p_{1,2} \). Thus from Eq. 5.37, we recover Jeffery’s equation for \( i = 1,2 \)

\[ \dot{p}_i = \Omega_{ij} p_j + \frac{\kappa^2 - 1}{\kappa^2 + 1} (S_{ij} p_j - p_i p_k S_{kl} p_l) \]

indicating that in the plane normal to gravity, the fiber executes Jeffery orbits. In the settling direction, there will exist a quasi-steady equilibrium where the torque due to fluid inertia will balance the torque due to turbulence.

\[ \frac{5 Re \ell p_3 W}{8 \ell \ln (2\kappa)} (p_3^2 - 1) + \delta_{i3} \Omega_{ij} p_j + \frac{\kappa^2 - 1}{\kappa^2 + 1} (\delta_{i3} S_{ij} p_j - p_3 p_k S_{kl} p_l) = 0 \]

Since \( p_i \ll 1 \), we can further simplify the above equation:

\[ p_3 \sim \frac{8 \ell \ln (2\kappa)}{5 Re \ell W} \left( \delta_{i3} \Omega_{ij} p_j + \frac{\kappa^2 - 1}{\kappa^2 + 1} \delta_{i3} S_{ij} p_j \right) \]

Thus, the variance, characterizing the “wiggling” motion of a fiber out of the horizontal plane, is:

\[ \langle p_3^2 \rangle = \left( \frac{8 \ell \ln (2\kappa)}{5 Re \ell W} \right)^2 \delta_{i3} p_j \delta_{m3} p_n \left( \frac{\kappa^2 - 1}{\kappa^2 + 1} \right)^2 (\Omega_{ij} \Omega_{mn}) + (S_{ij} S_{mn}) \]
where the $\langle \rangle$ denote the ensemble averages. Cross terms such as $\langle S_{ij} \Omega_{mn} \rangle$ are zero due to isotropy. $\langle S_{ij} S_{mn} \rangle$ and $\langle \Omega_{ij} \Omega_{mn} \rangle$ are fourth-order isotropic tensors whose form can be deduced using the property of symmetry, $\langle S_{ij} S_{mn} \rangle = \langle S_{ji} S_{mn} \rangle$; and continuity $\langle S_{ii} S_{mn} \rangle = \langle S_{ij} S_{mm} \rangle = 0$ (Brunk et al. 1997). This reduces the fourth-order tensors to the following general form:

$$\langle \Omega_{ij} \Omega_{mn} \rangle = \Omega_{ij} \Omega_{mn} = \Omega^2 \frac{6}{\delta_{im} \delta_{jn} - \delta_{in} \delta_{jm}} \quad (5.42)$$

$$\langle S_{ij} S_{mn} \rangle = S^2 \frac{10}{\delta_{im} \delta_{jn} + \delta_{in} \delta_{jm} - \frac{2}{3} \delta_{ij} \delta_{mn}} \quad (5.43)$$

where $\Omega^2 = \langle \Omega_{ij} \Omega_{ij} \rangle$ and $S^2 = \langle S_{ij} S_{ij} \rangle$. Plugging Eq. 5.42 and 5.43 in Eq. 5.41, we get:

$$\langle p^2 \rangle = \left( \frac{8 \ln (2 \kappa)}{5 R e W} \right)^2 \delta_{ij} \delta_{mn} \left[ \Omega^2 \frac{6}{\delta_{im} - \delta_{jm}} + \frac{\kappa^2 - 1}{\kappa^2 + 1} \right]$$

$$\langle S^2 \rangle = \left( \frac{8 \ln (2 \kappa)}{5 R e W} \right)^2 \frac{S^2}{10} \left[ \left( \frac{\kappa^2 - 1}{\kappa^2 + 1} \right)^2 \right] \quad (5.44)$$

$$\sim \left( \frac{8 \ln (2 \kappa)}{5 R e W} \right)^2 \left[ \frac{\Omega^2}{6} + \frac{\kappa^2 - 1}{\kappa^2 + 1} \right]^2 S^2 \quad (5.45)$$

$$\approx \left( \frac{8 \ln (2 \kappa)}{5 R e W} \right)^2 \left[ \frac{1}{12} + \frac{1}{20} \left( \frac{\kappa^2 - 1}{\kappa^2 + 1} \right)^2 \right] \quad (5.46)$$

$$= \left( \frac{8 \ln (2 \kappa)}{5 R e W} \right)^2 \frac{1}{\sigma_0^2} \left[ \frac{1}{12} + \frac{1}{20} \left( \frac{\kappa^2 - 1}{\kappa^2 + 1} \right)^2 \right] \quad (5.47)$$

where we have used the relation $\Omega^2 = S^2 = 1/2 \sigma_0^2$, typically used for homogeneous isotropic turbulence. Thus, we have for the rapid settling limit the following relation characterizing the departure of orientation from the horizontal plane due to turbulence

$$\langle p^2 \rangle = \left[ \frac{1}{12} + \frac{1}{20} \left( \frac{\kappa^2 - 1}{\kappa^2 + 1} \right)^2 \right] \frac{1}{\sigma_F^2} \equiv 2 \frac{1}{5} \frac{1}{\sigma_F^2}. \quad (5.48)$$

**5.3.3 Preferential Alignment of Triads**

We complement the alignment results from Sec. 3.4 by showing the alignment PDFs of sedimenting triads, Fig. 5.11, with their solid-body rotation rate vector, for different turbulence intensities. For high turbulence intensities, particles show a similar preferential alignment as crosses (see Fig. 3.4), the PDF has a peak at $\hat{p} \cdot \hat{\Omega}_s = 0$, meaning that triads are on average aligned with their orientation vector $\hat{p}$ perpendicular to their solid-body rotation vector $\hat{\Omega}_s$. This alignment of inertial triads is not as strong as compared to neutrally buoyant crosses. One reason for weaker alignment is that inertial triads experience inertial torques that tend to align $\hat{p}$ parallel to the direction of gravity. When the inertial torques become dominant over turbulent
torques, particles are strongly aligned with gravity and are only able to rotate around $\hat{p}$, i.e. $\hat{p} \cdot \hat{\Omega}_s = 1$. This shift of preferential alignment with the solid-body rotation rate can be seen very well in the PDFs of the low turbulence intensity cases of Fig. 5.11.

In addition to the alignment of $\hat{p}$ with the solid-body rotation vector, we present the alignment PDF of $\hat{p}$ with the coarse grained velocity gradients. The experiments were not intended to resolve the velocity gradients, so the results presented in Fig. 5.12 are noisy. Even though the preferential alignment is generally not very strong, there is a clear trend that shows preferential alignment of $\hat{p}$ parallel to $\hat{e}_3$, the compressional direction of the strain rate tensor. This alignment makes sense for the high turbulence cases, where our inertial particles should approach similar distributions as neutrally buoyant particles, where particles are aligned with their long axis along the extensional directions, $\hat{e}_1$ and $\hat{e}_2$, of the strain rate tensor. In the case of low turbulence intensity, this preferential alignment can not be explained using that argument because inertial particles are strongly aligned with gravity and the velocity gradients should be randomly oriented with respect to gravity. The PDF should be uniform.

More work needs to be done on this topic, but we assume that the two-way coupling of the particle might play a role. Sedimenting particles pull fluid with them, therefore inducing velocity gradients that are compressional along the direction of $\hat{e}_3$. We can do a simple analysis based on conservation of momentum to see if the magnitude of the induced velocity gradients is on of

**Figure 5.11:** The PDF of the alignment between a particle’s orientation and its solid-body rotation rate, $\hat{p} \cdot \hat{\Omega}_s$, for (a) small triads and (b) large triads. The colormap indicates turbulence intensities, from low to high (cold to hot).
order where it matters. The momentum of the particle is

\[ P_p = \Delta m g L / W_p \] (5.49)

where \( L/W_p \) is the time it takes a particle to sediment a distance \( L \) (one particle length). \( \Delta m = 3(\rho_p - \rho_f)\pi LD^2/4 \) is the relative mass of a triad and \( g \) is gravity. The particle transfers that momentum to the fluid it is sweeping through. We approximate that fluid volume as a cylinder with diameter \( L \). Setting the momentum of the particle, \( P_p \), equal to the momentum of the fluid, \( P_f = m_f W_f \), enables us to solve for the fluid velocity induced by the particle:

\[ W_f = \frac{(\rho_p - \rho_f)}{\rho_f} \frac{3 D^2 g}{2 L W_p} \sim 2.1 \text{ mm/s} \] (5.50)

This is on the order of the transverse velocity differences at the scale of the particle for low turbulence intensities and about 10% of the transverse velocity differences at high turbulence intensities. We assume the two-way coupling plays a non-negligible role in the alignment statistics of inertial particles. We also show the alignment of the velocity gradients with gravity, the \( \hat{z} \) direction in lab coordinates. Figure 5.13 shows no preferential alignment of within the measurement noise for the high turbulence cases, but we see similar preferential alignment of \( \hat{e}_z \) with \( \hat{z} \) for the low turbulence case. Reminder, the velocity gradients are only measured around particles, so we expect to see this trend.
Figure 5.12: PDF of the cosine of the angle between the particle orientation $\hat{p}$ and both vorticity $\hat{\omega}$ and the eigenvectors of the strain-rate tensor $\hat{e}_i$: (a) to (f) from high to low turbulence intensity.
Figure 5.13: PDF of the cosine of the angle between the direction of gravity $\hat{z}$ and both vorticity $\hat{\omega}$ and the eigenvectors of the strain-rate tensor $\hat{\varepsilon}_i$: (a) to (f) from high to low turbulence intensity.
Figure 5.14: The PDF of the mean-square tumbling rate of (a) small triads and (b) large triads. The color map indicates turbulence intensities, from low to high (cold to hot).

Table 5.1: Mean square rotation rates of inertial triads.

<table>
<thead>
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<th>Small Triads</th>
<th>Large Triads</th>
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<tr>
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<td>$\Delta -$</td>
<td>194</td>
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</table>

5.3.4 Rotation Rate Measurements

The PDF of the squared tumbling rate of inertial triads is shown in Fig. 5.14. All curves collapse relatively well within the measurement uncertainty. It seems that particle inertia and inertial torques do not change the shape of the PDFs, compared to neutrally buoyant particles in Fig. 3.6.

Figure 5.15 shows the mean-square rotation rate of triads for their specific aspect ratio. The rotation rates are summarized in Tab. 5.1.
5.3.5 How-to: Running the Vertical Water Tunnel

This section should serve as an instruction manual for the vertical water tunnel and includes useful information on how to set up an experiment and run the water tunnel with minimal frustration.

Before every experiment, the apparatus should be cleaned and washed out. Water that has been sitting in the pipes or hoses might have caused biological growth which can plug valves or screens. Running the water tunnel for at least 24 hours, with fresh water and 10 ml of algicide, through the filtration system can help to get rid of dirt and prevent more growth. When small, spherical tracer particles or larger particles have been used in previous experiments, rinsing the apparatus before the filtration process can help reduce the filtration time. If necessary, the bottom chambers (up to the contraction zone) can be easily removed from the water tunnel and therefore give access to the lower pressure plate and honeycomb flow straightener. For this purpose, we installed a spring system on the support frame. Before tightening the springs, which lift up the water tunnel, be sure to loosen the two rigid support brackets that connect the test section with the support frame at the upper end of the test section! Then, the lower four springs
can be tightened slowly and the bottom chambers can be removed. Similarly, there is a set of four springs at the upper end of the water tunnel which can be used to relief the weight on the test section or jet array, in case they have to be removed. Initially, we used cork gaskets between the flanges of each section of the water tunnel. This material forms a tight seal, however, it is not suitable for repeated use. The gaskets in the bottom chambers have been replaced with rubber gaskets, which are cut manually by hand from one big sheet of rubber (Multipurpose Neoprene Rubber Sheet, 36 inch width, 1/8 inch thickness, 30A durometer from McMasterCarr).

The clear surfaces of the test section should be cleaned with a soft sponge or lint-free tissues (e.g. Kimwipe). Dirt on these surfaces might show up on the camera images, which makes the image processing much more difficult. In the meantime, the fluid for the actual experiment should be degassed to avoid air bubbles. The degassing process usually takes at least 24 hours and using algicide to prevent biological growth in the degassing chamber is recommended.

After cleaning the apparatus and degassing the fluid for the experiments, the water tunnel can be filled. Degassed water should be pumped through the filter system. When the water reaches the lower pressure plate and is forced through the small holes, the water level can be used to judge the alignment of the apparatus. If water is pushed evenly through the holes of the pressure plate, the apparatus is leveled, if not, the leveling feet of the apparatus have to be adjusted accordingly. During the filling process, air will get trapped in some parts of the apparatus, e.g. the flow manifolds. Since the solenoid valves get hot when they are kept open continuously, I recommend filling the water tunnel until the water level reaches the middle of the test section. Then all solenoid valves can be opened and trapped air can escape.

Large particles can be added to the test section through the two removable windows on the test section and coarse screens can be used to confine them to the test section. After the water tunnel is completely filled, the variable speed pump can be used to pump water only through the jet array with all valves open, to get rid of the remaining trapped air. The water level will sink due to the escaped air and more degassed water has to be added to the system. The overflow container on top of the water tunnel should always be filled at least half way to avoid air being sucked into the return lines when running the variable speed pump at high speeds.

Small, spherical tracer particles can be conveniently inserted through the filtration system. The filters can be bypassed and the cartridge can be screwed of the mount, even when the water tunnel is filled. The filter can then be removed from the cartridge and tracer particles can be
added to the cartridge before screwing it back on the mount. That way, tracer particles can be pumped into the system. When small tracer particles are used, all filters should be bypassed to avoid clogging.

The LED backlights and camera triggers are controlled from a dedicated computer running LabVIEW (see \code-lib\labview\skramel\watertunnel\readme.txt) and signals are sent to the analog/digital BNC connector box from National Instruments. The 40 solenoid valves are triggered with five solid-state relay boards, which receive the signal from LabVIEW via USB connection through an optically isolated USB hub. Each relay can be activated manually through the LabVIEW interface or by reading in a .txt file containing hex numbers that define the state of each relay on each board. To create a randomly pulsed jet array, we use a MATLAB script (see \code-lib\matlab\wrees\readme.txt) that generates this .txt file, where the number of jets and the duration of each jet are input parameters. LabVIEW can also be used to log the flow rates from both flow meters. They are set up to output a current proportional to the flow rate, which is converted into a voltage measurement by adding a 50 Ω resistor. The BNC cable from the flow meter can be connected to an analog voltage input on the BNC connector box. The signal has to be calibrated to convert voltage to flow rate.

All technical drawings and AutoCAD files of the components of the water tunnel are located in \code-lib\autocad\skramel\.

5.4 Appendix D: Archived Data

The data, the camera calibration data and the results from the scanning system can be found in NAS2\data\rni\experiments\scanning\. The directory also includes the analysis codes used to process the data.

The data, the camera calibration data and the results from the chiral dipoles experiments can be found in NAS2\data\skramel\chiral_dipoles\. The directory also includes the analysis codes used to process the data.

The data, the camera calibration data and the results from the first experiments in the vertical water tunnel with triads can be found in NAS3\data\skramel\sedimentation\turbulence\. The data, the camera calibration data and the results of the experiments with fibers and triads
in quiescent fluid are located in NAS3\data\skramel\sedimentation\quiescent\. The codes used to analyze and process the data are located in the user directory NAS3\user\skramel\codes\sedimentation\. A summary of all my codes can be found in the NAS3\code-lib directory.
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