Rotational dynamics of rod particles in fluid flows

by

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Dedication

To Saman, who believed in me and sacrificed his future for mine.
Abstract

Rotational dynamics of rod particles in fluid flows
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The primary aim of this research is to study the dynamics of rods in fluid flows, quantify the alignment of rods with the flow and the effects of alignment on the rotation rates. We perform experimental measurements of the rotation of rods in three-dimensional turbulence and resolve the rotational properties.

In a 2D chaotic flow we measured the translational and rotational dynamics of rods along with the velocity of the carrier flow. Rods are strongly aligned with the stretching direction of the Cauchy-Green deformation tensor.

We report the first three-dimensional measurements of the rotational dynamics of rod-like particles as they are advected in a turbulent fluid flow. Tracer rods preferentially sample the flow since their orientations become correlated with the velocity gradient tensor. The probability distribution of the mean square rotation rate has a long tail which implies the presence of rare events with large rotation
rates. Rotation of particles is controlled by small scales of turbulence that are nearly universal, these measurements provide a rich system where experiments can be directly compared with theory and simulations.

In another set of experiments we measured the rotational statistics of neutrally buoyant rods with lengths $2.8 < l/\eta < 72.9$, where $\eta$ is the Kolmogorov length scale, in turbulence and quantify how their rotation rate depends on length. The mean square rotation rate of rods decreases as the length of the rods increases and for lengths in the inertial range. We derive an scaling of $l^{-4/3}$ for the mean square rotation rate and show that experimental measurements approach this scaling law. In comparison with the randomly oriented rods we see that all rod lengths develop alignment with the velocity gradient of the flow at the length of the rods. We have also measured the correlation time of the Lagrangian autocorrelation of rod rotation rate and find that the correlation time scales as the turn over time of eddies of the size of the rod. Measuring the rotational dynamics of single long rods provides a new way to access the spatial structure of the flow at different length scales.
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Chapter 1

Introduction

The dynamics of particulate material in fluid flows appear in a broad range of problems in nature and industry. In nature the locomotion of microorganisms [1, 2, 3, 4, 5] or prediction of the spread of an oil spill in the sea or the airborne ash in the atmosphere after volcanic eruptions [6] all require deep understanding of the dynamical properties of the flow and the way the motion of these complex particles are coupled with the flow. Dispersion of suspension of fibers in turbulent flow in paper industry [7] or dynamics of gas bubbles in pipes in underground oil recovery[8] are examples of dynamics of anisotropic or density mismatched particles in complex flows with significant economical impact. Equations of motion of such particles turbulent flow are not easily accessible so the role of experimental measurements is critical in modeling the dynamics of complex or inertial particles.

On the other hand, introducing new methods of measurements of the fundamental properties of turbulence in controlled laboratory environments that can be extended to complex industrial or geophysical flows is valuable experimentally.
We introduce an experimentally measurable dynamical property of single particles to probe the properties of velocity gradients in turbulence. Rotational dynamics of small rods is directly coupled with the velocity gradient of the flow, which are approximately universal in turbulence, and in contrast to spheres is easily measurable. Another important quantity to access experimentally is the Lagrangian evolution of coarse velocity gradients at a fixed length scale. If rods of any length rotate due to the velocity differences at the length of the rod, measuring the rotation rate of long rods gives us access to the structure of the flow at that length scale.

In the rest of this introduction I will give a brief introduction to turbulence theory and dynamics of anisotropic particles in turbulent fluid flows, where I review the current state of measurements and analytical work on the dynamics of rod-like particles in fluid flows. In the following chapters I will present the results from three different experiments I have performed. The first experiment in 2D chaotic flow where we quantify the correlation of orientation of rods with the carrier flow. The second experiment with tracer rods in 3D turbulence introducing the first experimentally measured time-resolved dynamics of rods in turbulence. Finally, we measured the rotation rate of long rods in turbulence and show how the rotational dynamics of rods can be used to probe the spatial structure of the flow at different length scales.

1.1 Introduction to Turbulence

The equation of motion of a fluid element is derived based on the conservation of mass and conservation of momentum. The resulting equation is known as the
Navier-Stokes equation. For an incompressible fluid:

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_j} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{1.1}
\]

where \( \vec{u} \) is the velocity of fluid, \( P \) is the pressure, \( \rho \) is the density and \( \nu \) is the kinematic viscosity. It is conventional to rewrite equation 1.1 in non-dimensional form and compare the magnitude of different terms in this equation. If we use the flow largest length scale, \( L \), and average velocity, \( \bar{u} \), to normalize the lengths and velocities in Eq. 1.1, a non-dimensional number appears which is known as Reynolds number \( Re = \bar{u}L/\nu \). Reynolds number is the ratio of the inertial forces to viscous force. Different fluid flows are categorized by their Reynolds number. Turbulent flows have large Reynolds numbers where the inertial forces are dominant and the non-linear term in Eq. 1.1 is large. Due to nonlinearity of the problem there is not a simple analytical solution to this problem. Kolmogorov’s hypothesis [9] for high Reynolds number turbulent flow, first proposed in 1941, has been a successful leading theory for approaching this problem. The main idea of the Kolmogorov theory is that at high enough Reynolds number a turbulent flow has a wide separation of scales from the energy containing scale (\( L \)) to the energy dissipation scale. In the scales in between, known as the inertial range, on average energy is passed down the scales with no loss.

The energy dissipation rate in the flow is defined by the velocity gradient of the flow at smallest scales. The velocity gradient is normally decomposed into it’s symmetric part, \( S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \) and antisymmetric part, \( \Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \) and energy dissipation is

\[
\langle \epsilon \rangle = 2\nu \langle S_{ij}S_{ij} \rangle \tag{1.2}
\]

The Kolmogorov length-scale also known as the dissipation length is \( \eta \). This
length-scale describes the size of the smallest eddies at which energy dissipates into heat. The corresponding time-scale is \( \tau_\eta \) and those scales are defined as

\[
\eta = \left( \frac{\nu^3}{\langle \epsilon \rangle} \right)^{1/4} \\
\tau_\eta = \left( \frac{\nu}{\langle \epsilon \rangle} \right)^{1/2}
\]

1.2 Particles in Turbulence

The transport of particulate material by fluids is a problem with far-reaching consequences, and a long history of study. When the particles are very small and neutrally buoyant, they behave as Lagrangian tracers and move with the local fluid velocity. However, particles with a density larger or smaller than the carrier fluid tend to show different dynamics, such as preferential concentration and clustering in turbulent flow fields [10]. Even particles that are neutrally buoyant can show dynamics different from the underlying flow when they are large compared to the smallest flow scales, since they filter the flow field in complex ways [11, 12, 13, 14].

A large number of researches have focused on the case of spherical particles. In many situations, however, including fiber processing in the paper industry [15, 7] and dynamics of ice in clouds [16, 17, 18, 19] and locomotion of many microorganisms [2, 1, 20, 5], the particles are not round. The case of ellipsoidal particles was first studied by Jeffery [21] and Taylor [22]; subsequently, Brenner addressed the case of general particle shape in a series of seminal papers [23, 24, 25, 26]. Here, we review the analytical model of the equation of motion of small and long rods in fluid flows.
1.2.1 Rotation Rate of Small Tracer Rods in Fluid Flows

There is an extensive literature on the motion of anisotropic particles in fluid flows. Anisotropic particles have fascinating dynamics even in simple flows [21, 27, 28, 29]. The dynamics of ellipsoidal particles in a simple shear flow was first studied by Jeffery [21] in 1922. When ellipsoidal particles are small compared with the smallest length scales in the flow, their rotation rate is determined by the velocity gradient tensor through Jeffery’s equation [21]:

\[ \dot{p}_i = \Omega_{ij} p_j + \frac{\alpha^2 - 1}{\alpha^2 + 1} (S_{ij} p_j - p_i p_k S_{kl} p_l) \]  

where \( p_i \) is a component of the orientation director and \( \alpha \equiv l/d \), is the aspect ratio of the ellipsoid given by the ratio of length (l) to diameter (d).

These insights about how rods couple to strain-rate, \( S \), and vorticity, \( \Omega \), have been extended in many different directions by recent work. Analytic studies of rod motion in flows with uniform velocity gradients have explored Jeffery orbits and deviations from them due to walls and fluid inertia [30, 31]. Numerical simulations provide access to the motion of the particles along with the velocity of the carrier flow and allow detailed study of the motion of particles in complex flows [32, 28]. Several simulations have addressed the turbulent case [33, 34, 35, 36, 37]; however, they must either use a drag model for the particle fluid interactions or fully resolve the particle boundary layer. Experimental studies have not been able to access both rod motion and the full fluid velocity field in flows more complex than uniform velocity gradients. Several groups have studied orientation dynamics in flows with uniform velocity gradients, where the effects of inertia [38], aspect ratio, and distance from solid boundaries [39, 40] have been investigated. In more complex flows, the rotational diffusivity [41], and orientation distribution in lab coordinates [42, 43, 44] have been measured. However, there are no available
experimental measurements of time-resolved rotational dynamics of anisotropic particles in 3D turbulence. The results of experimental study of rotational dynamics of tracer rods in 2D chaotic flow and 3D turbulent flow will be discussed in details in this thesis.

1.2.2 Rotation Rate of Long Rods in Turbulence

The rotational dynamics of long fibers with applications in paper making [15, 33] and drag reduction [45, 46, 47] are of great interest in industrial settings. The dynamics of long rods in complex fluid flows are complicated to study and experimental measurements at high Reynolds number flows are not available. The numerical simulations have studied the rotational and translational dispersion of long fibers [33]. The results of these simulations suggest that the rotation rate of rods scales with the eddies of the size of the rods. However, the simulations are at small Reynolds number with no inertial range. If the rotation rate of rods is correlated with the velocity gradient of the flow at the length of the rod, one expects to observe the transition from dissipative scales to the inertial scales. If an analytical model for the force on long fibers is available one would expect that simulations should be able to access the statistics of the rotation rate of long rods at high Reynolds number flows.

The slender body theory [48] is widely used for the force model on rods at low and moderate Reynolds numbers. Olson and Kerekes [15] introduce an analytical model for the equation of motion of long fibers which will be described in details here. The model is developed for high Reynolds number flow where the particle Reynolds number based on the length of the fiber $l$ is small. The fiber is supposed to be composed of small sections which are hydrodynamically independent and
each is smaller than $\eta$. The force on a long fiber in Stokes flow is proportional to the velocity between the fluid, $\bar{U}(r)$, and the fiber, $\bar{V}_f(r)$, where $r$ is the position along the fiber. So each section of the fiber follows this equation of motion:

$$\bar{f}(r) = \mathbf{D}[\bar{U}(r) - \bar{V}_f(r)] \quad (1.4)$$

where $\mathbf{D}$ is a constant drag tensor independent of the position along the fiber. Since the fiber is assumed to be rigid, the velocity at each point along the fibers is the sum of the translation velocity and the rotational velocity, $r\vec{\rho}_f$. The net force and torque on the particle is the sum of the forces and torques on each section.

$$\bar{F} = \int_{-l/2}^{l/2} \mathbf{D}[\bar{U}(r) - (\bar{V}_f + r\vec{\rho}_f)]dr \quad (1.5a)$$

$$\mathbf{M} = \int_{-l/2}^{l/2} r\vec{\rho} \times \mathbf{D}[\bar{U}(r) - (\bar{V}_f + r\vec{\rho}_f)]dr \quad (1.5b)$$

Since the fibers are neutrally buoyant and we assume that the inertial forces are small, the net force and torque must be zero. This results in,

$$\bar{V}_f = \frac{1}{l} \int_{-l/2}^{l/2} \bar{U}(r)dr \quad (1.6a)$$

$$\vec{\rho}_f = \frac{12}{l^3} \int_{-l/2}^{l/2} r\bar{U}(r)dr \quad (1.6b)$$

The translational and rotational velocities have a mean component and a fluctuating component as $\bar{V}_f = \bar{V} + \bar{v}$ and $\vec{\rho}_f = \bar{\rho} + \vec{\rho}$. Also the fluid velocity is composed of mean and fluctuating parts $\bar{U} = \bar{U} + \bar{u}$. The fluctuation component of the rotational dynamics can be derived from 1.6b after removing the mean rotation.

$$\vec{\rho}_i = \frac{12}{l^3} \int_{-l/2}^{l/2} (\delta_{ij} - p_ip_j) \bar{u}_j(r)rdr \quad (1.7)$$
This analytical model has been tested and compared with other solutions based on slender body theory [33] and is known to describe the dynamics of long fibers well.

1.2.3 Alignment

The alignment of rods with the flow is very important since it can affect the rotation of rods. We know that the rotation rate of rods in fluid flows are determined by the velocity gradient of the flow and consequently one would expect that the orientation of rods are correlated with this quantity. In recent numerical simulations [35] it has been shown that rods in turbulent flow develop alignment with the direction of vorticity and the intermediate eigenvector of the strain rate. In the limit of small rods with high aspect ratio, the rods approximate material lines, and one can use theoretical techniques developed for studying the evolution and alignment of material lines in turbulence [49, 50] to study the motion of rods. For analytically accessible flows researchers have looked at the patterns formed by the orientation of rods advected in fluid flows [51, 52, 28]. They compare the orientation of rods with the direction defined by the eigenvector of deformation gradient tensor. However, this quantity may have imaginary eigenvalues and consequently hard to study.

We utilize the Cauchy-Green deformation tensor used earlier to describe the Lagrangian coherent structures in fluid flows [6, 53, 54, 55] and compare the orientation of rods with the eigenvectors of this quantity. To quantify the Cauchy-Green deformation tensors, we use the flow map $\Phi(x, t_0, \Delta t)$, which specifies the position at time $t_0 + \Delta t$ of a fluid element that was located at position $x$ at time $t_0$; see Fig. 1.1. The deformation gradient tensor $F_{ij} = (\partial \Phi_i / \partial x_j)$ characterizes the
deformation of a fluid element by the flow map. Since $F_{ij}$ is not necessarily symmetric, its eigenvalues may not be purely real. We therefore use the left and right Cauchy-Green deformation tensors [56], which are the two possible symmetric inner products of $F_{ij}$ with itself:

$$C_{ij}^{(L)} = F F^T = \frac{\partial \Phi_i}{\partial x_k} \frac{\partial \Phi_j}{\partial x_k}$$ (1.8a)

$$C_{ij}^{(R)} = F^T F = \frac{\partial \Phi_k}{\partial x_i} \frac{\partial \Phi_k}{\partial x_j}.$$ (1.8b)

The eigenvalues of $C_{ij}^{(L)}$ are the same as the eigenvalues of $C_{ij}^{(R)}$. The square root of the maximum eigenvalue gives the stretching that the fluid element has experienced over the time $\Delta t$. To visualize this process, consider a fluid element that is initially circular and is stretched into an ellipse by the flow as in Fig. 1.1.

The stretching is the ratio of the semi-major axis of the ellipse to the radius of the circle. The spatial distribution of stretching in fluid flows is closely related to the finite time Lyapunov exponents, and is used to define Lagrangian coherent structures [53, 54, 57, 55, 6].

![Figure 1.1](image.png)

**Figure 1.1**: A fluid element at initial position $\vec{x}$ at time $t_0$ is mapped to final position $\Phi(\vec{x}, t_0, \Delta t)$ after time $\Delta t$ by the flow. The circular fluid element is also deformed by the flow to an ellipse. The eigenvectors of the left ($\hat{\vec{e}}_{1L}$ and $\hat{\vec{e}}_{2L}$) and right ($\hat{\vec{e}}_{1R}$ and $\hat{\vec{e}}_{2R}$) Cauchy-Green tensors are shown.

Even though the eigenvalues of the two Cauchy-Green tensors are identical, the eigenvectors are not. As shown in Fig. 1.1 the eigenvectors of the left Cauchy-
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Green tensor, \( C_{ij}^{(L)} \), indicate the direction of stretching in a coordinate system aligned with the fluid element at time \( t_0 + \Delta t \), so that the eigenvectors give the directions of the principal axes of the ellipse after stretching. The eigenvectors of the right Cauchy-Green tensor, \( C_{ij}^{(R)} \), on the other hand, indicate the direction of stretching in a coordinate system aligned with the fluid element at time \( t_0 \), so that material lines initially aligned with the right eigenvectors will end up aligned with one of the principal axes of the ellipse after stretching.
Rotation and Alignment of rods in 2D Chaotic Flow

Here, we study the motion of rod-like particles experimentally in quasi-two-dimensional flow. In this experiment we have access to the full velocity field of the flow around the rods. We compared the rotation rate of rods with the velocity gradient of the flow and quantified the correlation of orientation distribution of rods with the structure of the flow. We conclude that Jeffrey’s equation predicts the instantaneous rotation rate of rods up to 50% of the length scale of the flow very well and the rods are strongly aligned by the direction of the stretching of the left Cauchy-Green deformation tensor.

The results presented in this chapter has been published in Physics of Fluids 23-043302 (2011).
2.1 Experiment

The flow is produced in a shallow, electrolytic fluid layer that is driven electromagnetically using Lorentz forcing [54]. A sinusoidal current flows through the fluid layer, which interacts with the magnetic field provided by an arrangement of permanent magnets located beneath the plane of the fluid. This results in a body force on the fluid perpendicular to both the current and magnetic field. The experimental apparatus is shown in Fig. 2.1 The Reynolds number is defined as $\text{Re} = \frac{\tilde{u}L}{\nu}$, where $\tilde{u}$ is the root-mean-square fluid velocity, $L$ is the forcing length scale given by the typical magnet spacing, and $\nu$ is the kinematic viscosity of the fluid. The Reynolds number was moderate, $\text{Re} = 95$, and below transition to turbulence.

We use particle tracking methods to determine the trajectories and orientations of the rods. The time-resolved fluid velocity fields are also measured by tracking the motion of small tracer particles advected by the flow. The electrolyte solution is chosen to provide electrical conductivity and to render the rods and tracer particles neutrally buoyant, which ensures that they coincide in a single plane of the fluid.

2.1.1 Experimental Setup

The fluid density ($\rho=1.22 \text{ g/cm}^3$, 30% CaCl$_2$ in water) is 1.6% higher than the rods ($\rho=1.20 \text{ g/cm}^3$), so that the rods float at the upper surface. The rods are made from fluorescent plastic fiber optic cable (0.5 mm diameter) that is cut to the desired length, $l= 2.5 , 5 , 7.1 , 10 \text{ mm}$. The same material is used to make tracer particles (0.5 mm long cylinders), which
ensures that the tracers and rods float at the same height in the fluid. Ultra-violet lamps are used for fluorescence excitation. A random arrangement of permanent magnets with an average spacing of $L = 1.9$ cm is located beneath the shallow fluid layer (1.7 mm deep, $21 \times 21$ cm wide). A sinusoidal electric current, with frequency 0.1 Hz (period $T=10$ s), travels horizontally through the fluid, which leads to a time-periodic chaotic flow ($Re = 95, U = 0.91$ cm/s).
2.1.2 Image Analysis

We image a $16 \times 13 \text{ cm}^2$ ($1280 \times 1024 \text{ pixels}^2$) area in the center of the test section to avoid edge effects. Figure 2.2 shows a typical raw image of 10 mm rods taken at a frame rate of 40 Hz. Since the flow is time-periodic, the fluid velocity can be measured separately without the rods present.

For measurement of the fluid velocity field, the flow is seeded with tracer particles to an average concentration of 230 particles per image, and their motion is tracked over 115 periods resulting in about 27000 tracer particles per phase. The center of each tracer particle is measured with an uncertainty less than $\approx 30 \mu\text{m}$ (0.25 pixel). The particle velocities are measured by fitting a polynomial to the particle trajectories. The fluid velocity field is then extracted from all the tracer velocities occurring at the same phase by interpolation onto a square grid of 0.1
cm spacing.

In separate experiments, the rod motion is measured with a significantly lower particle concentration (10 to 40 depending on rod length) to avoid particle-particle interactions, and repeated 70 to 90 periods for each rod length. To determine the center and orientation of each rod, we find all bright pixels corresponding to a single rod. The rod position is determined using the intensity weighted center-of-mass of the pixels. A Hough transform gives a first guess for the orientation of the rod. Finally, we use a non-linear fitting algorithm to optimize the orientation measurements by minimizing the difference between an ideal, model rod image and the raw image. Using this method, the orientation of a rod is found to within $\pm 0.017$ rad accuracy.

2.2 Rotation Rate of Rods

The rotation rate of an ellipsoid in a 2D flow can be written in a simple form of the Jeffery’s equation (Eq.1.3) in two dimensions

$$
\dot{\theta} = \frac{1}{2} \left[ \omega + \left( \frac{1 - \alpha^2}{1 + \alpha^2} \right) \left[ \sin(2\theta) \left( \frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y} \right) 
- \cos(2\theta) \left( \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right) \right] \right],
$$

(2.1)

where $\theta$ is the inclination of the rod with respect to a fixed axis, $\alpha$ is the aspect ratio of the ellipsoid, and $u$ is the fluid velocity. If the velocity gradient changes in space, Eq. 2.1 is still the first term in a series expansion in higher spatial derivatives of the velocity. The coefficient of the strain-rate portion of the equation is the eccentricity of the ellipsoid, and is constrained to lie between zero (for spheres) and one (for lines). Even though the right circular cylinders we study have sharp
corners when compared with ideal ellipsoids, we expect any correction terms to Eq. 2.1 to be negligible at this order of approximation, and our measurements confirm this.
Figure 2.4: Probability density function (PDF) of the differences between measured and predicted rotation rate for different rod lengths: • 2.5mm, ▲ 5mm, + 7.5mm, ◊ 10mm

Equation 2.1 shows that the rotation rate of a rod in two dimensions can be estimated from the carrier fluid velocity gradient at the center of the rod. Figure 2.3 shows the measured rotation rate of several typical rods of different lengths. Also shown is the predicted rotation rate from Eq. 2.1 using experimentally measured velocity gradients at the position of the rod. Rod length increases from top to bottom. We measure the rotation rate of the rods from polynomial fits to the experimentally measured orientations.

As shown in Fig. 2.3, the predicted rotation rate is close to the experimentally measured rotation rate for all rod lengths we have studied. This may be surprising since our rods have lengths up to 53% of the length scale of the forcing and particle Reynolds numbers up to 74 (based on the rod length and the rms fluid velocity). Equation 2.1 gives good predictions despite the fact that the particles do not rigorously satisfy the conditions for which it was derived.
The probability distribution of the deviation between the predicted rotation rate and the experimentally measured rotation rate is shown in Fig. 2.4. The deviations of the prediction from measurement are about 20% of the root mean square rotation rate, and are mostly independent of rod length. The major contribution to these deviations is inaccuracy in determining the exact velocity gradient of the flow at the center of the rod. Also in measuring the rotation rate we fit a polynomial on tracks which can filter out some of the fast rotation rates. We have chosen an optimum fit length that ensures filtering noise from data.

2.3 Alignment

In our experiments we have access to high-resolution time-dependent velocity fields, allowing us to characterize both the Lagrangian and Eulerian flow dynamics. We can therefore directly compare the orientation of the rods with properties of the carrier flow. Below, we first study the alignment of rods with the strain-rate tensor \( s_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) \) measured at the position of the rod, and subsequently consider alignment with the Lagrangian history of the velocity gradients, defined by the Cauchy-Green deformation tensors.

2.3.1 Alignment of Rods with Strain-Rate

The orientation distribution of rods can be considered relative to different directions defined by the flow. First, we will consider alignment with the local strain-rate. Figure 2.5 shows the probability distribution of angles between the orientation of rods and the extensional direction of the strain-rate calculated at the center of the rods. This distribution shows that the rods tend to align with...
Figure 2.5: Probability Density Function (PDF) of the angles between rod orientation and extensional direction of strain-rate. The PDF shows weak alignment of rods with the strain-rate. Rod lengths are: • 2.5mm, ◦ 5mm, ◐ 7.5mm, ◌ 10mm.

the extensional direction of the strain-rate, although the alignment is fairly weak. For the rods studied, the alignment with the strain-rate does not show significant dependence on rod length.

2.3.2 Alignment of Rods with Stretching

Rods are weakly aligned with the strain-rate in these flows, but it is really the history of the velocity gradients along the trajectory of a rod that is responsible for its orientation distribution. As discussed in the introduction, this Lagrangian history of the velocity gradients can be quantified using the Cauchy-Green deformation tensors. In Fig. 2.6 we show snapshots of the past stretching field, defined by the eigenvalue of the Cauchy Green deformation tensor at each spatial point. The stretching field has many sharp maxima which are organized into lines. Su-
Chapter 2 - Rotation and Alignment of rods in 2D Chaotic Flow

Figure 2.6: Stretching fields with 10 mm rods superimposed in dark color. $\Delta t = T = 10$ s.

Figure 2.7: PDF of the alignment of rods with the direction of stretching defined by $\hat{e}_{1L}$. All rod lengths align with the stretching direction. Periodic flow, $\Delta t = T$, rod lengths are: $\bullet$ 2.5 mm, $\triangleright$ 5 mm, $+$ 7.5 mm, $\diamond$ 10 mm.

Perimposed on the stretching fields in Fig. 2.6, we show images of 10 mm rods taken at the same time.
The rods are preferentially aligned with the stretching lines which indicates the important role of stretching in orienting rods. As in Ref [54], we calculate stretching by integrating trajectories of virtual particles in measured velocity fields. The gradients in the definition of the Cauchy-Green tensors (Eq. 1.8a) are evaluated using finite differences of particle trajectories that are initially very close to each other. A rescaling method is used to keep the particles close to each other even as they experience exponential stretching.

In order to quantify the effect of stretching on the orientation of rods, we measure the distribution of angles between the orientation of each rod and the direction of past stretching, \( \hat{e}_{1L} \), at the center of the rod. Figure 2.7 shows that this distribution has a large probability around zero indicating that the rods preferentially align with the past stretching direction. The alignment is significantly stronger...
than the alignment with the strain-rate direction in Fig. 2.5. Surprisingly, the alignment is nearly independent of rod length even though the longest rods (10 mm) are 53% of the magnet spacing. Even for these relatively large rods, there is no measurable effect either from the rods averaging over the spatially varying velocity field or from the rotational inertia of the rods. Figure 2.8 shows the probability distribution of alignment of rods with the direction of past stretching for different integration times. The probability of alignment of rods with the direction of stretching increases with increasing integration time. This increase in alignment seems natural, as the bit of fluid that is accompanying the rod after longer integration has experienced more stretching. However, at some point the maximum probability saturates so that further increases in the integration time do not lead to additional alignment. This saturation may be a sign of limitations on the accuracy of the experimental measurements of the stretching direction of the fluid at the center of the rod.

We have also compared rod orientation in our flows to the direction defined by the eigenvectors of the deformation gradient tensor as used in previous studies [51, 28]. Even after two periods (20 s) of the forcing flow, there are many regions with complex eigenvalues. In these regions, the direction of the eigenvector of the deformation gradient is undefined, so the method based on the Cauchy-Green deformation tensors is more useful for comparing the alignment of rods with the deformation measured in our flows. It would be interesting for future work to make a more detailed comparison of these two methods.
Figure 2.9: (a) Deformation of a circular fluid element into an ellipse by the flow. (b) Initial circle in the coordinate system aligned with the principal axes of the right Cauchy-Green deformation tensor. (c) Final ellipse in the coordinate system aligned with the principal axes of the left Cauchy-Green deformation tensor.

2.3.3 Theoretical Prediction of Alignment with Stretching Direction

Here we use a simple model of deformation to predict the alignment of material lines due to stretching in our system. Some theoretical tools for solving the Fokker-Planck equation for the orientation distribution of microstructure in fluid flows have been developed [58], but we choose to solve a simple model that clearly reveals the connection of stretching to the orientation distribution. The effect of flow is to deform an infinitesimal circle of fluid into an ellipse. The ratio of the
semi-major axis of the ellipse to the radius of the circle is equal to the stretching that the circle has experienced which can be measured using the square root of the maximum eigenvalue of the Cauchy-Green tensor as described in section 2.3.2. If we consider straight material line segments through the center of the circle with a known initial distribution of angles, we can calculate the probability distribution of their angles after being stretched by the flow.

Figure 2.9(a) shows an initially circular fluid element that is then transported and stretched by the flow into an ellipse. In Fig. 2.9(b) we show the initial circle in the coordinate system aligned with the principal axes of the right Cauchy-Green tensor. Here a point on the circle of radius $r$ is given by the simple parametric equations, $x_0 = r \cos(\theta_0)$ and $y_0 = r \sin(\theta_0)$. After deformation by the flow over some time interval, the circle becomes an ellipse with the same area. In general the flow will have reoriented the ellipse, so in Fig. 2.9(c) we show the ellipse in the coordinate system defined by the principal axes of the left Cauchy-Green tensor. By choosing different coordinate systems in Fig. 2.9(b) and (c), we have used the Cauchy-Green tensors to account for rotation, leaving only the effect of stretching.

We can map any point $(x_0, y_0)$ on the circle to the corresponding point $(x, y)$ on the ellipse by

$$x = sx_0, y = \frac{y_0}{s}$$

where $s$ is the stretching.

The angle between the point $(x, y)$ and the semi-major axis of the ellipse is

$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{\tan(\theta_0)}{s^2}\right)$$

so the final and initial angles of any material line through the center of the material element are related by

$$\tan(\theta) = \frac{1}{s^2} \tan(\theta_0).$$
The number of lines in a range $d\theta$ is given by the number of lines that are mapped to this range from the initial distribution, so the probability distribution of angles, $P(\theta)$, is related to the initial distribution, $P_0(\theta_0)$, by

$$P(\theta)d\theta = P_0(\theta_0)d\theta_0. \quad (2.4)$$

From Eq. 2.3, the differentials are related by

$$\frac{d\theta_0}{d\theta} = \frac{d}{d\theta} \left( \arctan(s^2 \tan(\theta)) \right) = \frac{1}{s^2 \sin^2(\theta) + \frac{\cos^2(\theta)}{s^2}}, \quad (2.5)$$

so the final distribution of angles in range $d\theta$ for a given value of stretching, $s$, is

$$P(\theta)d\theta = \frac{P_0(\arctan(s^2 \tan(\theta)))}{s^2 \sin^2(\theta) + \frac{\cos^2(\theta)}{s^2}} d\theta. \quad (2.6)$$

Equation 2.6 implies that the final distribution of angles depends on the initial distribution $P_0(\theta_0)$ and the amount of stretching, $s$, that the material lines have experienced. If rods rotate as material lines we can use this theory to predict the final distribution of orientations of rods.

Rods in different regions of the flow experience different values of stretching. The probability distribution of orientations of rods is the sum over all stretching values weighted by the probability density of any particular value of stretching $P(s)$:

$$P(\theta)d\theta = \int P(s)ds \frac{P_0(\arctan(s^2 \tan(\theta)))}{s^2 \sin^2(\theta) + \frac{\cos^2(\theta)}{s^2}}. \quad (2.7)$$

### 2.3.4 Predicted alignment versus measured alignment

The stretching distribution $P(s)$ is measured from all rod positions to account for their sampling of the flow, and we further condition $P_0(\theta_0)$ on values of stretching $s$. 

To compare our measured orientation distributions with the prediction in Eq. 2.7, we need to know the PDF of initial orientations of rods, $P_0(\theta_0)$. Figure 2.10 shows this initial probability distribution from our experimental measurements, where $\theta_0$ is the angle between rod orientation and the extensional eigenvector of the right Cauchy-Green deformation tensor, $\hat{e}_{1R}$. Rods show weak alignment with the direction of future stretching, $\hat{e}_{1R}$. One might expect that there would be no alignment with the stretching that the rod will experience in the future, but we find a weak alignment which can be understood as a result of the time correlation of the velocity gradients of the flow.

From the measured initial distribution of rod orientation in Figure 2.10, we can
Figure 2.11: Comparison between the predicted (solid line) and measured (○) distribution of rod alignment with the stretching direction. Results are shown for 10mm rods in the periodic flow for four different integration times, $\Delta t = (a) \frac{T}{16}$, (b) $\frac{T}{8}$, (c) $\frac{T}{4}$, (d) $\frac{T}{2}$.

calculate the final probability distribution of orientation of rods using Eq. 2.7. Figure 2.11 compares this theoretical prediction of the final distribution of rod orientation with our measurements from the periodic flow (Fig. 2.8) for four different integration times. Both the predicted and measured distributions give angles measured from the extensional eigenvector of the left Cauchy-Green tensor, $\hat{e}_{1L}$. The predicted distribution shows alignment with the direction of stretching in fairly good agreement with our measurements; however, the theory predicts somewhat stronger alignment than is observed. The deviation is largest for long integration
times where the theory predicts that material lines are strongly aligned by the stretching to produce a sharp peak near zero in Fig. 2.11 (d). Inaccuracies in our measured velocity fields may lead to slightly inaccurate measurements of the stretching direction. These inaccuracies would have the largest effect in regions with nearly perfect alignment leading to smaller probability in the experimental distribution near $\theta = 0$. Another factor could be that rods are not material lines. Either their length or aspect ratio could cause the measured alignment to differ from the prediction for material lines. However the lack of rod length dependence in the alignment distribution (see Fig 2.7) suggests this is not a large effect.

2.3.5 Effects of deviations of integrated trajectories from fluid trajectories

At longer integration times in these chaotic flows, small inaccuracies in the velocity field can lead to large deviations between virtual particle trajectories and the trajectories of the rods. We can correct for the deviations of virtual particle trajectories from rods by forcing the center of a group of virtual particles to follow the center of a rod. This way we are sampling the same stretching that the rod has experienced. Figure 2.12 shows the effect of forcing the virtual particles to follow the rods on the probability distribution of alignment of rods with the direction of past stretching. The distribution in Fig. 2.12 (a) shows that the benefit of forcing the virtual particles to follow the rods is quite small. However, if we only look at rods that experience large stretching (Fig. 2.12 (b)) the effect of forcing the virtual particles to follow the rods is to create significantly stronger alignment. We conclude that integration errors can have some effect in regions of large stretching, but the overall effect on the orientation distributions we measure
Figure 2.12: PDF of alignment of rods with the direction of stretching, showing the effect of forcing virtual particles to follow the trajectories of rods. Data is for the periodic flow with 10mm rod length and integration time ($\Delta t = T$). (a) Probability distribution calculated for all stretching values, (b) Probability distribution conditioned for stretching values larger than one rms. • virtual particles follow the rod trajectory, + virtual particle trajectory may deviate from rod trajectory.
is not significant.
Experimental Methods for tracking rods in 3D turbulence

3.1 Turbulent Flow Measurement

Advances in imaging technology have made it possible to obtain time-resolved trajectories of particles in turbulence using high speed stereoscopic imaging, and these measurements have produced many new insights about Lagrangian translational dynamics of particles [10]. We have performed an experimental study of the dynamics of rods in an oscillating grid turbulence. In this chapter I introduce the specifications of the experimental apparatus, measurement techniques and material used in these experiments. I have contributed new methods and analysis codes for the measurements of the orientation of rods in 3D.
3.1.1 Experimental Apparatus

The turbulence apparatus is an octagonal Plexiglas tank ($1 \times 1 \times 1.5 \text{ m}^3$) and is filled with approximately 1,100 liters of degassed fluid. The fluid is filtered to remove particulate down to 0.2 $\mu$m. Two grids generate the turbulence. The grids have a mesh size of 8 cm and are equally spaced from the top and bottom of the tank with a 56 cm spacing between the grids as shown in Fig. 3.1. The amplitude of the oscillation was 12 cm peak to peak. The grid frequencies were 1.5 and 3 Hz. Since it is important to maintain the same experimental condition for an accurate measurement and comparison of the data, we maintain a constant temperature of the fluid within 0.2 $^\circ$C using a water cooling system during the entire set of runs in each day.

The fluid used in these experiments is a solution of 19% by weight CaCl$_2$ in water. The density of the fluid is $\rho = 1.15\text{g/cm}^3$. The high density fluid was necessary to ensure that rods are density matched with the fluid and there are no inertial effects such as settling of rods. I added 175 kg of ice-melting salt to 400 gallons of water and filtered it several times to obtain a clear and transparent fluid. The density matching was exceptionally good. Between experiments I filtered the solution as the highly corrosive salt reacts with the aluminum plates and the grids in the experimental apparatus. This solution was kept in a plastic tank before and after each experiment to avoid any further corrosion. The apparatus was rinsed and washed several times after each experiment to make sure there are no residues of salt.

The illumination was made possible by a 50 W pulsed Nd-YAG laser. The wavelength of the laser is 532 nm. We used a single pulse for each frame. The laser beam shown in the Fig. 3.1 is in the $x$ direction of the lab coordinates. We have
two different sets of illuminations. In one experiment one laser beam and in the final experiment we used four beams for better resolution in orientation measurements. We used four cameras for all these measurements. All four cameras were used at a resolution of 1Mb (1280 × 1024 pixels) and run at 450 Hz. In these experiments I took advantage of the ICC (image compression circuit) which was designed and built in Professor Voth’s lab. These ICC’s provide the ability of taking many hours of data. The data is streamed to the hard disks continuously and large data sets are acquired. The gain and aperture of the cameras were adjusted based on the experiment for best resolution and least saturation. The two A540K Basler cameras located at the top row in Fig. 3.1, camera one and
two, are at 10 degrees from horizontal plane. The two MC1362 Mikrotron cameras on the bottom row in Fig 3.1, camera three and four, are at 17 degrees from horizontal. These numbers can be obtained geometrically using the dimensions of the apparatus and aiming for the center of the tank. I designed the location of the cameras in these experiments at the widest possible angle between pairs of cameras. This ensures our best measurements of the orientation of rods. The Wesleyan machine shop made specific windows with flat surfaces parallel to the lenses so the abbreviation effects are minimized.

3.1.2 Tracer Particles Preparation

In order to obtain the flow parameters we need to measure the velocity of tracer particles. I used 200 $\mu$m PMMA fluorescent particles. These particles have a density of $\rho = 1.19$ g/cm$^3$. Tracer particles are custom made by Cospheric, specialized in making microsporoses. We required specific parameters for the tracers. The diameter of the tracers must be smaller than the Kolmogrov length-scale of the flow while large enough to be detected by the cameras. The florescent dye should be excited at the frequency of the laser ($\lambda = 532$ nm) and emit at a considerably different frequency so the optical filters on the cameras pass all the fluorescent line and filter the laser. I have tested Rhodamine-B in lab using an Ocean Optic spectrometer from Professor Hüwel’s lab. The absorption for Rhodamine-B is between 520 and 560 nm depending on the medium and emits at $580\pm20$ nm measured from dyed rods.
3.1.3 Measurements of the Flow Characteristics

The flow is seeded with tracer particles. The average number of particles in the detection volume is between 50 and 80. This seeding density ensures no interactions between particles. We use Particle Tracking Velocimetry (PTV) to measure the velocity of tracer particles in the flow. The positions of the center of the particles are measured in the 2D images of the cameras. Using the camera calibrations we stereomatch these 2D positions and obtain the 3D position of the particles in lab coordinates. In order to measure the velocity of the particles, we first find the tracks by comparing each frame with it’s previous one and find the corresponding points on each track. Choosing tracking parameter requires careful tests on the original data. If the search parameters are not properly chosen we may filter out the large velocities or introduce noise to the measurements when particles are close. We measure the velocity of the particles by fitting a quadratic polynomial on different sections of the tracks. The detection volume is then divided into smaller bins and the mean flow is measured in these bins. We remove the mean flow from the measured velocities and obtain the turbulence fluctuating velocity.

3.2 Experimental Measurements of the Rods in Turbulent Flow

For measuring the rotation of rods in fluid flows, we need particles with precisely measured lengths and diameter in the range we can optically detect. The commercial fibers available are not all of the same lengths and are usually too thin (less than 10µm) and hard to detect. At the time of my experiments the camera
lenses available in the lab had limitations. This problem is now solved for future work since different lenses were purchased. Due to these circumstances and the need for a very precise measurement for first experiment on rods, we decided to produce rod particles in the lab with specifications we require.

My contribution in this section is the method we developed for extracting the orientation of rods from two-dimensional images by multiple cameras.

3.2.1 Rods, Particle Preparation

The rods are made from nylon threads. We purchased monofilament nylon threads from The Thread Exchange, a domestic distributor in the textile industry. The density of the thread is $\rho = 1.15 \text{ gr/cm}^3$. They are available at several diameters ranging from 80 to 300 $\mu\text{m}$. In all experiments I used 200 $\mu\text{m}$ threads. The threads come on spools and we bundle them on poles and let relax for a few hours. The tensile strength of nylon is high, but caution in stretching is required since they can relax back to their original length and weaken the bundles during cutting. The bundles are wrapped in heat-shrink tubes of lengths 20 cm. The tubes are then shrunk by heat gun at 450° F. The final product is fairly solid at this point. These bundles are then attached to a translation stage and advanced by a micrometer to be cut to the desired length with a single-beveled razor. The Wesleyan machine shop built the platform and razor holders specifically for this work.

The nylon rods are transparent and I developed methods for dying them with fluorescent dye. We used Rhodamine-B as discussed in Section 3.1.2. The dye is dissolved in water and the threads are soaked in for at least 5 hours at temperature between 60 and 90 °C. The dye diffuses in the rods at high temperature, and in
my experience, hardly diffuses out at room temperature. This is not true for all polymers. After dying the rods it is necessary to rinse them as Rhodamine-B is a category 2 health hazard material. The waste from this process need to be collected and disposed properly.

### 3.2.2 Orientation Measurements in 3D

![Figure 3.2: Raw image of 1mm rod in the turbulent flow and also zoom in for better visualization](image)

Measuring the orientation of rods in 3D lab coordinates require multiple cameras. Each camera reports a two-dimensional image of the rod as shown in Fig. 3.2. From these images we measure the position and orientation of the rods in 2D using the method introduced in Section 2.1.2 for 2D flow. After finding the position and orientation in 2D images, we stereomatch the center of the particles using the calibration parameters of cameras.

Using the information of the position of the rod in 3D we define a vector that points
from the camera to the center of the rod. The orientation vector of the rod in the plane of the image defines another vector. With these two vectors we can define a plane that contains the rod. Each camera that detects the rod defines a plane that includes the rod. In an ideal situation these planes should intersect at one line which lies on the rod as illustrated in Fig. 3.3(a). However, the experimental uncertainty in calibration, finding the position in 2D and stereomatching results in a case that these planes do not intersect all at one line. We need to find the orientation that best matches all these planes. If the unit vector that defines each plane is $\hat{n}_i$ and the orientation vector of the rod is $\hat{p}$ as in Fig. 3.3 (b), the rod lies in the plane when the angle between the vector $\hat{p}$ and $\hat{n}$ is $\pi/2$ or in other words $\hat{n}_i \cdot \hat{p} = 0$. For the case of multiple cameras we require a least square of this quantity, $\sum_{i=\text{cameras}} (\hat{n}_i \cdot \hat{p})^2$. The residual of this least square represents the accuracy of the orientation measurements.

**Figure 3.3:** (a) Schematic of planes defined by set of cameras, the rod which appears on images of this set of cameras should be at the intersection of these planes. (b) The unit vector ($\hat{n}$) of a plane defined by one camera that includes the rod labeled as $\hat{p}$.
We have measured the rotational dynamics of small rods, in the tracer limit, in turbulent flow experimentally. These experimental measurements are compared with simulation of rods in homogenous and isotropic DNS turbulent flow, $R_{\lambda} = 180$, by our collaborators. We find that the rotation rate of rods is suppressed compared to randomly oriented rods due to the alignment of rods with the velocity gradients of the flow.

The results presented in this chapter were published in Physical Review Letters 109, 134501 (2012).

4.1 Turbulent Flow

The characteristic parameters of the flow are measured using particle tracking velocimetry. From these measurements we obtain the flow r.m.s velocity and en-
ergy dissipation rate. The experiments were performed at two frequencies of the grid, \( f = 1.5 \) Hz and \( 3\) Hz. Velocities of the tracer particles were measured as introduced in Section 3.1.3. The r.m.s turbulence velocities at 1.5 and 3 Hz are \( \tilde{u} = 32.4 \) and \( 63.2 \) mm/s respectively. We measure the Eulerian second and third order velocity structure functions to find the energy dissipation rate. First, we find the longitudinal velocity difference between pairs of particles at separation distance \( r \) as \( \delta u(r) = (\tilde{u}(x) - \tilde{u}(x + r)) \cdot \vec{r} \). The longitudinal velocity structure function is defined as \( D_p = \langle (\delta u(r))^p \rangle \), where \( p \) is the order of the structure function. In the inertial range, the second order longitudinal velocity structure function \( D_{LL}(r) = C_2 \langle (\epsilon) r \rangle^{(2/3)} \), where \( C_2 \) is an approximately universal constant [59, 60]. The third order structure function can be derived from Navier-Stokes as \( D_{LLL}(r) = (-4/5) \langle (\epsilon) r \rangle \), known as the 4/5 law [60]. With measured structure functions we can access the energy dissipation rate from inertial range scaling.

Figure 4.1 shows the second and third order longitudinal velocity structure function measured from tracer velocity measurements. The energy dissipation rate at 1.5 Hz and 3Hz are \( \langle \epsilon \rangle = 3.55 \times 10^{-4} \) and \( 2.9 \times 10^{-3} \) m\(^2\)/s\(^3\). The kinematic viscosity of the fluid is measured at \( \nu = 1.76 \times 10^{-6} \) m\(^2\)/s. Independent measurement of \( \tilde{u} \) and \( \epsilon \) provides access to the integral length-scale \( L = \tilde{u}^3/\langle \epsilon \rangle = 96 \) mm and 87 mm. The Kolmogorov length scale and time scale are respectively \( \eta = 375 \mu m \) and \( \tau_\eta = 70 ms \), and \( \eta = 210 \mu m \) and \( \tau_\eta = 25 ms \). The corresponding Taylor Reynolds numbers are \( R_\lambda = 160 \) and \( R_\lambda = 214 \).

### 4.2 Rotation Rate of Tracer Rods in Turbulence

I present the first experimental measurements of the time-resolved 3D orientation of rods in turbulent flow. The rods are 1 mm in length by 200\( \mu m \) in diameter. The
Figure 4.1: Eulerian second (top row) and third (bottom row) order longitudinal velocity structure function shown as a function of pair separation r. (a,c) are for 1.5 Hz or $R_\lambda = 160$, (b,d) are for 3 Hz or $R_\lambda = 214$.

position of the center of the rods was measured with an uncertainty of $\approx 40\mu\text{m}$.

We have measured the orientation of rods in three dimensional (3D) space using images from multiple cameras as described in detail in Section 3.2.

### 4.2.1 Tracking Rods in 3D

Figure 4.2 shows an experimentally measured trajectory of a 1mm rod at $R_\lambda = 214$. This track is 284 ms long. The projection of the center of the rod is shown
Figure 4.2: Three-dimensional view of a rod trajectory with a large rotation rate from the experiment at $R_{\lambda} = 214$. The color of the rod represents the rotation rate. This rod is tracked for 284 ms. The green lines show the projection of the center of the rod onto the y-z and x-y planes. The rod is a circular cylinder with length 1 mm and diameter 0.2 mm.

on two reference planes. The colormap represent the magnitude of the rotation rate of this rod. This example illustrates several of the important properties of the rotation of rods. First, this rod has bursts of high rotation rate where the rotation rate squared is up to 30 times its mean reflecting the intermittency of rod rotations. Second, in the upper right, the rod is caught in a vortex, but its rotation rate is not large because the rod has become aligned with the vorticity
reflecting the tendency of anisotropic particles to become aligned by the velocity
gradients in the flow.

4.2.2 Measuring Rotation Rate from Experimental Data

Figure 4.3 shows the components of the orientation vector of the rod shown earlier
in Fig. 4.2. We require the orientation vector to be of unit length \((p_x^2 + p_y^2 + p_z^2 = 1)\). The measured orientation of rods shows that this quantity can have large
fluctuations over a small range of time. The accuracy of this measurement is
critical in finding the rotation rates.

The rotation rate of rods is measured from quadratic fits to the orientation versus
time. As shown in the example of a typical orientation measurement for 1 mm rod
in Fig. 4.3, experimental tracks usually have noise. The quadratic fit to a part of
the track has a contribution from the noise. The ideal case would be to choose the
proper window for each part of the track that minimizes the effects of the noise
and represent the true rotation rate of an instant in the track. However, finding
this proper window is not simple. We use an established method introduced by
Voth et. al [61] that systematically measures the rotation rate over different fit-
times. Around each point on the track we choose fit-times (or windows) ranging
from a few time-steps up to at least 50 time-steps on either side and fit a quadratic
polynomial on this window. The value of the rotation rate at the midpoint of this
window is reported for different fit-times. One expects that the rotation rate at
short fit-time is mostly dominated by noise and at long fit-time the contribution of
noise is minimized, however choosing a long fit-time smooths over the trajectory
and sometimes rejects the large rotation rate. We plot the mean square rotation
rate as function of fit-time and fit the data \(\langle \dot{p}_i \dot{p}_i \rangle\) versus fit-time \((t/\tau_\eta)\) by an
exponential function \( f(t) = A \exp(\lambda t) \). The best estimate of the means square rotation rate is found by evaluating this function when \( t/\tau_\eta \to 0 \).

Figure 4.4 shows the mean square rotation rate for different fit-times for 1mm rods at two Reynolds numbers. We have extrapolated each of this data with two possible range of fit-times. In this measurement the extrapolation to zero fit-time has a small dependence on what ranges of fit-times we choose and it appears as
a measurements uncertainty, this will be discussed further in details and is solved in the next experiment. We report the two extrapolations as upper and lower bounds of the measurement. The measurements in Figure 4.4 (b) are extended to longer time with respect to $\tau_\eta$ since the frame rate of cameras are constant but the Kolmogorov time-scale is shorter at higher Reynolds number. The extrapolation overestimates the measured value slightly and this systematic uncertainty can be quantified. (See Section 4.3.6).

4.3 Results

4.3.1 Probability Distribution of Rotation Rate of Tracer Rods

The probability distribution function (PDF) of the rotation rate squared, $\hat{p}_i\hat{p}_i$, of rods is shown in Fig. 4.5. The PDF has a long tail, this indicates the presence of rare events with rotation rates squared up to 60 times the average value ($\langle \hat{p}_i\hat{p}_i \rangle$). The agreement between DNS and experiment is very good for the core of the distribution up to 20 $\langle \hat{p}_i\hat{p}_i \rangle$. At larger rotation rates the experimental PDFs are slightly below the DNS. This difference is not much larger than the systematic errors in the experimental data represented by the error bars, but it may reflect the effect of finite length of the rods in the experiment. The rod lengths are $2.6\eta$ at $R_\lambda = 160$ and $4.7\eta$ at $R_\lambda = 214$. Ref. [33] indicates that rods less than about $7\eta$ should not have a measurable change in their rotation rate variance from the tracer limit, but it is possible that the rare events are more sensitive to rod length.
4.3.2 Rotation Rate of Finite Aspect Ratio Ellipsoidal Particles

The rotation rate of ellipsoidal particles is predicted by Jeffrey’s equation for finite aspect ratio, $\alpha$.

$$\dot{\theta}_i = \Omega_{ij}p_j + \frac{\alpha^2 - 1}{\alpha^2 + 1}(S_{ij}p_j - p_i p_k S_{kl}p_l)$$  (4.1)
We have used this equation and predicted the mean square rotation rate of randomly oriented ellipsoids with different aspect ratio: For simplicity replace $b = \frac{a^2 - 1}{a^2 + 1}$. The second moment of rotation rate can be found as following:

$$\langle \dot{p}_i \dot{p}_i \rangle = \langle (\Omega_{ij}p_j + b(S_{ij}p_j - p_ip_kS_{kl}p_l))(\Omega_{im}p_m + b(S_{im}p_m - p_ip_qS_{qn}p_n)) \rangle$$

$$= \langle \Omega_{ij}\Omega_{im}p_jp_m \rangle + b^2 \langle p_jp_mS_{ij}S_{im} \rangle + b^2 \langle p_ip_kS_{kl}p_ip_qp_qS_{qn}p_n \rangle$$

$$- b^2 \langle S_{ij}S_{qn}p_jp_ip_qp_n \rangle - b^2 \langle S_{im}S_{kl}p_ip_kp_ip_m \rangle + b\langle \Omega_{ij}p_jS_{im}p_m \rangle$$

$$+ b\langle \Omega_{im}p_mp_j \rangle - b\langle \Omega_{ij}p_ip_qS_{qn}p_n \rangle - b\langle \Omega_{im}p_mp_ip_kS_{kl}p_l \rangle$$

Assuming that the fiber orientation is uncorrelated with the direction of local strain-rate and rate of rotation, we can decompose each average into the product
of averages over orientation and velocity gradients [33].

\[
\langle \dot{p}_i \dot{p}_i \rangle = \langle p_j p_m \rangle (\langle \Omega_{ij} \Omega_{jm} \rangle + b^2 \langle S_{ij} S_{im} \rangle) + b^2 \langle p_k p_l p_q p_n \rangle \langle S_{kl} S_{qn} \rangle
\]

\[-2b^2 \langle p_i p_j p_q p_n \rangle \langle S_{ij} S_{qn} \rangle + 2b \langle p_j p_m \rangle \langle \Omega_{ij} S_{im} \rangle - 2b \langle p_i p_j p_q p_n \rangle \langle \Omega_{ij} S_{qn} \rangle\]

Since the fiber orientation distribution is isotropic, the moments of the orientation can be related to the general isotropic second and fourth-order tensors as following:

\[
\langle p_j p_m \rangle = c_0 \delta_{jm}
\]

\[
c_0 = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} d\theta d\phi \cos^2 \theta = \frac{1}{3},
\]

and

\[
\langle p_i p_j p_l p_m \rangle = c_1 \delta_{ij} \delta_{lm} + c_2 \delta_{im} \delta_{jl} + c_3 \delta_{il} \delta_{jm}
\]

(4.2)

As \( \langle p_i p_j p_l p_m \rangle \) is symmetric under exchange of indices \( c_1 = c_2 = c_3 \). The coefficient, \( c_1 \), can be calculated by similar integration as above, which results in \( c_1 = c_2 = c_3 = 1/15 \).

\[
\langle \dot{p}_i \dot{p}_n \rangle = c_0 \delta_{jm} (\langle \Omega_{ij} \Omega_{jm} \rangle + b^2 \langle S_{ij} S_{im} \rangle) + b^2 c_1 (\delta_{kl} \delta_{qn} + \delta_{kn} \delta_{ql})
\]

\[+ \delta_{kj} \delta_{ln} \langle S_{kl} S_{qn} \rangle) - 2b^2 c_1 (\delta_{ij} \delta_{qn} + \delta_{in} \delta_{jq} + \delta_{iq} \delta_{jn}) \langle S_{ij} S_{qn} \rangle
\]

\[+ 2bc_0 \delta_{jm} \langle \Omega_{ij} S_{im} \rangle - 2bc_1 (\delta_{ij} \delta_{qn} + \delta_{in} \delta_{jq} + \delta_{iq} \delta_{jn}) \langle \Omega_{ij} S_{qn} \rangle
\]

Now apply the Kronecker delta and collect terms. We also know that terms including the product of the symmetric tensor, \( S_{ij} \), and the antisymmetric tensor, \( \Omega_{ij} \) are zero. The trace of the strain-rate tensor is zero so the terms including
\( \langle S_{ii}S_{jj} \rangle \) are also zero. By replacing these terms we have

\[
\langle \dot{p}_i \dot{p}_i \rangle = c_0 \langle \Omega_{ij} \Omega_{im} \rangle + b^2 (c_0 - 2c_1) \langle S_{ij}S_{ij} \rangle
\]

so the second moment of the rotation rate for a finite aspect ratio ellipsoid is:

\[
\langle \dot{p}_i \dot{p}_i \rangle = \frac{1}{3} \langle \Omega_{ij} \Omega_{jm} \rangle + \frac{1}{5} \left( \frac{\alpha^2 - 1}{\alpha^2 + 1} \right)^2 \langle S_{ij}S_{ij} \rangle
\]

In isotropic turbulence \( \langle S_{ij}S_{ij} \rangle = \langle \Omega_{ij} \Omega_{ij} \rangle = \langle \epsilon \rangle / 2\nu \) where \( \langle \epsilon \rangle \) is the energy dissipation rate and \( \nu \) is the kinematic viscosity. Replacing these values in Eq. 4.3 we will have the following:

\[
\frac{\langle \dot{p}_i \dot{p}_i \rangle}{\langle \epsilon \rangle / \nu} = \frac{1}{6} + \frac{1}{10} \left( \frac{\alpha^2 - 1}{\alpha^2 + 1} \right)^2.
\]

This equation describes the mean square rotation rate of randomly oriented ellipsoids as a function of aspect ratio, \( \alpha \), in turbulence.

### 4.3.3 Mean Square Rotation Rate

Figure 4.6 shows the effect of the shape of particles on their rotation rate in turbulence. The rotation rate variance of disk shaped particles \( (\alpha < 1) \) is much larger than that of the spheres \( (\alpha = 1) \). This can be qualitatively understood as the additional contribution of strain \( (S_{ij} \) in Jeffery’s equation Eq. 1.3) to the rotation rate. However, the rotation rate of rods \( (\alpha > 1) \) is much smaller than spheres even though the rate-of-strain contributes to their rotation as well. At large \( \alpha \), our simulations agree with earlier work on thin rods at lower Reynolds number by Shin and Koch [33]. The experimental measurements at \( \alpha = 5 \) are consistent with the simulations considering the measurement uncertainties.

Understanding the rotation rate data in Fig. 4.6 requires considering the preferential alignment that occurs between particles and the velocity gradient tensor.
Equation 4.4 describes the mean square rotation rate of randomly oriented rods. The results from Eq. 4.4 is shown as the green solid curve in Fig 4.6. As particles are advected by the flow, they become oriented so that their rotation rates are very different than the randomly oriented case, with the largest difference occurring for thin rods \((\alpha >> 1)\). Thin tracer rods are material line segments, so the large decrease in rotation rate for thin rods can be qualitatively understood as resulting from the known preferentially alignment of material lines with the vorticity vector \([35, 49, 62]\). Since only the vorticity perpendicular to the rod contributes to its rotation rate, aligned particles have greatly reduced rotation rates.

![Figure 4.6: Rotation rate variance as a function of aspect ratio.](image)

Figure 4.6: Rotation rate variance as a function of aspect ratio. (a) Blue + is the DNS at \(R_\lambda = 180\). Black □ is the experiment at \(R_\lambda = 214\). Red ◦ is the experiment at \(R_\lambda = 160\). The error bars on the experimental points represent systematic error due to extrapolation from the finite fit time required to measure the rotation rate [61]. The purple ◦ shows the result for infinite aspect ratio rods from simulations by Shin and Koch [33] at \(R_\lambda = 53.3\). The Green line is the analytic prediction for randomly oriented rods from Eq. 4.4. The dashed line indicates the rotation due to vorticity.
4.3.4 Flatness of the Rotation Rate

Figure 4.7: Fourth moment of the rotation rate of ellipsoids. Blue + is the DNS at $R_\lambda = 180$. Black ■ is the experiment at $R_\lambda = 214$. Red ○ is the experiment at $R_\lambda = 160$. Green dashed line is the prediction for spheres uncorrelated with the direction of $\Omega_{ij}$. The purple dashed-dot line is the fourth moment of $S_{ij}$ and the black dashed line represents the case of a Gaussian distribution.

For a more quantitative evaluation of the shape of the pdf of rotation rate squared, we report in Fig. 4.7 a normalized fourth moment of the rotation rate, $\langle (\dot{p}_i \dot{p}_i)^2 \rangle / \langle \dot{p}_i \dot{p}_i \rangle^2$ as a function of aspect ratio. The experimental measurements are in fairly good agreement with the simulations considering that the differences are on the order of the experimental measurement errors and particle size may affect the tails of the experimental distribution as discussed above. The fourth moment for a sphere can be related to the fourth moment of the vorticity tensor by assuming that the orientation director is uncorrelated with $\Omega_{ij}$. Since the rotation rate of sphere is described as $\dot{p}_i = \Omega_{ij} p_j$ and is equivalent to the solid body rotation of the sphere with an orientation vector, $\hat{p}$. This orientation is not defined for spheres due to symmetry. This orientation of $\vec{p}$ is chosen randomly in simulations...
as $\Omega \times \dot{\vec{p}} = |\Omega||\dot{\vec{p}}|\sin(\theta)$. The normalized fourth moment of rotation of sphere has a coefficient that arises from the choice of the $\vec{p}$ and is found as

$$\frac{\langle (\dot{p}_i\dot{p}_i)^2 \rangle}{\langle \dot{p}_i\dot{p}_i \rangle^2} = \frac{\langle \sin^4(\theta) \rangle \langle (\Omega_{ij}\Omega_{ij})^2 \rangle}{\langle \sin^2(\theta) \rangle \langle \Omega_{ij}\Omega_{ij} \rangle^2}$$

$$= \frac{\pi/2}{\pi/2} \int_{\phi=0}^{\pi/2} \int_{\theta=0}^{\pi/2} \sin^4(\theta) \sin(\theta) d\theta d\phi \frac{\langle (\Omega_{ij}\Omega_{ij})^2 \rangle}{\langle \Omega_{ij}\Omega_{ij} \rangle^2}$$

$$\frac{6 \langle (\Omega_{ij}\Omega_{ij})^2 \rangle}{5 \langle \Omega_{ij}\Omega_{ij} \rangle^2}$$

and is in good agreement with the simulations. All these fourth moments are much larger than the value of $5/3$ that is obtained if the components, $\dot{p}_i$, each have a gaussian distribution. Rods ($\alpha >> 1$) and disks ($\alpha << 1$) have nearly identical normalized fourth moments, and the variation with aspect ratio is less than 20%, indicating that the dependence on particle shape of the normalized probability distribution is much weaker than that of the variance of the rotation rate in Fig 4.6.

### 4.3.5 Anisotropy in Measurements

Different experimental and analytical effects contribute to anisotropy. The anisotropy of the fluid flow is a real phenomena to be studied. However, the anisotropy in measurements and analysis technique must be quantified and if possible minimized. We detected a measurement anisotropy in this data which has a large contribution to the components of the rotation rate. As mentioned in Section 3.1.1, we use one laser beam in the $x$ direction for illumination. After analyzing the data and measuring orientations of 1mm rods, an anisotropy in orientation measurement was detected. This problem was almost entirely removed entirely in the next experiment with four beams.
Figure 4.8: Rod orientations measured experimentally in 3D space at $R_\lambda=160$. The area around $x$-axis (green line) is empty due to no measurements.

Figure 4.8 shows all measured orientation vectors in the data set at $R_\lambda=160$. If we plot all orientations on a sphere for an isotropic measurement we expect that the area of the sphere is covered uniformly. However, in Fig. 4.8 an area around the orientation of the laser beam is not covered. This indicates that we were not able to find rods appearing at these orientations. This is mostly due to shadowing effects. If the laser beam illuminates the rod from either ends of the rod, the end caps are much brighter than the sides of the rod. Such images do not have sufficient information to measure orientations. The nonuniform illumination along particles results in anisotropy in orientation measurements.
The anisotropy in sampling orientation results in anisotropy in rotation rate. The mean square rotation rate as a function fit-time for each component of the orientation vector is shown in Fig. 4.9. Since orientations along the $x$-axis are sampled at orientations where $p_x$ is small, the rotation rate, $\dot{p}_x$ on average is larger along this axis compared to other components as shown in Fig. 4.9. The $y$ and $z$ components of the rotation rate have similar behavior at all fit-times. This shows symmetry in $\langle \dot{p}_y \dot{p}_y \rangle$ and $\langle \dot{p}_z \dot{p}_z \rangle$ measurements. In order to remove single component bias in the measured rotation rates, we report the sum of all components
\( \langle \dot{\rho}_i \dot{\rho}_i \rangle = \langle \dot{\rho}_x \dot{\rho}_x + \dot{\rho}_y \dot{\rho}_y + \dot{\rho}_z \dot{\rho}_z \rangle \). The accuracy of this method will be discussed in more details in Section 5.4.1 as we compare these results with the experiment with four beams. We show that the measurements of the individual components of the rotation rate are improved and also the magnitude of the mean square rotation rate, \( \langle \dot{\rho}_i \dot{\rho}_i \rangle \), is measured accurately even though the components are sampled isotropically.

### 4.3.6 Numerical Simulation of Rods

Quantifying the extrapolation uncertainty in measuring the mean square rotation rate of rods requires an indirect way of accessing the actual value of the rotation rate. In order to isolate the effects of data analysis on the same quantity, we use a method first introduced by Voth et al. [61] to quantify the systematic error in mean square rotation rate.

We studied the motion of tracer rods in direct numerical simulations (DNS) of homogeneous and isotropic turbulence at \( R_\lambda =180 \). The translational motion of tracer rods matches that of fluid particles, so we use a database of previously simulated Lagrangian trajectories [63] to integrate Jeffery’s equation (1.3) and obtain the time evolution of particle orientations. The spatial spectral resolution was \( 512^3 \) points. These simulations stored the full velocity gradient tensor along Lagrangian trajectories at time intervals of about \( 1/10 \tau_\eta \). We integrate Equation 1.3 for rods of \( \alpha=5 \) using a fourth order Runge Kutta integration routine. The mean square rotation rate of rods is then measured from this data directly. We use the camera calibration parameters from the experimental data and create images of these rods similar to the experiment. These images are then analyzed using the same analysis code for the experiment.
We measure the rotation rate of these simulated rods and plot the mean square rotation vs. fit-time for these rods as shown in Fig. 4.10. The extrapolation overestimation is measured from this data and is 6%, so the method of analysis overestimates the mean square rotation rate by 6%. We will shift the measured mean square rotation rate measured for rods by this amount and also include this systematic error in the error-bars. We have studied cases with noise added to the simulated images and shown that this difference increases with experimental noise.

**Figure 4.10:** Mean square rotation rate from DNS for different fit-time. The purple □ is the directly measured rotation rate from simulations, red ◊ is the simulations analyzed with experimental method and solid blue line is the extrapolation of the simulated data to zero fit-time.
Rotational Dynamics of Long Rods in Turbulence

We measure the dynamics of long rods with lengths ranging from $2.8\eta$ in the tracer limit up to $70\eta$ in the inertial range. We find that long rods are correlated with eddies of the size of the rod, and the correlation time of the rotation rate scales with the eddy turn-over time of length of the rod. We show how single particle measurements of rods are capable of revealing statistics of the velocity field at different length scales of the flow. We also find that the correlations between the velocities and rotation rate of rods does not depend on the length of rods.
Chapter 5 - Rotational Dynamics of Long Rods in Turbulence

5.1 Experiment

5.1.1 Turbulent Flow

The experiments were performed at two grid frequencies, \( f = 1.5 \text{ Hz} \) and \( 3 \text{Hz} \). Consistency of experiments in measuring the flow parameters and rod motion is required for detailed and exact comparisons of the dynamics of rods with the carrier flow. The density of the fluid was \( \rho = 1.15 \text{gr/cm}^3 \). Matching the density of the fluid with rods is critical since these rods are much longer than Kolmogorov length scale of the flow. Even small differences between densities could introduce inertial effects. The flow parameters are measured using PTV. Velocities of the 200\( \mu \text{m} \) PMMA tracer particles were measured with particle tracking technique as introduced in Section 3.1.3. The rms velocity is \( \bar{u} = \sqrt{u_i u_i / 3} = 30.4 \text{ mm/s} \) at 1.5 Hz and 62.8 mm/s at 3Hz. The Eulerian second and third order longitudinal velocity structure functions are measured and the energy dissipation rate is obtained from the Kolmogorov 4/5 law. Figure 5.1 shows the seconds order velocity structure functions at both grid frequencies. The energy dissipation rate measured from the structure function for 1.5 and 3 Hz are \( \langle \epsilon \rangle = 319 \) and 2800 \( \text{mm}^2/\text{s}^3 \) respectively. Table 5.1 show the flow parameters measured from tracer particle tracking.

5.1.2 Transverse Velocity Structure Function

The second order transverse velocity structure function in these experiments are required at all scales in order to apply a model for the rotation rate of rods. The
second order transverse structure function, \( D_{NN}(r) = \langle \left( \left( \bar{u}(x) - \bar{u}(x + r) \right) \times \vec{r} \right)^2 \rangle \), at separation distance \( r \). Figure 5.2 shows the second order transverse structure function, \( D_{NN}(r) \), for the two experiments at \( R_\lambda =150 \) and 210. The second order transverse structure function is compensated by \( r^{2/3} \), the inertial range dependence.
Chapter 5 - Rotational Dynamics of Long Rods in Turbulence

\[ \lambda = \left(15 \bar{u} L / \nu \right)^{1/2} \]

\[ \bar{u} = \bar{u}^3 / \langle \epsilon \rangle \]

\[ L = (\nu^3 / \langle \epsilon \rangle)^{1/4} \]

\[ \eta = (\nu / \langle \epsilon \rangle)^{1/2} \]

<table>
<thead>
<tr>
<th>Grid Freq.</th>
<th>( R_\lambda )</th>
<th>( \bar{u} )</th>
<th>( \langle \epsilon \rangle )</th>
<th>( L )</th>
<th>( \eta )</th>
<th>( \tau_\eta )</th>
</tr>
</thead>
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<td>Hz</td>
<td>mm/s</td>
<td>mm²/s³</td>
<td>mm</td>
<td>mm</td>
<td>s</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
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<td>30.4</td>
<td>319</td>
<td>87.9</td>
<td>0.36</td>
<td>0.074</td>
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<tr>
<td>3</td>
<td>210</td>
<td>62.8</td>
<td>2800</td>
<td>84</td>
<td>0.21</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table 5.1: Table of flow parameters: Grid Freq., the frequency of oscillation of the grids in the apparatus; \( R_\lambda \), Taylor Reynolds number; \( \bar{u} \), rms velocity of the flow; \( \langle \epsilon \rangle \), energy dissipation rate; \( L \), energy input length scale; \( \eta \), Kolmogorov length scale; \( \tau_\eta \), Kolmogorov time scale. \( \nu \) is the fluid kinematic viscosity and is \( 1.75 \times 10^{-6} m^2/s \).

of the structure function on separation distance. As shown in Fig. 5.2 resolving the structure function at small scales is not possible experimentally. The minimum separation distance that can be measured experimentally is at least twice the diameter of the tracer particles (200 \( \mu \)m) plus the stereomatching uncertainty, so the minimum separation distance is 2.5 time the tracer diameter. So there are no experimental measurements available in the dissipation range \( r \leq \eta \). There is an analytical expression for the transverse structure function in dissipation range [60]:

\[ D_{NN}(r) = \frac{2}{15} \frac{\langle \epsilon \rangle}{\nu} r^2 \]

for \( r < \eta \)

We use Batchelor parametrization [64, 65] to determine the form of the structure function in the region between the dissipation range and experimentally well resolved region. The form of the parameterizing function is

\[ D_{NN}(r) = \frac{\langle \epsilon \rangle}{\nu} \frac{r^2}{(1 + (r/a\eta)^x)^{(2-\xi)/x}} \]

Here \( a \) is the limit of the crossover length. \( \xi = 2/3 \) and is known from the inertial
range scaling. The only fit parameter is the exponent $x$ which is determined using the experimental data and the dissipation range form. The cross over length is defined to be the intersection of the velocity structure function of dissipation range and inertial range as $\frac{2}{15} \langle \epsilon \rangle / \nu r^2 = \frac{4}{3} C_2 (\langle \epsilon \rangle r)^{2/3}$, so we find that $a = r / \eta = (10 C_2)^{4/3}$. Figure 5.2 shows the $D_{NN}$ for both Reynolds number, including the experimental data, dissipation range form and the transition form found by using Eq. 5.2. The Batchelor parametrization matches the dissipation range and the inertial range. The parametrization is not meant to match the large scales as also seen in Fig. 5.2.

5.1.3 Measurements of Rotation of Long Rods

The rods are nylon thread with diameter of 0.2 mm and are cut to different lengths ($l = 1, 3, 6.8, 15.2$ mm). All particles are dyed fluorescent for better detection. The rotational dynamics of rods are measured using stereoscopic images from four high speed cameras[66]. Each camera that detects a rod defines a plane in which the rod should exist. We require at least three cameras to detect each single rod and the orientation of the rod is the intersection of the planes these cameras define as discussed in Section 3.2.

In these experiments the maximum detection volume is 160 cm$^3$, while the effective detection volume is different for each rod length and is the smallest (85 cm$^3$) for the longest rods at 15.2mm. The effective detection volume is smaller to ensure that the entire rod is in the illuminated detection volume so the position and orientation of rods are measured more accurately based on the full length of rods. The number density of rods is small so particle-particle interaction is negligible. The particle concentration is 0.025 cm$^{-3}$ for 1 mm rods, and 0.0075 cm$^{-3}$ for the
longest rods at 15.2 mm. The uncertainty in measuring the center of rods is 60µm for the 1 mm rods and 180µm for 15.2 mm rods. This uncertainty is determined based on the stereomatching accuracy. The accuracy of measuring the orientation for 1 mm particles is 0.01 rad and increases with rod length. The uncertainty in the orientation of the rods is determined from the residual of the intersection of planes defined by multiple cameras.

Figure 5.3: Images of 1 mm rods as viewed by camera 3 (Fig. 3.1). The images are presented in the spherical coordinates of the camera, as θ is measured from the axis connecting the center of the camera to the center of the detection volume and ϕ is the polar angle in the plane parallel to the camera. The colors represent the pixel intensity of the images.

Figure 5.3 shows the raw images of the 1 mm rods as detected by one camera. This figure shows the images at different orientations as viewed by the camera. We present the orientation of the rods in the coordinate system. The angle θ measures the orientation away from axis of the camera and the angle ϕ is the polar angle in the plane parallel to the camera. So θ=0 and ϕ=0 means that the
rod points towards the camera. The full length of the rod is seen by the camera at $\theta = \pi/2$. The intensity of light along the length of the rod is not uniform and depends on the relative orientation of the rod with respect to the camera. However, the image analysis codes are built with layers to handle different cases of lengths and image intensities. These parameters are saved along with results and used as filters on the data. Figure 5.3 shows that we are able to detect rods at all orientations in the camera coordinates.

5.1.4 Improved Illumination

I used four laser beams for illumination to remove the anisotropy in the orientation measurements mentioned in Section 4.3.5. The laser beam is split into two perpendicular beams and we used a set of front surface mirrors to reflect the beams back to the center. We have optimized the orientation of mirrors to avoid the interference patterns at the center of the detection volume. Figure 5.4 shows the mean square rotation rate of 1 mm rods comparing the results of the first experiment with a single beam and the second experiment with multiple beams. The extrapolation over similar range of fit-times are presented for comparison. The mean square rotation rate for one beam data is clearly affected by several sources of measurement uncertainties as we see the extrapolation to zero fit time depends on the range chosen for fit. However, the data for four beams is very close to converging to a single extrapolated value over any range. An important note in Fig 5.4 (c) and (d) is that the anisotropy in the components of the rotation rate measured in single beam experiment is very large and this limitation is completely removed by using multiple laser beams.

Here, we clearly demonstrate the improvement of the experimental measurements
of the rotation rate with illuminating the particles from different directions. Figure 5.4 also shows that even for anisotropic sampling of the experiment with one laser beam, we were able to minimize the anisotropy of each component by reporting the sum of all components as $\langle \dot{p}_i \dot{p}_i \rangle$.

**Figure 5.4:** Comparison of mean square rotation rate for single laser beam and four beam data. (a) $\langle \dot{p}_i \dot{p}_i \rangle$ for 1.5 Hz (b) $\langle \dot{p}_i \dot{p}_i \rangle$ for 3Hz. Red $\circ$ is the single beam and brown solid line is the extrapolation to zero; black $\bullet$ is the four beam and magenta solid line is the extrapolation to zero. Comparison of the components of the mean square rotation rate of single beam with four beams for (c) 1.5Hz (d) 3Hz. Single beam Red $\Box$: $\langle \dot{p}_x \dot{p}_x \rangle$, green $\triangle$: $\langle \dot{p}_y \dot{p}_y \rangle$, black $+$: $\langle \dot{p}_z \dot{p}_z \rangle$; 4 beam, blue $\circ$: $\langle \dot{p}_x \dot{p}_x \rangle$, $\blacktriangle$: $\langle \dot{p}_y \dot{p}_y \rangle$, $\blacktriangleleft$: $\langle \dot{p}_z \dot{p}_z \rangle$. 
5.2 Results

5.2.1 Measuring the Rotation Rate of Long Rods

The mean square rotation rate for all rods is measured as a function of fit-time. Figure 5.5 shows the \( \langle \dot{\hat{p}}_i \dot{\hat{p}}_i \rangle \) for two different Reynolds numbers \( R_\lambda = 150 \) and 210 and different rod lengths. The mean square rotation rate at small fit-time is the largest which means that the contribution of noise is considerable. As discussed earlier we can not separate the noise from rotation rate due to turbulence so we use the extrapolation method. The mean square rotation rate decreases as we increase the fit-time. At very long fit-time the quadratic fit starts to filter the large rotation rates.

The extrapolation to zero fit-time is done by fitting a part of the data within \( 1 \tau_\eta \) to \( 5 \tau_\eta \) with \( f(t) = A \exp(-\lambda t) \). For all rod lengths the extrapolations converge closely for different ranges of \( t/\eta \) for most of the rod lengths. The noise in the orientation measurements depends on the length of rods as we can see from \( \langle \dot{\hat{p}}_i \dot{\hat{p}}_i \rangle \) in Fig. 5.5. At short rod lengths the orientation measurement uncertainty is larger than long rods so we expect better measurements for longer rods. Comparing the data for different rod lengths we can see that at short fit-time the contribution of noise to measurements is decreasing as the length of rods increases from 1 mm to 6.8 mm, Fig.5.5 (a)-(c)-(e). Unfortunately the statistics for 15.2 mm rods is much smaller than other rod lengths for comparison.

An interesting note about the 6.8 mm rods in Fig.5.5 (e), (f) is that the mean square rotation rates between \( 0.5 \tau_\eta \) and \( 1.5 \tau_\eta \) is close to a plateau which one would expect if there was no measurement uncertainty in the data. Resolving the short fit-time for \( R_\lambda = 210 \) is not possible compared to \( R_\lambda = 150 \). The two
Figure 5.5: Mean square rotation rate as a function of fit-time for different rod lengths. Measured mean square rotation rate (red ▶). The solid blue line is the extrapolation to zero fit length. Right column is for $R_\lambda = 150$ and left column is $R_\lambda = 210$ and the length of rods is increasing from top to bottom as (a,b) 1 mm, (c,d) 3 mm, (e,f) 6.8 mm, (g,h) 15.2 mm.

Reynolds numbers have different time-scales $\tau_\eta = 25$ and 74 ms, but the camera frequency is the same for all data sets which results in different time resolutions.
in the data.

**Figure 5.6:** Effect of applying filters on the orientation accuracy on the means square rotation rate. a) 1 mm rods b) 6.8 mm rods at $R_A = 150$. Filter1 = 0.1 black ●. Filter2 = 0.05 green ○ Filter3 = 0.01 red +.

As mentioned earlier the uncertainty in orientation measurements produces uncertainty in measured rotation rates. We use the residual of the function that optimizes the orientation in 3D to reject some of the bad measurements. This quantity $\sum_{i=cameras}(\hat{n}_i \cdot \hat{p})^2$ has an average value less than $10^{-4}$ and a maximum of 1. Figure 5.6 compares the effects of two filters on the mean square rotation rate of 1 mm and 6.8 mm rods. The first filter is 0.1 and means that we reject all data with $\sum_{i=cameras}(\hat{n}_i \cdot \hat{p})^2 > 0.1$. The third filter is 0.01. The mean square rotation rate is slightly affected by these filters and converge to the same extrapolated value between the second and third filters. We used filter1 (0.1) for rod lengths of 3, 6.8 and 15.2 mm for mean square rotation rates in Fig. 5.5 and filter3 (0.01) for 1 mm rods. The uncertainty in measuring the orientation of the rods depends on the length of the rods and is largest for short rods, the smallest filter is used for shortest rods.
5.2.2 Effects of Length of Rods on Rotation Rates

The mean square rotation rate as a function of rod length is shown in Fig. 5.7. Our experimental measurements show that by increasing the length of rods, the mean square rotation rate decreases. As the rotation rate of short rods is due to the smallest eddies with the largest velocity gradients, one would expect that the mean square rotation rate should be largest for shortest rods. Longer rods are expected to experience smaller rotation rate compared to tracer rods. We have corrected the measurements by shifting it down 6% from the extrapolated value at zero fit-time. As the extrapolation method overestimates the results by this amount. These results agree within measurement uncertainty with an earlier simulation[33] of long fibers in turbulent flow at $R_\lambda = 53.3$.

![Figure 5.7](image)

**Figure 5.7:** Mean square rotation rate as a function of rod lengths. Experimental data at $R_\lambda = 150$ (○) and 210, (□)

The error bars on the experimental measurement of the rotation rate in Fig. 5.8
represent both statistical error of measuring the rotation rates from experimental data and systematic error due to extrapolation. We have obtained the systematic error due to extrapolation of the mean square rotation rate by simulation of particles as described in Section 4.3.6.

5.2.3 Model for Rotation of Randomly Oriented Rods

For a more detailed understanding of the dependence of the rotation rate on the length of rods, we compare the results of our measurements in Fig. 5.7 with the rotation rate of randomly oriented rods. Olson and Kerekes [15] have proposed a model to describe the rotational velocity of fibers which was introduced in Section 1.2.2. They show that the rotation rate of a neutrally buoyant fiber of length $l$ is described as

$$ \dot{p}_i = \frac{12}{l^3} \int_{-l/2}^{l/2} (\delta_{ij} - p_ip_j) u_j(r) r dr $$

(5.3)

where $\vec{p}$ is the orientation unit vector of the fiber and $\vec{u}$ is the turbulence fluctuating velocity along the fiber. If we assume that the orientation of rod is uncorrelated with the velocity of the flow in Eq. 5.3, we can obtain an expression for the mean square rotation rate of randomly oriented rods. The following derivation is based on the results by Olson and Kerekes [15].

$$ \langle \dot{p}_i \dot{p}_j \rangle = \frac{12}{l^3} \int_{-l/2}^{l/2} (\delta_{ik} - p_ip_k) u_k(r) r dr \cdot \frac{12}{l^3} \int_{-l/2}^{l/2} (\delta_{jm} - p_jp_m) u_m(r) r dr $$

(5.4)

assuming that the orientation of fibers is uncorrelated with the velocity of the flow

$$ \langle \dot{p}_i \dot{p}_j \rangle = \frac{144}{l^6} (\delta_{ik} - p_ip_k) (\delta_{jm} - p_jp_m) \langle \int_{-l/2}^{l/2} u_k(r) r dr u_m(r') r' dr' \rangle $$

(5.5)

The integrand is symmetric around a square which can be defined with the limits of the integrals, [-l/2 l/2] and [-l/2 l/2]. The integral over a square is equal to
twice the integral over a triangle as \( \int_{-l/2}^{l/2} \int_{-l/2}^{l/2} drdr' = 2 \int_{-l/2}^{l/2} \int_{-l/2}^{l/2} dr'dr. \)

\[
\langle \dot{p}_i \dot{p}_j \rangle = \frac{288}{l^6} \langle (\delta_{jm} \ - p_j p_m) (\delta_{ik} \ - p_i p_k) \rangle \int_{-l/2}^{l/2} \int_{-l/2}^{l/2} u_m(r')r'dr'u_k(r)rdr \tag{5.6}
\]

We apply a change of variables as \( r' = r + s \).

\[
\langle \dot{p}_i \dot{p}_j \rangle = \frac{288}{l^6} \langle (\delta_{jm} \ - p_j p_m) (\delta_{ik} \ - p_i p_k) \rangle \int_{-l/2}^{l/2} \int_{(r+l/2)}^{0} u_m(r + s)u_k(r)(r + s)dsdr \tag{5.7}
\]

We know that \( u_m(r + s)u_k(r) \) is related to the velocity correlation function at separation distance \( s \) as \( \tilde{u}^2 R_{km}(s) = u_m(r + s)u_k(r) \). The integral in Eq. 5.7 is integrated below.

\[
\tilde{u}^2 \int_{-l/2}^{l/2} \int_{-l/2}^{0} R_{km}(s)r(r + s)dsdr = \tilde{u}^2 \left[ \frac{r^3}{3} \int_{-l/2}^{0} R_{km}(s)ds \right]^{l/2}_{r = -l/2} - \int_{-l/2}^{l/2} \frac{r^3}{3} dr
\]

\[
R_{km}(-r - \frac{l}{2})dr + \frac{r^2}{2} \int_{-l/2}^{0} sR_{km}(s)ds \left. \right|^{l/2}_{l/2} - \int_{-l/2}^{l/2} \frac{r^2}{2} (-r - \frac{l}{2})R_{km}(-r - \frac{l}{2})dr
\]

\[
= \tilde{u}^2 \left[ \frac{l^3}{24} \int_{-l}^{0} R_{km}(s)ds - \int_{-l/2}^{l/2} \frac{r^3}{3} R_{km}(-r - \frac{l}{2})dr \frac{l^2}{8} \int_{-l}^{0} R_{km}(s)ds
\]

\[
- \int_{-l/2}^{l/2} \frac{r^2}{2} (-r - \frac{l}{2})R_{km}(-r - \frac{l}{2})dr \right] \tag{5.8a}
\]

Now change variable as \( x = -r - \frac{l}{2} \) and simplify Eq. 5.8a

\[
= \tilde{u}^2 \left[ \frac{l^3}{24} \int_{-l}^{0} R_{km}(s)ds - \int_{0}^{-l} \frac{1}{3} (x + \frac{l}{2})^3 R_{km}(x)dx + \frac{l^2}{8} \int_{-l}^{0} R_{km}(s)ds + \int_{0}^{-l} \frac{x}{2} (x + \frac{l}{2})^2 R_{km}(x)dx \right] \tag{5.9}
\]

Since \( x \) and \( s \) are variables in the integrals, we can replace them by \( r \)

\[
= \tilde{u}^2 \left[ \int_{0}^{-l} \left( -\frac{l^3}{24} - \frac{x^2}{3} - \frac{r^2}{8} + \frac{r}{2} (-r - \frac{l}{2})^2 \right) R_{km}(x)dx \right]
\]
\[ \dot{\mathcal{p}}_i \dot{\mathcal{p}}_i = \frac{24}{l^3} \bar{u}^2 \left[ \int_0^l \left( 1 - 3 \frac{r}{l} + 2 \left( \frac{r}{l} \right)^3 \right) \delta_{mk} R_{km}(r) dr \right] \] (5.11)

As \( R_{22} = R_{33} = R_{NN} \) and rods are rigid, the mean square rotation rate of randomly oriented long rods is \([15]\)

\[ \langle \dot{p}_i \dot{p}_i \rangle = \frac{48 \bar{u}^2}{l^3} \int_0^l \left[ 1 - 3 \frac{r}{l} + 2 \left( \frac{r}{l} \right)^3 \right] R_{NN}(r) dr \] (5.12)

where \( \bar{u}^2 \) is the rms velocity of the fluid flow and \( R_{NN}(r) \) is the fluid transverse velocity correlation function at separation distance of \( r \). \( R_{NN}(s) \) is related to the velocity structure function as \( \bar{u}^2 R_{NN}(r) = \bar{u}^2 - \frac{1}{2} D_{NN}(r) \). We are able to apply this model to the experimental measurements since we have measured the flow transverse velocity structure function.

In Fig. 5.8 the mean square rotation rate of rods is compared with the model for randomly oriented rods by applying Eq. 5.12 to our experimental measurements of \( D_{NN} \) at \( R_\lambda = 150, 210 \). The mean square rotation rate of short rods is much smaller than randomly oriented rods of the same length \( (l/\eta) \). However, this difference decreases as the length of the rod is increased with respect to \( \eta \). Previous studies of tracer rods \([49, 33, 35, 66]\) have shown that as rods are carried by the flow their orientation becomes correlated with the directions defined by the velocity gradient tensor of the flow and this alignment results in suppression of the rotation rate of short rods compared to randomly oriented rods \([66]\). The smaller differences between measured rotation rate of long rods and randomly oriented rods suggests that the alignment of long rods with the velocity gradient of the flow at their length scale is weaker than short rods.
Figure 5.8: Mean square rotation rate as a function of rod lengths. Comparison of the experimental data at $R_\lambda = 150$ (red ◦) and 210, (red □) and model of randomly oriented rods at $R_\lambda = 150$ (purple ◊) and $R_\lambda = 210$ (black ◳). The inset is the ratio of the mean square rotation rate of randomly oriented rods to the experiment.

The ratio of rotation rate of randomly oriented rods to experimental measurement is shown in the inset of Fig. 5.8. The relative value of the rotation rate for random rods to the experiment confirms that the correlation of long rods with the flow is weaker than short rods but not negligible. In contrast to this observation, the previous simulations of long fibers [33] the correlation of long fibers with the flow is much weaker and close to randomly oriented rods. We expect this discrepancy to be the result of much smaller Reynolds number in the simulations.

The mean square rotation rate for randomly oriented rods calculated from experimental measurements does not have Reynolds number dependence within measurement uncertainty. The small differences between two Reynolds number
in the model for randomly oriented rods are due to uncertainties in measuring $D_{NN}$.

### 5.2.4 Scaling Law for Rotation Rate

We propose a scaling law for the mean square rotation rate in the Kolmogorov inertial range. This scaling is derived with two methods. We first use dimensional analysis to find the scaling. We know that the rotation rate of tracer rods scales by the Kolmogorov time scale $\langle \dot{p}_i \dot{p}_i \rangle \sim \tau_\eta^{-2}$. Rotation rate has dimensions of $s^{-1}$ and the time scales of small scales is $\tau_\eta$. Assuming that long rods are only responding to the eddies of their size, the mean square rotation rate for rod at length $l$ will scale like $\tau_l^{-2}$, where $\tau_l$ is the time scale of eddies of size $l$. In the inertial range the time scale $\tau_l$ can be defined as $\tau_l = l/u_l = l/(l\langle \epsilon \rangle)^{1/3}$, where $u_l$ is the velocity at length $l$. This argument results in an inertial range scaling for mean square rotation rate as $\langle \dot{p}_i \dot{p}_i \rangle \sim l^{-4/3}$.

We can derive the same scaling argument for the rotation rate in the inertial range using Eq. 5.12 and assuming that at all scales the velocity structure function have the form of the inertial range. This assumption is equivalent to an infinitely large Reynolds number. The velocity structure function in the inertial range is defined as $D_{NN}(s) = 4/3D_{LL} = 4/3C_2\langle \epsilon \rangle^{2/3}$, where $C_2$ is an approximately universal constant [59, 60, 67]. By replacing this form in Eq. 5.12, the mean squared rotation rate for randomly oriented rods in the inertial range is

$$
\langle \dot{p}_i \dot{p}_i \rangle = \frac{48}{l^3} \int_0^l \left[ 1 - 3 \frac{r}{l} + 2 \left( \frac{r}{l} \right)^3 \right] \left[ \dot{u}^2 - \frac{2}{3} C_2 \langle \epsilon \rangle^{2/3} \right] dr \quad (5.13)
$$
and replace $\eta = (\nu^3/\langle \epsilon \rangle)^{1/4}$,

$$\langle \dot{p}_i \dot{p}_i \rangle / (\langle \epsilon \rangle / \nu) = \frac{108}{35} C_2 \left( \frac{l}{\eta} \right)^{-4/3}$$

(5.14)

Figure 5.9 shows the scaling law in the inertial range (Eq. 5.14) and compares it with the mean square rotation rate of randomly oriented rods and our experimental measurements. Both the measured rotation rates and the prediction of Eq. 5.12 approach an $l^{-4/3}$ scaling for large $l$. The experimental data has a smaller coefficient compared to randomly oriented rods as expected due to alignment effects.

![Figure 5.9: Mean square rotation rate as a function of rod lengths. Comparison of the mean square rotation rate of rods (red ◦ and □) and model for randomly oriented rods (○ and □) with $l^{-4/3}$ scaling argument (solid green line) in the inertial range.](image)

Within error bars of the experimental data for $l > 30\eta$, one could also fit the data with a different exponent slightly steeper than $-4/3$. In this same range, the
prediction of Eq. 5.12 is also steeper than $l^{-4/3}$. The cause of the steeper scaling can be found in the fact that the prediction of Eq. 5.12 overshoots the power law scaling in the range $20\eta < l < 50\eta$. This overshoot occurs in the range of scales slightly larger than the dissipative range because scales smaller than the length of the rod contribute to the rotation rate. For $l > 50\eta$, well into the inertial range, the prediction converges with the scaling law from Eq. 5.14, because here the contributions from the dissipation range are becoming negligible. The overshoot also appears to exist in the experimental rotation rate data, although the effect is of the same size as the measurement uncertainties.

We now investigate this overshoot in more detail and quantify how the contribution of all scales smaller than the length of rods should be considered.

### 5.2.5 Which Scales Affect the Rotation Rates?

As seen in Fig. 5.9, the mean square rotation rate of randomly oriented rods seem to be larger at scales between $20\eta$ and $50\eta$. The experiments at $R_\lambda = 150$ and $210$ have a small inertial range that spans less than a decade of $l/\eta$, but we have access to $D_{NN}$ for a very large inertial range from the Batchelor parametrization. In Eq. 5.14 we use the $D_{NN}$ of Batchelor parametrization as shown in Fig. 5.2, and integrate for high Reynolds number or equivalently large inertial range.

Figure 5.10 compares the scaling law with the mean square rotation rate of rods at high Reynolds number flow. The results shows that the overshoot at scales smaller than $50\eta$ is happening at high Reynolds numbers. However, the mean square rotation rate of randomly oriented rods approaches the scaling law at $100\eta$ and agrees beyond that. Results in figure 5.10 suggest that the scales smaller than the length of the rod contribute to the rotation rate but only scales close to
the length of the rod are relevant. For rods well into the inertial range the small scales are negligible.

We confirm this proposal by finding the two regimes of length scales, far from $l$ and close to $l$, in the integral of Eq. 5.14 below

$$\langle \dot{p}_i \dot{p}_i \rangle = \frac{48}{l^3} \int_0^l \left[ 1 - 3 \frac{r}{l} + 2 \left( \frac{r}{l} \right)^3 \right] R_{NN}(r) dr$$

We apply a change of variables as $x = \frac{r}{l}$ and replace $\bar{u}^2 R_{NN}(r) = \bar{u}^2 - 1/2D_{NN}(r)$, so

$$\langle \dot{p}_i \dot{p}_i \rangle = \frac{48}{l^2} \int_0^1 \left[ 1 - 3x + 2x^3 \right] \left[ \bar{u}^2 - \frac{1}{2}D_{NN}(xl) \right] dx \quad (5.15)$$
\[
\begin{align*}
\int_0^\delta [1 - 3x + 2x^3] \left[ \tilde{u}^2 - \frac{1}{2} D_{NN}(xl) \right] \, dx + \int_0^1 [1 - 3x + 2x^3] \left[ \tilde{u}^2 - \frac{1}{2} D_{NN}(xl) \right] \, dx \\
= 24 \int_0^\delta [1 - 3x + 2x^3] \left[ -\frac{2}{15} \langle \epsilon \rangle (xl)^2 \right] \, dx + \int_0^1 [1 - 3x + 2x^3] \left[ -\frac{4}{3} C_2 (\langle \epsilon \rangle xl)^{2/3} \right] \, dx \\
= 24 \left( -\frac{2}{15} \int_0^\delta [x^2 - 3x^3 + 2x^5] \, dx - \frac{4l^{-4/3}}{3} C_2 (\langle \epsilon \rangle)^{2/3} \int_\delta^1 [x^{2/3} - 3x^{5/3} + 2x^{11/3}] \, dx \right)
\end{align*}
\]

\[
\frac{\langle \dot{\hat{p}}_i \dot{\hat{p}}_i \rangle}{\langle \epsilon \rangle / \nu} = \frac{108}{35} C_2 \frac{l^{-4/3}}{\eta^{-4/3}} + 24 \left[ -\frac{2}{15} \left( \frac{\delta^3}{3} - \frac{3\delta^4}{4} + \frac{\delta^6}{3} \right) + \frac{4}{3} C_2 \frac{l^{-4/3}}{\eta^{-4/3}} \left( \frac{3}{5} \delta^{5/3} - \frac{9}{8} \delta^{8/3} + \frac{13}{7} \delta^{14/3} \right) \right]
\]

For \( \delta \to 0 \) all terms including \( \delta \) will be zero and it will result in the inertial range scaling of \( l^{-4/3} \). While if \( \delta \) is not negligible there is contributions from the scales smaller than \( \delta \) to the rotation rate and the results will deviate from predictions of Eq. 5.9.

### 5.2.6 Probability Distribution of Rotation Rate

Figure 5.11 shows the probability distribution function of the rotation rate squared, \( \dot{\hat{p}}_i \dot{\hat{p}}_i \), normalized by the mean for different rod lengths. The PDF shows a weak dependence on the rod length as seen in the tail of the distribution. For all rod lengths there are rotation rates, \( \dot{\hat{p}}_i \dot{\hat{p}}_i \), larger than 10 times the mean, \( \langle \dot{\hat{p}}_i \dot{\hat{p}}_i \rangle \), which implies that even for long rods there are rare events with large rotation rates. As seen in Fig. 5.11 the tail of the distribution for long rods \( (l/\eta > 20) \) is below the tracer rods \( (l/\eta < 7) \). The error bars represent the random statistical error and the systematic error in measuring the rotation rates due to extrapolation in measuring the mean square rotation rates.

It is helpful to compare the PDF of the rotation rate of long rods with the PDF of acceleration of large spheres in turbulence. Recent experimental results by Volk
et. al [14] shows that the tail of the distribution becomes narrower as the size of the spheres become larger. This same behavior is seen for the PDF of the rotation rate of rods.

### 5.2.7 Lagrangian Autocorrelation of Rotation Rate

Figure 5.15 (a) shows the Lagrangian autocorrelation of rotation rate measured for different rod lengths. Our measurements show that the autocorrelation time of rotation rate depends on the length of rods and increases with rod length. The rotation rate of longer rods remain correlated longer than short rods. These
results agree with simulation [33] at $R_\lambda=39.9$. If the rods are rotating due to eddies of their size, $l$, then the decay time for the correlation of rotation rate of various rod lengths should scale as the turn over time at the length of rods.

Figure 5.12: Lagrangian auto-correlation of rotation rate for different rod lengths. a) The zero-crossing time. The length of rods in Kolmogorov length-scales are $l/\eta= 2.8$ (green +), 4.9 (black •), 8.5 (🔗), 14.5 (open □), 19.1 (red ◇), 32.8 (blue ◇), 42.2 (brown ◇) and 72.9 (purple ◇). b) The horizontal axis is normalized by the time-scale of the size of the rods $\tau_l$. The symbols are displayed at every other data point.
In Fig. 5.15(b) the horizontal axis \((t)\) is normalized by \(\tau_l\), time scale of eddies with length scale \(l\). The Lagrangian auto-correlation of rotation rate for all rod lengths collapse on a single curve within measurement uncertainty and shows that the autocorrelation of rotation rate of rods scales with the time-scale of the eddies of the length of the rod for all rod lengths. The time scales for short tracer rods \((l/\eta < 5)\) is the Kolmogorov time-scale \((\tau_\eta)\) and the time scales for longer rods are measured from longitudinal second order velocity structure function of the fluid particles \((\tau_l = \frac{1}{\sqrt{15}} l/\delta u_l = \frac{1}{\sqrt{15}} l/\sqrt{D_{LL}(l)})\). Measuring the autocorrelation function for long rods \((l = 42.4\eta \text{ and } 72.9\eta)\) is difficult as the effective detection volume is small compared to the length of these rods so the trajectories are not long enough to measure long time auto-correlations.

### 5.3 Rods and Coupling Between Small Scales and Large Scales

Rotation rate of rods scales with the eddies with size of the length of the rods as we have shown in Sections 5.2.2 and 5.2.7, and velocity of the rods is a large scale quantity. We measured two quantities, rod helicity and conditional rotation rates, which have correlations between velocity and the rotation rate of rods.

#### 5.3.1 Rod Helicity

We measured the helicity corresponding with the rotation rate of rods. Helicity measures the alignment between the velocity and vorticity and in isotropic turbulence exhibits similar cascades to the energy [68]. We define the rod helicity as \(H_{rod} = \vec{u}_{rod} \cdot \vec{\omega}_{rod}\) where \(\vec{u}_{rod}\) is the translational velocity of the center of the rod
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and $\vec{\omega}_{\text{rod}} = \hat{p} \times \vec{p}$ is the component of the angular velocity of the rod perpendicular to the rod. Due to symmetry of the rods along major axis of the axis the full rotation rate of rods is not measurable and the angular velocity normal to the orientation of the rod is the only component we can measure. For all rod lengths the average helicity, $\langle H_{\text{rod}} \rangle$, is close to zero as expected. There is no preferred handedness in the sample.

Figure 5.13 shows the root mean square helicity for different rod lengths. Our measurements show that the rms of rod helicity normalized by the absolute value of the velocity and rotation rate, $\langle H_{\text{rod}}^2 \rangle^{(1/2)} / (\langle u_{\text{rod}}^2 \rangle^{(1/2)} \langle \omega_{\text{rod}}^2 \rangle^{(1/2)})$, does not depend on the length of the rods within our measurement uncertainty. This result suggests that there is no preferred alignment between the rotation rate of rods and velocity of rods of different length. Since this quantity depends on the relative orientation of the rotation and velocity vectors as

$$\frac{\langle H_{\text{rod}}^2 \rangle^{(1/2)}}{\langle u_{\text{rod}}^2 \rangle^{(1/2)} \langle \omega_{\text{rod}}^2 \rangle^{(1/2)}} = \frac{\langle u_{\text{rod}}^2 \omega_{\text{rod}}^2 \cos^2(u_{\text{rod}}, \omega_{\text{rod}}) \rangle^{(1/2)}}{\langle u_{\text{rod}}^2 \rangle^{(1/2)} \langle \omega_{\text{rod}}^2 \rangle^{(1/2)}} \quad (5.16)$$

We have also measured the rod helicity for simulated tracer rods (Section 4.3.6) at $R_\lambda = 180$ which is also close to our experimental measurements. We have seen in Fig. 5.7 that the mean square rotation rate of rods depend on the length of rods. Since the magnitude of the rotation rate is included in the normalization of Eq.5.16, the effects of the length of rods on the magnitude of Helicity is removed in this quantity.

The probability distribution of rod helicity is shown in Fig. 5.15. The PDF does not show any dependence on the length of the rods. The statistics are smaller for the longest rods compared to tracers but even at smaller statistics there is no significant rod length dependence. The PDF of rod helicity measured here has more rare events in the tails than the fluid helicity although the rms helicity is
Figure 5.13: RMS rod helicity measured for different rod lengths. Comparison of experiment at $R_\lambda = 150$ (red ◦), $R_\lambda = 210$ (red □) and simulated tracer rods at $R_\lambda = 180$ (green ◂).

not larger. Such behavior would arise due to larger intermittency in rod rotation rates. The experimental measurements also agree with the simulation of rods at $R_\lambda = 180$. In simulations of tracer rods [66] it has been shown that rod rotation rate is more intermittent than both vorticity and strain. Wider tails of the PDF of rotation rate is expected compared to fluid helicity since $\vec{\omega}_{rod} = \hat{p} \times \vec{p}$ is more intermittent than $\omega_{ij}$, fluid vorticity.

5.3.2 Conditioning Rotation Rate on Velocity

The signatures of large scales in the properties of small scales have been observed in many turbulent flows. Blum et al [69] and Wijesinghe [70] have shown that the conditional longitudinal velocity structure function of this same turbulent flow has some anisotropy between the statistics and the direction of energy injection (z-direction) is different from other directions. We condition the rotation rate of
Figure 5.14: Probability distribution of rod helicity for different rod lengths. The length of rods in Kolmogorov length-scales are $l/\eta = 2.8$ (green $+$), $4.9$ (black ●), $8.5$ (○), $14.5$ (open □), $19.1$ (red ○), $32.8$ (blue ○), $42.2$ (brown △), $72.9$ (purple ⋄) and dash-dotted line is the simulations of tracer rods at $R_\lambda = 180$.

Tracer rods on the velocity. This quantity reveals the dependence of the rotation rate on the magnitude and orientation of the velocity. The rms of the rotation rate of 1 mm rods conditioned on different components of the velocity is shown in Fig. 5.15. The conditional rotation rate on $u_x$ and $u_y$ have the same values in most bins of the velocity. However the dependence of the rotation rate on the $z$-component of the velocity is weaker. We know that this is the direction of the energy injection into the flow at large scales. The dependence of the rotation rate on the velocities measured here is comparable to the conditional velocity structure function of this same turbulent flow. The rotation rate of rods is correlated with the transverse velocity structure function while the previous measurements are done for the longitudinal structure function. The $z$-dependence of the rotation rate is smaller than $x$ and $y$ dependence but for the longitudinal structure function is the opposite.
Chapter 5 - Rotational Dynamics of Long Rods in Turbulence

Figure 5.15: Rotation rate of rods at $R_\lambda = 210$ conditioned on the components of the velocity a) 1 mm rods, b) 3 mm rods. Blue □: conditioned on $u_x$, green ▼ conditioned on $u_y$: red + conditioned on $u_z$.

These measurements show that there is anisotropy in the flow and the $z$-direction is slightly different from the other directions. The flow is approaching isotropy at small scales [70] but the effects of large scales appear in the statistics of small scales. The conditional rotation rate for 1 mm and 3 mm rods have similar dependence on the velocities. This shows that the dependence of rotation rate on the velocity scales up to 15 $\eta$ is similar to small scales. To resolve this quantity we need large statistics that are not available in other rod lengths.

5.4 Anisotropy Effects

We remove the significant orientation measurement anisotropy due to illumination by using multiple laser beams. As discussed in Fig. 5.4 the samples in measured orientation are affected by the direction of the laser beam.
5.4.1 Measurement Anisotropy

Figure 5.16: Components of the mean square rotation rate for different fit times. Black o is the sum of all components, x-component of the rotation rate: blue □, y-component: green ▼ and z-component: red +. Right column is for $R_\lambda = 150$ and left column is $R_\lambda = 210$ and the length of rods is increasing from top to bottom as (a,b) 1 mm, (c,d) 3 mm, (e,f) 6.8 mm, (g,h) 15.2 mm.
The anisotropy in measurements should be identified and the effects of the anisotropy should be quantified in the data. We have shown the effects of measurements anisotropy in rotation rate can be identified by comparing components of the rotation rate. Figure 5.16 shows the components of the mean square rotation rate for different fit-times for all rod lengths at two Reynolds numbers. The measurements of different components of the rotation rate for 1 mm to 6.8 mm rods have a very small dependence on the direction of the components. This suggests that the measurements of the rotation rate are very close to isotropic. The differences between different components become larger at very long fit-times for 6.8 mm rods. Some anisotropy is expected for long rods since the detection volume is only a few rod length wide in each direction and the number of tracks that span more than a few \( \tau_\eta \) is very limited so at long fit-times the number of samples are very small and the statistics are not converging. Also our measurements for the 15.2 mm rods are less isotropic than all other rod lengths even at small fit-times. Unfortunately, these measurements have the smallest number of samples besides the skewness of the detection volume that hard to separate for this particular length.
Conclusions

Our measurements of the rotational dynamics of the rods in two dimensional flow show that the rotation rates of rods are in good agreement with the rotation rate predicted for ellipsoidal particles without inertia in a flow with a uniform velocity gradient. This agreement holds even for rods with lengths up to 53% of length scale of the velocity field. We find that rods align weakly with the extensional direction of strain-rate; however, the alignment with the eigenvectors of the Cauchy-Green deformation tensor is much stronger. In both periodic and non-periodic flows the alignment of rods with the direction of the Cauchy-Green deformation tensor is almost independent of rod length.

We performed the first experimental measurements of the time resolved rotational dynamics of rods in turbulent flow. The mean square rotation rate of tracer rods compared with the randomly oriented rods is much smaller because of the strong alignment of rods with the direction defined by the velocity gradient of the flow. The probability distribution of the rotation rates has a long tail due to intermittency of the rotation rate inherited from velocity gradient of the flow. Our experimental measurements of the rotation rates of tracer rods agrees with
the numerical simulations of tracer rods well.

The measured mean square rotation rates of long rods, ranging from $2.8\eta$ to $72\eta$, decreases as the length of the rod increases. We propose an inertial range scaling for the mean square rotation rate and show that our results approach this $l^{-4/3}$ scaling in the inertial range. The PDF of rotation rate shows only a weak dependence on rod length. We find that rods develop preferential alignment so that their rotation rates are considerably smaller than that predicted for randomly oriented rods for all rod lengths. This alignment depends on rod length as rods in the inertial range show a smaller effect of alignment than tracer rods. The Lagrangian autocorrelation time of the rotation rate depends on the length of rods and scales with the eddy turn over time at a scale equal to the rod length. Helicity measured from the rotation rate of rods is the alignment of the direction of the rotation rate of rods with their velocities and does not have any rod length dependence.

The research on the dynamics of rods has great potential to contribute to fundamental theory of turbulence and also be applied to numerous applications of anisotropic particles in fluid flows. We quantify the strong correlation of the orientation of tracer rods with the fluid velocity. Long rods provide a promising path for studying the dynamics of large particles in turbulence since the dynamics of slender bodies can be related to the flow structure analytically. Single particle measurements of long rods can be directly used to probe the spatial structure of the flow at different length scales.
Bibliography


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