Effects of Fluctuating Energy Input on the Small Scales in Turbulence

by

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Abstract

In the standard cascade picture of 3D turbulent fluid flows, energy is injected at a constant rate at large scales. It is then transferred to smaller scales by triad interactions that result from the non-linearity of the Navier-Stokes equation. The down-scale transfer is intermittent, and a vast literature has explored the signatures of this internal intermittency on statistics of the small scales in turbulence. However, the energy injection at large scales is not constant in most real turbulent flows. We explore the signatures of these fluctuations of large scale energy input on small scale turbulence statistics. Measurements were made in a flow between oscillating grids, which produces $Re_\lambda$ up to 271. By modulating the oscillating grid frequency we can introduce temporal variations in the large scale energy injection. We find that the Kolmogorov constant depends on the degree of large scale energy fluctuations, and we can quantitatively predict this change with a refined model from Kolmogorov hypotheses by measuring time dependence of the energy at large scales. The effects of fluctuations of large scale energy input can also be observed by conditioning structure functions on the large scale velocity. Quantifying this dependence provides an alternative measurement of the change of Kolmogorov constant and it is in agreement with the experimental measurement and refined model.
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3.16 The relationship of the curvature $b$ of the conditional second order structure function with the experimental measurements of Kolmogorov constant, our refined model, and Monin and Yaglom model
Turbulent fluid flow is a chaotic and non-linear phenomenon. It is complicated and hard to understand, but at the same time it is important to our daily life because the turbulent flows are everywhere from oceans to atmosphere. Many scientists and engineers have endeavored to gain understanding on turbulent flows to control and predict this phenomenon. In the 19th century, Claude-Louis Navier and George Stokes put together the Navier-Stokes equation that can solve many important problems of fluid dynamics. A major breakthrough in the understanding of the turbulent flow happened in the 1940s when Russian scientist Andrey Kolmogorov proposed Kolmogorov’s hypotheses in 1941 to explain the turbulent motions in incompressible flow.

1.1 Energy Cascade

Turbulence in a fluid flow consists of dynamics at many different length scales. Consider a diagram of energy cascading down the turbulent flow at high Reynolds
number with its largest eddy at characteristic length scale $l_0$ and characteristic velocity $u(l_0)$. The energy is injected into the flow at the largest scale. Large eddies are anisotropic and affected by boundary conditions. They are unstable so they break up and transfer the energy down to smaller scale until it reaches a sufficiently small length scale that the viscosity becomes effective in dissipating the energy.

### 1.2 Kolmogorov’s Hypotheses

Kolmogorov argued that the directional information of the large scales is lost in the process of eddies being broken down, and therefore the small scale should be isotropic. In the energy cascade, there are two dominant processes: the energy transfer when large eddies break down into smaller ones, and the viscous dissipation in the smallest scales. Based on this understanding, Kolmogorov’s first similarity hypothesis states that in a turbulent flow at sufficiently high Reynolds number, the statistics of the small scale ($l \approx \eta$) motions have a universal form that is uniquely determined by the kinematic viscosity $\nu$ and energy dissipation rate per unit mass $\varepsilon$. This range is known as the dissipative range. The smallest length scale is called the Kolmogorov length scale $\eta$, which can be represented as

$$\eta = (\nu^3/\varepsilon)^{1/4}. \quad (1.1)$$

The ratio of the smallest to largest scale decreases as $Re^{-3/4}$ [1]. Therefore, as the Reynold’s number becomes higher, there is a larger length separation of small and large scale. As a result, there exists a range of length scale $l$ that is
very small compared to the largest eddy size $l_0$, but still very large compared to Kolmogorov length scale $\eta$. These eddies in this range still have a large enough Reynolds number compared to those at the dissipative range, so their motions are hardly affected by viscosity. As a consequence, Kolmogorov’s second similarity hypothesis states that in a turbulent flow at sufficiently high Reynolds number, the statistics of the motions of scale $l$ ranging $l_0 \gg l \gg \eta$ have a universal form that is uniquely determined by $\varepsilon$ and independent of $\nu$. This range is commonly known as the inertial range. A summary of different length scales and ranges are shown in Fig. 1.1.

**Figure 1.1:** A diagram showing the process of energy cascade for different length scales and ranges of a high Reynold’s number turbulence.

### 1.3 Structure Function

Kolmogorov uses the second order velocity structure function to illustrate the similarity hypotheses [2]. To obtain the structure function, we have to know the longitudinal velocity difference of two particles with distance $r$ denoted as $\Delta u_r = [u(r) - u(x + r)]_L$. The subscript $L$ denotes that it is the longitudinal velocity difference. The structure function is therefore defined as $D_p = \langle (\Delta u_r)^p \rangle$ where $p$ represents the order of the structure function. When $p = 2$, the second order structure function is the mean square of velocity difference between two points.
with separation distance $r$. According to the second similarity hypothesis, the statistics of the motion are independent of $\nu$ in the initial range, so the longitudinal structure function can be written as

$$\langle (\Delta u_r)^p \rangle = C_p (\varepsilon r)^{p/3},$$  \hspace{1cm} (1.2)

where $C_p$ is a constant and $\varepsilon$ is the energy dissipation rate [2]. Under constant energy input, we can obtain the second order structure function in the inertial range being $\langle (\Delta u_r)^2 \rangle = C_2 (\varepsilon r)^{2/3}$. Previous publication suggests that the experimental value of $C_2$ is within 15% of 2.0. [3] As for the third-order structure function, one can use Navier-Stokes equation to solve for the exact solution as $\langle (\Delta u_r)^3 \rangle = \frac{4}{5} (\varepsilon r)$. This is known as the four-fifths law [2]. We will use these second and third-order structure functions repeatedly in our data analysis.

### 1.4 Fluctuations of Large Scale Energy Input

We have now only considered constant energy input, but many natural phenomenon have different degrees of fluctuations in large scale energy injection. One of the earliest recognitions of the importance of fluctuations in the energy dissipation rate in turbulence can be found in a footnote by Landau in the textbook on fluid mechanics published with Lifschitz in 1944 [4]. The footnote explains that universal formulas for the small scales of structure functions do not exist because the energy dissipation rate will fluctuate on long time scales, and these fluctuations will be different in different flows. Frisch [5] provides an extended discussion of the footnote. In the refined similarity theory by Kolmogorov [6] and Obukhov [7] in 1962, this insight on universality is extended to include fluctuations that result
from the random character of the transfer of energy between scales, which is often
the importance of internal intermittency. However, this credit seems to be some-
what misplaced since the available published text by Landau observes only that
large scale fluctuations in the energy dissipation will destroy universality of small
scales [5, 8]. During the intensive effort to understand internal intermittency over
the past 50 years, the direct application of Landau’s insight about the importance
of large scale fluctuations has often been obscured.

The refined similarity theory by Obukhov [7] and Kolmogorov [6] proposed that
in the inertial range the moments of velocity differences between two point \( \Delta_r u \)
are universal functions when they are conditioned on the locally averaged value
of the energy dissipation rate, \( \varepsilon_r \), defined as the instantaneous energy dissipation
rate averaged over a sphere of radius \( r \). The conditional moments are

\[
\langle (\Delta_r u)^p | \varepsilon_r \rangle = C_p (\varepsilon_r r)^{p/3},
\]

where \( C_p \) are universal constants [1]. Averaging this expression over a distribution
of \( \varepsilon_r \) yields

\[
\langle (\Delta_r u)^p \rangle = C_p \langle \varepsilon_r^{p/3} \rangle r^{p/3} = C_p \langle \varepsilon_r^{p/3} \rangle \langle \varepsilon_r r^{p/3} \rangle,
\]

where \( \varepsilon = \langle \varepsilon_r \rangle \) is the mean energy dissipation rate. Since the moments of \( \varepsilon_r \)
depend on \( r \), this means that the inertial range scaling laws are modified by
internal intermittency. Kolmogorov proposed that the fluctuations of \( \varepsilon_r \) could be
described with a power law scaling

\[
\langle \varepsilon_r^p \rangle = C_{\varepsilon r} \left( \frac{L}{r} \right)^{\xi_p}.
\]
In Kolmogorov’s paper published in 1962, a log-normal model was used to relate $\xi_p$ for all $p$ to $\xi_2 = \mu$, which is commonly called the intermittency exponent. An extensive literature has explored the $r$ dependence of statistics of $\varepsilon_r$ in order to understand anomalous scaling exponents in the inertial range [9].

However, the effects of fluctuations in the energy dissipation rate due to the large scales has been given much less attention, even though this is the direct application of Landau’s original comment. Kolmogorov did state that the coefficients in the scaling laws should not be universal, presumably because he recognized that large scale fluctuations would not be universal [6]. Monin and Yaglom [10] provide a simple model at the beginning of their section titled ‘Refined Treatment of the Local Structure of Turbulence, taking into account fluctuations in the dissipation rate. (see also [11]). They consider averaging together equal numbers of samples from two different turbulent states: state 1 with energy dissipation rate $\varepsilon_1 = (1 + \gamma)\langle \varepsilon \rangle$ and another state 2 with $\varepsilon_2 = (1 - \gamma)\langle \varepsilon \rangle$. Here $\langle \varepsilon \rangle$ is the mean energy dissipation rate and $\gamma$ is a measure of the difference in energy dissipation between the two states. Then Eq.(1.4) implies that a measured second order structure function in the inertial range averaged over equal contributions from each state would be

$$D_2(r) = \langle (\Delta_r u)^2 \rangle = \frac{C_2}{2} \left[ (1 + \gamma)^{2/3} + (1 - \gamma)^{2/3} \right] (\varepsilon r)^{2/3}. \tag{1.6}$$

So the large scale fluctuations in the energy dissipation are predicted to change the coefficients of inertial range scaling laws without changing the power law scaling. In this model, $\gamma$ must be less than one, and the coefficient of the second order structure function can decrease to as low as $C_2/2^{1/3} \approx 0.794 C_2$ for the case $\gamma = 1$ where there is no energy injection in state 2. This model is easily extended to the
case where samples are included from state 1 with probability $\beta$ and from state 2 with probability $1-\beta$. Now the energy dissipation rates are $\varepsilon_1 = (1+(1-\beta)\gamma/\beta)\langle \varepsilon \rangle$ and $\varepsilon_2 = (1-\gamma)\langle \varepsilon \rangle$. For this extended model, the measured structure function of order $p$ would be
\[ D_2(r) = \kappa(\beta,\gamma) C_p (\langle \varepsilon \rangle r)^{p/3}. \] (1.7)

where the correction factor of the coefficient is
\[ \kappa(\beta,\gamma) = \left[ \beta \left( 1 + \frac{1-\beta}{\beta} \gamma \right)^{p/3} + (1-\beta)(1-\gamma)^{p/3} \right]. \] (1.8)

In the limiting case $\gamma = 1$ and $\beta \to 0$, the coefficient for $p = 2$ goes to zero, and the coefficients for $p > 3$ go to infinity, so the effects of large scale fluctuations on the small scale statistics can be very large. In this limiting case, the flow consists of brief pulses of large energy injection between long periods of no energy injection.

In both Monin and Yaglom’s [10] and Davidson’s [11] remark, the presentation of the model in Eq. (1.4) is followed with the observation that in typical situations this effect is not large. Fig. 1.2 shows contour plots of the correction factor as a function of the fluctuations in the energy input, $\gamma$, and the fraction of the time spent in the high energy input state, $\beta$. The observation that the correction is not large in most cases is apparent in the figure. For $p = 2$ the correction is less than 2.4% for half of the parameter space. However, the correction can be very large in some flows. There is always a divergence for $\gamma = 1$ and $\beta \to 0$, and for large $p$, the correction is larger. Although this two state model does not intend to describe real turbulent flows, we will show that it provides a reasonably good description of some of our data.
**Figure 1.2:** This graph illustrates the correction factor $\kappa$ from Eq. (1.8) for $p = 2$. It shows the change in Kolmogorov constant in different duty cycle and fluctuations in the energy input $\gamma$. The change is more significant when we have small duty cycle and large energy injection fluctuation.

In real flows, energy dissipation rate and $\varepsilon_r$ have continuous distributions. In the continuous case, Eq. (1.4) can be used to predict the behavior of structure functions, but there are now contributions to the distribution of $\varepsilon_r$ from both internal intermittency and fluctuations in the energy input. In particular, $\varepsilon_r$ for $r \geq L$, has a distribution, which is determined not by cascade processes, but by the mechanisms creating the turbulence. In cases where internal intermittency can be ignored, we can estimate the fluctuations in the energy input and use that to predict the coefficients of scaling laws. If it is possible to determine the time dependence of the turbulent kinetic energy, $u^2$, and the integral length scale, $L$, then the estimate $\varepsilon \propto u^3/L$ can be used to obtain:
\begin{equation}
\langle (\Delta_r u)^p \rangle = C_p \frac{\langle u^3 / L \rangle^{p/3} }{\langle u^3 / L \rangle^{p/3} } \langle \varepsilon r \rangle^{p/3}.
\end{equation}

In our flow, where $L$ has a weak dependence on the variations in the energy input, this simplifies to

\begin{equation}
\langle (\Delta_r u)^p \rangle = C'_p \frac{u^p}{\langle u^3 / L \rangle^{p/3} } \langle \varepsilon r \rangle^{p/3}.
\end{equation}

If internal intermittency is also important, then the two effects may be combined as

\begin{equation}
\langle (\Delta_r u)^p \rangle = C''_p \frac{\langle u^3 / L \rangle^{p/3} }{\langle u^3 / L \rangle^{p/3} } \left( \frac{L}{r} \right)^{\xi_p} \langle \varepsilon r \rangle^{p/3}.
\end{equation}

It is important to determine the size of the effects of fluctuations in the large scale energy input in real turbulent flows. Surprisingly, there is no published result that we know of that documents a dependence of coefficients of inertial range scaling laws for structure functions on systematic changes in the large scales of the flow. A compilation of experimental [9] and simulation [12] results have given credence to the notion that the second order coefficients are close enough to be independent of the flow that they can be treated as universal constants. At least two experimental studies have explored this issue in detail. Praskovsky et al [13] study two high Reynolds number flows, a mixing layer and a return channel. They find a conditional dependence of the second order structure functions on the instantaneous velocity and connect this with spatial and temporal variability of the energy flux passing through the cascade. They emphasize that the conditional dependence they observe is not in violation of the assumptions of the refined Kolmogorov theory [see Eq. (1.3)] since the local instantaneous energy flux should change in the small scales. More recently, Mouri et al [8] explored the effects of large scale fluctuations of the turbulence energy dissipation rate. They measure
grid and boundary layer turbulence and clearly confirm that the large scale en-
ergy fluctuations exist and that they affect small-scale statistics. They explicitly
state the the large scale fluctuations do not affect the power law scaling or the
coefficients of second order structure functions in the inertial range.

There is another set of literature exploring time dependent energy input in turbu-
ulence that has identified the presence of response maxima when the energy input
oscillates with a period on the order of the large eddy turn-over time. This effect
was first predicted in a mean field theory [14]. It has been explored in a variety
of models, numerical simulations and experiments [14–19]. However, this work
has focused on modulation periods near the turn-over time and seems not to have
considered the effects on structure functions, which are most prominent for long
modulation periods.

In this thesis we present a series of experimental measurements of the effects
of time-dependent energy input on the small scales of turbulence. In Chapter
2, we will give a brief explanation of the experimental set-up and procedures.
The endeavor on upgrading the real-time image compression circuit with nearest
neighbor algorithm is also discussed in this chapter. In Chapter 3, we focus
on second order structure functions where the effects of internal intermittency
are small to discuss the interaction between large scales and small scales. An
extended analysis of the conditional structure function is also included. We find
that the coefficients of inertial range scaling laws depend on the fluctuations in the
large scale energy input, and measure second order coefficients of inertial range
scaling laws that are more than 20% below the value for the continuously driven
case. We can use the refined model to quantify this change in the coefficients.
This work also provides a theoretical foundation for the analysis of conditional
structure functions in Blum’s Ph.D. thesis. A concluding remark of this work
will be in Chapter 4. This thesis is based on the experimental data collected by Dan Blum, and there are several figures and analysis from his Ph.D. thesis [20]. The introduction on literature review and result section has also been edited and polished by Prof. Voth for future publication.
Chapter 2

Experiment

This work is based on the data collected by optically tracking tracer particles in a turbulent flow. The 3D positions are determined by four cameras as shown in Fig. 2.1. We track each particle by comparing adjacent frames and identifying a particle by its possible trajectory as it moves through the frame. The velocity of a particle can be determined by calculating the distance traveled over time after it has been identified over several frames. We incorporate the real-time image compression circuit with compression factors of 100 to 1000, which enables us to acquire data continuously and endlessly without running out of storage memory [21].

My main contribution is on upgrading the real-time image compression system. Therefore, this chapter will briefly describe the existing set-up of the experiment, and will give a more detailed description on the real-time image compression system and the nearest neighbor algorithm.
Figure 2.1: Experimental apparatus diagram. Two oscillating grids were held 56.2 cm apart in an 1100 l octagonal prism Plexiglas tank. Four high speed cameras were used to stereoscopically image an illuminated volume in order to record 3D particle positions. Illumination was provided by a Nd:YAG laser with 50 W average power. This figure is obtained from Dan Blum’s Ph.D. thesis [20].

2.1 Experimental Apparatus

The tank that we use to generate turbulence is an octagonal Plexiglas tank that is 1 x 1 x 1.5 m$^3$ filled with approximately 1100 l of filtered and degassed water. Two identical octagonal grids oscillating in phase generate the turbulence. The grids have 8 cm mesh size, 36% solidity, a 56.2 cm spacing between grids and a 1 cm gap between the grids and the tank walls. The grid oscillation has 12 cm amplitude and is powered by an 11kW motor. The grid frequency could be raised up to 5 Hz to increase the Reynolds number to up to 271 [22].

The data is acquired using 3D particle tracking measurements. 4 cameras are used in the experiments: 2 Bassler A504K video cameras capable of 1280 x 1024 pixel resolution at 480 frame per second, and 2 Mikrotron MC1362 cameras with the same pixel resolution and data rates, but with greater sensitivity. This is
approximately a data rate of 625 Mbyte per second per camera.

It is difficult to record such high data rates with 4 cameras because of the amount of data that need to be processed and stored. Our research group, therefore, has developed a real-time image compression circuit to threshold images in real time so that only pixels above a brightness threshold are stored, while the dark background pixels are discarded [21]. However, the real-time image compression circuit has the problem of decreasing the accuracy with which the center of the image can be located. This is because when the particles do not evenly reflect the laser light due to its reflection angle or scattering, dimmer pixels of the particles will be filtered out. When we need to take data for rods [23] or thin fibers, any missing pixels might result in breaking the images into two parts and might be mistaken as two different particles when it is actually one. We have attempted to improve the current version of real-time image compression circuit with a nearest neighbor algorithm. We will discuss this algorithm in section 2.3.

Images were processed by a stereo matching method to find the 3D position of particles in space [24]. We use known stereo matched pairs from the cameras and run a nonlinear optimization to minimize the stereo matching error and find the optimal camera position parameters, thus increasing the accuracy of particle location. [22].

2.2 Real-time Image Compression Circuit

Real-time image compression circuits have two important features. The first feature is that it can process images in real time, meaning that we can process the images as the cameras are still running and taking new images. The second fea-
ture is that the images we store are compressed. We only store the pixels with brightness value above the threshold so that we can save a lot of memory space. Since our tracer particles are small, the bright pixels only account for less than 1% of the total image; therefore, in a typical experiment the compression ratio is around 100 to 1000 [21].

The input clock cycle data from the camera is 67.58 MHz. In each clock cycle, the circuit receives ten pixels of 8-bit brightness data in parallel. The circuit also receives from the cameras a frame valid bit indicating frame breaks, a line valid bit indicating line breaks, and other camera configuration and trigger signals. The pixels are received line by line in a left-to-right, top-to-bottom manner. The maximum resolution of a frame is 1280 x 1024 pixels.

We use an Altera Cyclone FPGA, Field-Programmable Gate Array, as our programmable circuit for image processor. We use Quartus II, a software provided by Altera, to program the FPGA to design a circuit layout that will filter out the pixels below a certain threshold value and output the bright pixels with its brightness and position information saved as vectors [21]. This FPGA is capable of working above 200MHz, so it has no problem handling the camera signal at 67.58 MHz. Fig. 2.2 shows several basic components of this image processor within the FPGA circuit.

The basic function of this processor is that we take the images from the four cameras, compare the brightness value of each pixel [Fig. 2.2(b)] with a preset threshold value [Fig. 2.2(c)] and store the pixel position and brightness if brightness is above threshold. We set the threshold as 30 (256 is the brightest value, and 0 is complete darkness) for data collection. The position counter [Fig. 2.2(a)] will store the position information as the pixels come in. The buffer [Fig. 2.2(d)] which consists of ten FIFOs is used to store the brightness and position value of
the bright pixels with 25-bit vector data. The first 8 bits are used to store the brightness value, which ranges from 0 to 256. The next 7 bits are for the x position. Because we have 10 FIFOs to store a total of 1280 pixels for the x position, each FIFO only needs to store 128 pixels. Therefore 7 bits will be sufficient to represent the x position. We have the remaining ten bits used for storing y position to distinguish the 1024 lines of each frame. The output controller [Fig. 2.2(e)] recreates a new camera frame containing only the data that is above the threshold. The output multiplexer [Fig. 2.2(f)] passes the data from the FIFOs to the output bus. It takes in the data from the ten FIFOs and does the final calculation.
of the values of brightness and positions. The information comes out as a 29-bit array with 8 bits for the brightness, 11 bits for the x position and 10 bits for the y position. The vector data now needs 11 bits to represent the 1280 pixels in the x direction so 11 bits are allocated for storing the x position. The data is saved in a custom video file (.cpv files) and our research group has designed a CPV reader to process the compressed data.

2.3 Nearest Neighbor Algorithm

In order to increase the accuracy of defining the position of the particle, we devise a nearest neighbor algorithm for our real time image compression circuit that will store not only the pixel which has the brightness value above threshold, but also its four neighboring pixels as shown in Fig. 2.3. The idea is simply to store four times as many pixels as had been previously stored. But the real-time property of our image compression circuit complicates the implementation of this algorithm. The advantage of the real-time image compression circuit is that it allows us to process data in real time, and store only the bright pixels and discard the dark pixels to make more storage memory available for more data. But the nearest neighbor algorithm requires the information of the neighboring pixels which could have been discarded during the filtering. Therefore, some buffering of pixel data is necessary preceding the filtering in order to implement our nearest neighbor algorithm.

There are several advantages of this algorithm in improving the accuracy of positioning our tracer particles. Firstly, with the nearest neighbor pixels being captured, we would be able to obtain a better image of our tracer particles. The algorithm enables us to include the slightly dimmer pixels in the edges of the
particle which would otherwise be neglected. Secondly, it can reduce the chances of falsely filtering out some of the dim pixels which are actually part of the particles. Thirdly, we can increase the threshold value to avoid including noise signals such as water bubbles or other dust particles. By increasing the threshold, it also reduces the possibilities of data overflows in our system. Currently we can only store 5000 bright pixels due to our usage of the FIFOs in the set-up, and by raising the threshold we can reduce this problem.

**Figure 2.3:** (a) A diagram of brightness data of 4 pixels x 3 pixels frame before and after going through the real-time image compression circuit (b) A diagram of the same data set before and after going through the real-time image compression circuit with nearest neighbor algorithm.
2.3.1 Algorithm

The fundamental idea of this nearest neighbor algorithm is that when we find a bright pixel, we not only store the pixel itself (center pixel) but also all its four nearest neighboring pixels (top, bottom, left and right pixels). Fig. 2.4 shows the relative positions of the pixels and the order which they come into the circuit. When processing the data coming in from the camera in real time, we need to store at least its top and left pixels, which come in earlier than center pixels, before they are discarded.

![Diagram showing pixel positions]

**Figure 2.4:** A diagram showing the relative positions of the top, left, center, right, and bottom pixels, and the order of them coming into the circuit.

The straightforward way to implement this nearest neighbor algorithm is to store three lines of pixels and then compare them together at the same time. Top pixel will be the first line; center, left and right pixel will be the second line; bottom pixel will be the third. We compare the brightness values of all these five pixels to the threshold. As long as the brightness values of any of these five pixels are above the threshold, we then save the center pixel. We can scan through the whole frame and store all the bright pixels and their nearest neighbors correctly.
2.3.2 Code Implementation and Architecture

In order to keep the nearest neighbors of the bright pixels, we create a sequence of buffers as pre-buffers that take input from the cameras and their output goes to the original compression circuit [21]. Fig. 2.5 shows the architecture of the implementation of the real-time image compression circuit of the nearest neighbor algorithm. We have to prepare five groups of ten FIFOs as pre-buffers for temporary storage.

![Diagram of circuit architecture](image)

**Figure 2.5:** This is a conceptual diagram of the circuit architecture programmed in FPGA including the pre-buffers for nearest neighbor algorithm.

Ideally, we could only store two lines of data, and then compare them at the same time as the third line coming in. However, the time delay of the data after being
written in and read out of FIFOs does not synchronize with the data coming in directly from the camera. The time delay comes from the time it takes for electric current flowing through specific parts of the circuit. Therefore we have to channel the bottom pixel data through FIFOs to ensure the same time delay in order to synchronize all three lines of data to read it out simultaneously for filtering.

It is of most importance to read out all the five pixels from FIFOs at the correct timing. We need to program to temporarily store the top pixel data for 2048 clock cycles (two lines) until the bottom pixels being read in. We also have to store the left, right, and center pixel for 1024 clock cycles (one line). We also have to consider the boundary issues when we implement this algorithm such as the first line of a frame will not have any top pixels, and the last line of the frame will not have any bottom pixels. Besides, the first pixel of the line will not have any left pixel, and similarly the last pixel of the line will have no right pixel.

For the top FIFO group [Fig. 2.5(j)], we start writing in from the first line of the frame, and read out when we start writing in the third line. For the center FIFO group [Fig. 2.5(g)], we start writing in from the first line, and read out when we start writing in the second line. For the bottom FIFO group [Fig. 2.5(k)], we start writing in from the second line, and read out immediately after it is written in. The purpose of writing in and reading out the bottom FIFO is to create the same time delay as the top and center for synchronization. For the top, center and bottom FIFO groups, we read in pixel 1 into FIFO 1, pixel 2 into FIFO 2 until pixel 10 into FIFO 10.

For the left FIFO group [Fig. 2.5(h)], we start writing in from the first line, and read out when we start writing in the second line. We will write pixel 1 into FIFO 2, pixel 2 into FIFO 3, until pixel 9 into FIFO 10 and pixel 10 into FIFO 1. This set-up enables the circuit to automatically configure the left pixels in the correct
line-up relative to the center pixels for comparison. We need to be careful to insert a zero into FIFO 1 in the beginning of each line because the first pixel of each line has no left pixel. For the right FIFO group [Fig. 2.5(i)], we start writing in from the first line, and read out when we start writing in the second line as we did for left FIFO group. We write pixel 1 into FIFO 10, pixel 2 into FIFO 1, until pixel 9 into FIFO 8 and pixel 10 into FIFO 9. This set-up will automatically read out the right pixel of the center pixel. We need to insert a zero into FIFO 10 in the end of the line because the last pixel of the line has no right pixel.

The real-time image compression circuit takes actions according the clock, line valid, and frame valid signal generated by the camera. So we also have to control the line valid and frame valid signals to delay them for the time of processing two lines, so that the position counter [Fig. 2.5(a)] will only start counting when the data start going through the filter for comparison.

After the pre-buffer, the five pixels will come out synchronized and ready for comparison in the filter [Fig. 2.5(d)]. We make a minor adjustment on the original filter code so that as long as any of the five pixels is above the threshold, we will record the information of the center pixel. Until this step, we have controlled the signal coming out of the filter being equivalent to the original real time image compression circuit, with the only difference being the pixels with brightness value lower than the threshold will be stored as well if their nearest neighbor is a bright pixel. Then the information will be passed to the FIFO buffer in preparation for extracting the information on brightness and x, y position for output as the original real-time image compression circuit works. [21]

With this set-up, we are able to run a Quartus II built-in simulation to generate a simulation file. The simulation adjusts for the time delay based on the circuit configuration, and it is a reliable testing ground for whether the implementation
is correct considering the effect of potential time delay. We manually create an image input in the simulation to test whether this architecture correctly implement our nearest neighbor algorithm. Our simulation file successfully producing the expected result of nearest neighbor algorithm is a positive indication that the implementation of the nearest neighbor algorithm is on the right track. We can then download the code into our FPGA and read the CPV file to verify real image data.

2.3.3 Challenges and Problems

One would expect that the actual pixels being saved will be the same as the simulations, since the main function of the simulations is that it will help us diagnosed the problem before we downloaded the program into the circuit. However, the images captured by our nearest neighbor algorithm do not produce the same result as the simulation has shown. The successful downloading rate also become a lot lower, and the stored neighboring pixels are randomly scattered. The unsuccessful download means that the FPGA circuit does not configure according to our architecture, so we cannot receive and store the information about images properly. Much effort has been put in to figure out the discrepancy between the simulation and the actual outcome. A major problem comes from the time delay when pixel data goes through FIFOs or flip-flops. These temporary storages are intended for controlling the five pixel groups to have pixel data coming out simultaneously for comparison in real time. However, there was unexpected time delay that was not shown in the simulation making our pixel data come out at different times. The camera clock oscillates fast at 67.58 MHz, therefore a very small unexpected time delay could mess up the implementation of the algorithm. We tried to clean up our architecture to decrease the possible time delay, but it
Chapter 2 - Experiment

does not show significant improvement.

The approach we took to fix the problem is to re-design the architecture from its simplest form with only the center FIFO group. We tested if this architecture produces the correct outcome. Afterwards, we added the left FIFO group to test whether all the bright pixels have their left pixels recorded. Not until the circuit produces the correct outcome did we include the right, top or bottom FIFO groups. The test runs showed that the left and right pixels are actually two clock cycles behind the center pixel. We were able to solve this problem by decreasing the delay for two more clock cycles, and we would have the bright pixels recorded correctly with their left and right neighbors.

As for the top and bottom pixels, at this moment we are yet to figure out the reason leading to the discrepancy between the simulation and the actual outcome. There seems not to be a pattern of where top and bottom pixels are being stored. Besides, when we added in the top and bottom FIFO groups, the successful rate of downloading the program into the FPGA circuit dropped by more than 50%. We have figured out that it is not a problem with insufficient memory of the FPGA since we only use up around 4% of the total usable space. After many systematic tests, we reached a conclusion that there was something about the time delay when we tried to delay for one or more than one lines that we did not understand. We have to gain a deeper knowledge on how the signals being transmitted between the camera and circuit, and the time delay inside the circuit in order to understand how to proceed with this problem. Currently we are able to store the left and right neighbors successfully. Nevertheless, unless both the top and the bottom pixels are stored correctly as well, only the left and right neighbors being stored does not increase the accuracy in positioning the tracer particle. Including only the left and right neighbors will produce an asymmetry.
in the 3D space and therefore is not beneficial.

The original threshold compression algorithm for our tracer particle finding degrades the position accuracy by 0.1 pixel [20]. Therefore, the lack of a functioning nearest neighbor algorithm does not invalidate the result of this work. However, when the experiments require a better resolution and more accurate position for particles of different shapes, such as long rods, thin fibers, or jacks, etc., the benefit of nearest neighbor algorithm is likely to be greater.
Chapter 3

The Effects of Fluctuating Energy Input on Small Scales

3.1 Data

Previous work sheds light on that structure functions are affected by the large scale energy fluctuations [22]. To isolate the role that large scale energy fluctuation plays in small scale, we augment its effect by modulating the driving frequency of the oscillating grids. We modulate the driving frequency of the motors to enhance fluctuations of large scale energy input. There are many ways to introduce large scale energy fluctuations in the turbulence. For example, rather than driving the grids continuously at 3Hz, we can drive it at 3Hz for 15 s, and then halt for 15 s, and repeat. This produces a periodic time dependence in the energy input with a longer time scale than the grid oscillation period. Fig. 3.1 shows a schematic of the frequency modulation with variable definitions. There are several variables that we control to augment the large scale energy fluctuations in a controlled way.
In this paper, we explore three different ways to augment the large scale energy fluctuations: (1) change $T$, the time to complete one modulation cycle (2) change the frequency modulation by holding $f_{\text{high}}$ constant and changing $f_{\text{low}}$ from 0 up to $f_{\text{high}}$, and (3) changing the duty cycle $t_{\text{high}}/T$.

Figure 3.1: The sketch illustrates the position and frequency of the grids over time. $t_{\text{high}}$ is the time when the motor is at relatively high frequency, $t_{\text{low}}$ is the time when at relatively low frequency. $T$ is the cycle period, the time to complete one modulation cycle. $f_{\text{high}}$ is the high frequency of motor and $f_{\text{low}}$ is the low frequency. $\Delta f$ is the frequency differences between $f_{\text{high}}$ and $f_{\text{low}}$ of the motor. Note that when the motor driving frequency being changed, it requires a short period of time to reach the desired frequency.

Fig. 3.2 is a specific example when $T = 24$ s, and $(f_{\text{high}} - f_{\text{low}}) = (3,0)$ Hz and 50% duty cycle. It gives us a better idea of the relationship of the grid position, frequency and energy over time. It takes time for energy to propagate from the grid to the detection volume, so the energy lags several seconds after the grid frequency changes. The inertial of the system used to drive the grids limits the rate at which the driving frequency could be changed. We were able to reduce the time required to stop or start to less than 1/3 of a second by minimizing the inertia in the experiment. The original version of this apparatus [22] used a flywheel to improve symmetry between the up and down stroke of the oscillating grids. For this experiment we replaced the flywheel with a coupler. For the run with period $T = 24$ s shown in Fig. 3.2, the start time is less than one oscillation
and accounts for less than 3% of the data. However, limitations from the inertia of the drive system did limit our experiments to periods of $T = 3\, \text{s}$ and greater which resulted in the period of the modulation of the energy input always being longer than the large scale turn over time, $\tau$ in Table 3.1.

![Diagram of grid position, frequency, and energy over time](image)

**Figure 3.2:** The sketch illustrates a specific case of $f_{\text{high}} = 3\, \text{Hz}$, $f_{\text{low}} = 0\, \text{Hz}$, period $T = 24\, \text{sec}$, and with 50% duty cycle. The three rows are position, frequency, and energy of the grid over time. The energy is calculated in the unit (mm/s)$^2$. The energy is the time averaged data. Both the first and second cycle are averaged statistics over the whole experiments and hence identical.

We conducted three specific sets of experiments to explore the effects of fluctuations of large scale energy injections on small scales. In the first set of experiments we changed the period $T$ from 3, 6, 12, 24, 48, to 384 seconds while always modulating the grid frequency with $(f_{\text{high}} - f_{\text{low}}) = (3 - 0)\, \text{Hz}$, with a duty cycle of 50%. We will refer to these experiments as “varying the period”. In the second set of experiments, we hold $f_{\text{high}} = 3\, \text{Hz}$ and change $f_{\text{low}}$ from 3, 2, 1, to 0 Hz to get $(f_{\text{high}} - f_{\text{low}}) = (3 - 3), (3 - 2), (3 - 1), (3 - 0) \, \text{Hz}$ with $T = 30\, \text{s}$ period and 50% duty cycle. We will refer to these experiments as “varying the frequency”. In the third set of experiments we change the duty cycle from 0%, 25%, 50%, 75% to
100% while always modulating the grid frequency with \((f_{\text{high}} - f_{\text{low}}) = (3 - 0)\text{Hz}\)
and a period of \(T = 30\) s. We will refer to these experiments as “varying the duty
cycle”. In the end, we continuously drive our grids at frequency ranging from 1Hz
to 4Hz to vary Reynolds number as our control group to show that the effects we
observe cannot be simply attributed to the changes in Reynolds number. Settings
and measured parameters for each of the experiments are given in Table 3.1.

In Fig. 3.3a we show the time dependence of the fluctuating energy measured
by phase averaging over many cycles. The energy, \(\langle u_i u_i \rangle\), is shown for the set
of experiments varying the period. Time zero is defined as the time when the
energy input halts. For all the different experiments, the energy dissipates at
approximately the same rate, so the decay curves nearly collapse. After half a
period, the energy input resumes; so for the \(T = 24\) s data we see the energy rise
rapidly around 14 s since the grid was turned back on at 12 s. For the longer
period experiments, the energy has only decayed to 10% of its initial value after
25 s. In Fig. 3.3b, this data is shown with time normalized by the period. One
additional data set with \(T = 384\) s is added in this plot. Only for this data set
with a very long period does the fluid become approximately quiescent before the
energy input is resumed.

\section*{3.2 Results}

\subsection*{3.2.1 Coefficients of Inertial Range Scaling Laws}

First we look at the coefficients of scaling laws of different sets of experiments.
This coefficient is commonly regarded as the Kolmogorov constant. The change of
the coefficients of scaling laws calculation from the second order structure function
Chapter 3 - The Effects of Fluctuating Energy Input on Small Scales

Figure 3.3: (a) Time dependence of the fluctuating energy measured by phase averaging over many cycles for experiments with varying period. The motor is halted at time $t = 0$, and turned on when $t$ reaches half a cycle period $T/2$. Symbols represent the cycle period, $T$ of $+= 3$ s, $\circ = 6$ s, $\ast = 12$ s, $\times = 24$ s, $\square = 48$ s. The symbols $\circ$ and $+$ are only plotted every four data points, and other symbols show the full data set. (b) The fluctuating energy versus $t/T$ with additional data set $T = 384$ s ($\diamond$). The motor is halted at $t/T = 0$, and turn on at $t/T = 1/2$. All symbols are only plotted every other data points. Part b of this figure is edited from Dan Blum’s Ph.D. thesis. [20]
shows as large as a 20% decrease. From the refined model in Eq. (1.9), we can also obtain a prediction for the change of the coefficients of scaling laws.

Varying Period

Fig. 3.4 shows the third order structure functions from the experiments varying the period. The energy dissipation rate is determined from this data and the 4/5 law. When compensated by $\varepsilon r$ (see Eq. (1.2)), the inertial and dissipation ranges of these third order structure functions collapse fairly well, suggesting that the small scales of these turbulent flows are similar.

However, the compensated second order structure functions shown in Fig. 3.5a do not collapse well at all. The maximum of these compensated structure functions,

![Graph](image)

**Figure 3.4:** Third order compensate structure functions for the experiments with varying period. Symbols represent the cycle period, $T$ of $\oplus =$ 3 s, $\circ =$ 6 s, $\ast =$ 12 s, $\times =$ 24 s, $\square =$ 48 s, $\Diamond =$ 384 s. Driving frequency modulations is $(f_{\text{high}} - f_{\text{low}}) =$ (3 - 0)Hz and the duty cycle is 50%.
which is an estimate of the coefficient in the inertial range scaling laws, shows a 20% decrease as the period increases. Increasing the fluctuations in the energy input does have a significant effect on the small scales of the flow. The shape of the second order structure functions shows little change consistent with the idea that fluctuations in the energy injection at large scales primarily changes the coefficients in scaling laws while leaving the scaling exponents unchanged.
Fig. 3.5b shows the second order structure functions scaled by the prediction of Eq. (1.9). The good collapse of these curves after scaling indicates the effects of fluctuations in the energy input are largely captured by the refined model.

Fig. 3.6 shows the measured coefficient of the inertial range scaling of the second order structure functions, commonly labelled $C_2$. The decrease in the 'constant' $C_2$ as the period increases is obvious indicating that the previous assessment [8,13] that large scale fluctuations do not affect second order structure functions is wrong. Fig.3.6 also shows the prediction of our refined model from Eq. (1.9) with the model value of $C_2 = 2.0$. The experimental measurement and the refined model are in fairly good agreement. There are many possible factors that contribute to the difference between the measurements and the model, including the difficulty in measuring scaling coefficients at modest Reynolds number and limi-

![Figure 3.6: Experimental measurements of Kolmogorov constant $C_2$ (●), and the $C_2$ predicted by the refined model (×) for the experiments of varying period. The dotted line represents the prediction of Monin and Yaglom model.](image)
tations of the estimate $\varepsilon = u^3/L$. The dotted line is the prediction of the model by Monin and Yaglom. Our experimental value of $C_2$ approaches this dotted line when the period is long as it should since in that case we are approaching the situation they consider where the energy passing down the cascade goes to zero in the off state.

Measuring scaling coefficients from this data at modest Reynolds numbers has some difficulties. From the third order structure functions we extracted the energy dissipation rate by averaging the three bins at the maxima between $r/\eta = 15$ and 68. For the second order structure functions, we used this same definition of the inertial range even though the peak of the second order compensated structure functions are at slightly larger $r$. This results in measured second order scaling coefficients that are well below the peak value. We tried using a different inertial range for the second order data. This makes small changes in the magnitude of the scaling coefficients, but has no effect on the conclusions we draw. Data at larger Reynolds numbers will be necessary to provide more precise quantitative measurements of how scaling coefficients depend on fluctuations in the energy input.

**Varying Amplitude**

Similar effect of the large scale energy fluctuations on small scales are also seen in the data sets with different amplitude of the fluctuations in the energy input. Fig. 3.7a shows the second order structure function compensated by $\varepsilon r$ for the data sets where amplitude of the energy input is varied by changing the grid oscillation frequency. Similar to the experiments varying the period, the curves do not collapse, indicating that the the coefficients of the scaling law depend on
the large scales. Fig. 3.7b shows the second order structure functions scaled by the prediction of Eq. (1.9). The better collapse of these curves after scaling again indicates that the refined model is accurately describing the effects of fluctuating energy input.

Fig. 3.8 shows the measured Kolmogorov constants $C_2$ along with predictions from the refined model and Monin and Yaglom model. The main point is that increasing

![Graph showing the relationship between $\frac{\langle \Delta u^2 \rangle}{(\varepsilon r)^{2/3}}$ and $\frac{\langle \Delta u^2 \rangle \langle \varepsilon^{23} \rangle}{(\varepsilon r)^{2/3} \langle \varepsilon \rangle^{2/3}}$ vs. $r/\eta$.](image)

**Figure 3.7:** (a) Second order compensated structure function for the experiments with varying amplitude. (b) Second order structure functions scaled by the ratio of moments of the energy dissipation rate predicted by the refined model. Symbols represent different frequency modulations of $(f_{\text{high}} - f_{\text{low}}) \times = (3 - 3) \text{ Hz}, \ast = (3 - 2) \text{ Hz}, \square = (3 - 1) \text{ Hz}, \circ = (3 - 0) \text{ Hz}$. Cycle period $T$ is 30 s, and the duty cycle is 50%.
the amplitude of the fluctuations in the energy input systematically decreases the constant as predicted. Quantitatively, the refined model has coefficients larger than those measured meaning that it underestimates the effect of the large scale fluctuations. This deviation is likely because the refined model uses the time dependence of the rms velocity to estimate the fluctuations in the energy input and this does not capture all of the fluctuations in the energy input. Monin and Yaglom’s model works well for small amplitude of the energy input fluctuations, but for the largest fluctuation amplitude (3-0)Hz, it predicts a much larger effect of the large scale fluctuations than are observed experimentally. This is expected since this data is for period $T = 30s$ and at large amplitude of the energy input fluctuations there is not enough time for the energy to decay to the constant values assumed by the Monin and Yaglom model.

![Figure 3.8: Experimental measurements of Kolmogorov constant $C_2$ (●), the $C_2$ predicted by the refined model (×), and the prediction of Monin and Yaglom model (□) for the experiments of varying amplitude. The predictions of refined model and Monin and Yaglom model assume $C_2 = 2$ when $(f_{\text{high}} - f_{\text{low}}) = (3 - 3) \text{ Hz}$]
For the experiments varying the amplitude of the fluctuations in the energy input, we did not directly measure the phase averaged fluctuating velocity needed in the refined model. To make predictions with this model, we had to model the fluctuation velocity using the known values for continuous driving at different frequencies and the decay rate data in Fig 3.3.

**Varying Duty Cycle**

The set of experiments varying the duty cycle in Fig. 3.9 also shows that the compensated second order structure functions show strong dependence on fluctuations in the energy input. We show the measured Kolmogorov constants in Fig. 3.10. When the duty cycle is smaller, we observe a smaller Kolmogorov constant. For 25% duty cycle we see the smallest value of the Kolmogorov constants of any data

![Figure 3.9: Second order compensated structure function for the experiments with varying duty cycle. Each curve shows the driving frequency modulations \((f_{\text{high}} - f_{\text{low}}) = (3 - 0)\text{Hz}\), with period of 30 seconds, and duty cycle of \(+ = 100\%\), ○ = 75\%, × = 50\%, * = 25\%.](image)
set with $C_2 = 1.58$. Note that 25% duty cycle and 75% duty cycle do not have the same constants. Because times with large energy input dominate the moments of the energy dissipation rate, the effects on the Kolmogorov constant are largest for low duty cycle where bursts of large energy injection are followed by a long quiescent period. The predictions of the Monin and Yaglom model shown in Fig. 3.10 are consistently below the measured constants. We expect that if the experiments were performed for larger period rather than $T = 30$ s they would approach the Monin and Yaglom predictions.

Figure 3.10: Experimental measurements of Kolmogorov constant $C_2$ (●), and the prediction of Monin and Yaglom model (□) for the experiments of varying duty cycle. Each curve shows the driving frequency modulations $(f_{\text{high}} - f_{\text{low}}) = (3 - 0)$Hz, with period of 30 seconds, and duty cycle of $\circ = 100\%$, $\bullet = 75\%$, $\times = 50\%$, $\ast = 25\%$. 
Varying Reynolds Number

The set of experiments varying Reynolds number for constant energy input in Fig. 3.11 shows that the Kolmogorov constants for the second order structure functions do no show strong dependence on Reynolds number. We vary Reynolds number from $Re_\lambda = 139$ at 1Hz continuous driving to $Re_\lambda = 271$ at 4Hz continuous driving. The shape of the structure function changes at the lowest Reynolds number as expected, but after using the third order structure functions to determine the energy dissipation rate, the peak value remains relatively constant. This confirms that the variation we observe in the Kolmogorov constant is not simply the result of different effective Reynolds numbers in different experiments.

Figure 3.11: Second order compensated structure function for the experiments with varying Reynolds number. Symbols represent different Reynolds number, $Re_\lambda$ of $\circ = 262$, $\ast = 271$, $\times = 250$, $\blacksquare = 237$, $\triangledown = 163$. 
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<td>N/A</td>
<td>N/A</td>
<td>100%</td>
<td>7.14</td>
<td>6.87</td>
<td>0.96</td>
<td>52.9</td>
<td>271</td>
</tr>
</tbody>
</table>

**Table 3.1:** Experimental parameters and resulting statistics for different sets of experiments. Note that the case of $f_{\text{high}} = 3$ Hz, $f_{\text{low}} = 3$ Hz, Duty Cycle 50% is the same as the case of $f_{\text{high}} = 3$ Hz, $f_{\text{low}} = 0$ Hz, Duty Cycle 100%.
3.2.2 Conditional Structure Functions

Previous work has used conditional structure functions to quantify the effects of the large scales on small scales in turbulent flows [13, 22, 25]. Velocity differences between two points separated by \( r \) are dominated by structures near scale \( r \) while velocity sums of two points are dominated by the large scales in the flow, so moments of velocity differences conditioned on the sums provides a convenient way to observe the effects of the large scales. We find that conditional structure functions provide a more sensitive measurement of the existence of fluctuations in the large scale energy input than the coefficients of inertial range structure functions. However, theoretical tools to predict the effects of large scale fluctuations on conditional structure functions are not yet available. In this section we present measured conditional structure functions as we systematically change the fluctuations in the energy input.

Varying Amplitude

Fig. 3.12 shows conditional structure functions for the data sets varying the amplitude of the fluctuations in the energy input. We condition the structure function on the velocity component that is transverse to \( r \) denoted \( \Sigma u_\perp \). In order to compare the conditional structure function for different length scales, we normalized the vertical axis by the unconditioned structure function. The horizontal axis is normalized by the characteristic velocity \( u = (\langle u_i u_i/3 \rangle)^{1/2} \). In Fig. 3.12a for constant driving of the oscillating grids, we see the results published in Ref. [25] that the conditional structure functions for all length scales are very similar. There is a slight dependence on length scale with the smallest length scales showing the strongest dependence on the large scale velocity. This result remains unexplained.
Figure 3.12: The velocity dependence of conditional second order conditional structure functions for the experiments with varying amplitude. The frequencies modulated were \((f_{\text{high}} - f_{\text{low}})\) = (a) \((3 - 3)\) Hz (b) \((3 - 2)\) Hz, (c) \((3 - 1)\) Hz, (d) \((3 - 0)\) Hz. Each curve represents the following separation distances \(r/\eta\): \(+ = 2.67\) to \(5.33\), \(\circ = 5.33\) to \(10.67\), \(* = 10.67\) to \(21.33\), \(\times = 21.33\) to \(42.67\), \(\Box = 42.67\) to \(85.33\), \(\Diamond = 85.33\) to \(170.67\), \(\triangle = 170.67\) to \(341.33\), \(\triangledown = 341.33\) to \(682.67\). This figure is obtained from Dan Blum’s Ph.D. thesis [20].
since it is the opposite of what is expected that the small scales are approaching universality. The same effect is seen in DNS data in Ref. [25]. However, in this paper we are focusing on fluctuations of the energy input and we will see that these produce much bigger effects than the small differences for different length scales.

Fig. 3.12b-d show that increasing the fluctuations in the energy input produces a large increase in the dependence of the conditional structure functions on the large scale velocity. In each sub-figure, the curves for different length scales remain very similar which demonstrates the fact observed earlier that fluctuating energy input does not change the length scale dependence. It primarily changes a pre-factor scaling the entire structure function. Note that Fig. 3.12a still has dependence on the large scale velocity even though the oscillating grid is driven at a constant 3 Hz frequency. We interpret this as fluctuations in the energy input that remain even in the case of constant driving [25]. To more directly compare the effects of changing the energy input fluctuations, we extract curve for \( r/\eta = 10.7 \) to 21.3 from Fig. 3.12(a, b, c and d) and plot them on one graph as shown in Fig. 3.13a. In Fig. 3.12 and Fig. 3.13 the symmetry around the mean large scale velocity is due to the inherent isotropy in the measurement of the transverse component of large scale velocity. Since the particles chosen are randomly distributed, the transverse direction \( r \) is randomly oriented, and \( \Sigma u_\perp \) transverse to \( r \) is isotropic. The difference of the value in the minima of the conditioned structure function is due to normalization.

To quantify the observed dependence of the conditional structure function, we fit all the curves in Fig. 3.12 to the functional form \( au^4 + bu^2 + c \). Fig. 3.13b shows the fit coefficient \( b \) as a function of the separation distance \( r/\eta \). The coefficient \( b \) measures the curvature of the conditional structure functions at the origin and
Figure 3.13: (a) The velocity dependence of conditional second order conditional structure functions of one separation distance $r/\eta = 10.67$ to $21.33$ for the experiments with varying amplitude. $(f_{\text{high}} - f_{\text{low}}) = (3-3)\text{Hz (}, (+), (3-2)\text{Hz (}, (3-1)\text{Hz (}, (3-0)\text{Hz (×) with 50% duty cycle and } T = 30 \text{ s. (b) The coefficient } b \text{ is a measure of the dependence of the conditional structure function on the large scales of all separation distances as in Fig. 3.12 for the experiments with varying amplitude. Symbols are the same as (a). This figure is edited from Dan Blum’s Ph.D. thesis [20].}

captures the primary dependence on the large scale velocity. Measuring the coefficient of the second order term $b$ is also in keeping with previous studies [26]. There is an increase by more than a factor of 5 in the curvature, $b$, as the fluctua-
tions in the energy input increases from driven at 3 Hz continuously to alternating between 3 and 0 Hz. The degree to which all length scales show similar dependence on the large scales can also be evaluated from Fig. 3.13b. In section 3.2.3 we will show that changes in $b$ are closely related with a fit parameter to the changes in Kolmogorov constants that we presented in section 3.2.1.

### Varying Period

Fig. 3.14a shows the conditional second order structure functions for the experiments with varying period. When period $T$ increases, there is a stronger dependence on large scale velocity. The two shortest periods $T = 3$ s and 6 s have similar and relatively low curvatures. Increasing the period allows the turbulence to decay closer to quiescent before the energy input resumes, so the conditional dependence on the large scale velocity is stronger at longer periods. For the very long period, $T = 384$ s, the conditional structure function has a different shape with a sharp minimum at the center of a region with less curvature. This is the result of the high energy state providing the samples with large velocity sum while the low energy state provides only samples with velocity sum near zero. For this data at $T = 384$ there is also a much stronger dependence on the length scale as shown in Fig. 3.14b.

### Varying Duty Cycle

Fig. 3.15a shows the second order conditional structure functions for the experiments with varying duty cycle. It shows that reducing the duty cycle produces a large increase in the dependence of the conditional structure functions on the large scale velocity. The result is consistent with our previous findings that increasing
Chapter 3 - The Effects of Fluctuating Energy Input on Small Scales

Figure 3.14: (a) The velocity dependence of second order conditional structure functions of one separation distance $r/\eta = 10.67$ to 21.33 for the experiments with varying period. Symbols represent the cycle period, $T$ of $+= 3$ s, $\circ = 6$ s, $* = 12$ s, $\times = 24$ s, $\Box = 48$ s, $\diamond = 384$ s. Driving frequency modulations is $(f_{high} - f_{low}) = (3 - 0)$Hz, and the duty cycle is 50%. This figure is edited from Dan Blum’s Ph.D. thesis [20].

The fluctuations of the large scale energy injections increases the dependence of the second order conditional structure functions. Here all length scales show fairly similar dependence on the large scales as seen in Fig. 3.15b.
Figure 3.15: The velocity dependence of conditional second order conditional structure functions of one separation distance $r/\eta = 10.67$ to 21.33 for the experiments with varying duty cycle. Each curve shows the duty cycle of $+$ = 100%, $\circ$ = 75%, $*$ = 50%, $\times$ = 25% with driving frequency modulations ($f_{\text{high}} - f_{\text{low}}$) = (3 - 0)Hz and period $T = 30$ s. This figure is edited from Dan Blum’s Ph.D. thesis [20].
3.2.3 Connecting Conditional Structure Functions and Coefficients of Inertial Range Scaling Laws

The curvature $b$ of the conditional structure functions increases as the fluctuations of the large scale energy input increases. This suggests that it might be possible to connect $b$ with changes in the coefficients of inertial range scaling laws presented in section 3.2.1.

A simple linear parameterization $C_2 = 2(1 - 0.15b)$ seems to match the measured scaling coefficients fairly well as shown in Fig. 3.16. However, we do not have a solid theoretical foundation for choosing this functional form and the value of 0.15 is a rough fit. For weak fluctuations in the energy input, which includes most turbulent flows of interest, this parameterization seems to work fairly well. But for extreme cases it fails. At low duty cycles in Fig. 3.16b, this parameterization is well above the measured coefficient. In the limit where one of the states is actually quiescent ($\gamma = 1$ in Fig 1.2), the curvature $b$ should go to infinity while the coefficient of the scaling law would not go to zero. Conditional structure functions and coefficients of inertial range scaling laws are both modified by fluctuations in the large scale energy input of turbulence. A more complete understanding of the relationship between these two could be very useful since the effects of fluctuations in the large scale energy input are much easier to measure using conditional structure functions.
Figure 3.16: The relationship of the curvature $b$ of the conditional second order structure function with the experimental measurements of Kolmogorov constant, our refined model, and Monin and Yaglom model with experiments of (a) varying amplitude (b) varying period (c) varying duty cycle. • is the experimental Kolmogorov constant $C_2$, * is the refined model, □ is the Monin and Yaglom model, and ◊ is the parameterization 2(1-0.15$b$).
Chapter 4

Conclusion

Previous research has established that small scales in turbulence are not entirely independent of the instantaneous large scale behavior [2, 6, 8, 22, 27]. One of the large scale properties, the fluctuation of large scale energy injection, or the large scale intermittency, was highlighted. In this thesis, we devise three sets of experiments to vary large scale energy input to understand the effects of large scale energy fluctuations on small scales. Kolmogorov’s paper on refined similarity hypotheses and Landau’s remark both suggest that the large scale fluctuations in the energy dissipation will destroy the universality of small scales, and the coefficient of the scaling law will not be universal. [4, 6] However, the recent effort of understanding the turbulence has been focusing on the internal intermittency to obtain a more precise scaling law exponent [6]. On the other hand, the coefficient of the inertial range scaling law in second order structure function, commonly known as the Kolmogorov constant, is regarded as constant in most of the recent studies.

In this thesis, through systematically varying the large scale energy injections, we
find that when we increase the fluctuations of large scale energy input, the Kolmogorov constant for the second order structure function decreases. We extend the idea behind Kolmogorov’s refined similarity hypotheses to propose a refined model to quantify the change of the second order coefficients of inertial range scaling laws. Our refined model and the experimental measurement of Kolmogorov constant are in fair agreement in predicting the change of Kolmogorov constant. Besides looking at the second order structure function, we also look into making use of the conditional structure function to measure the effect of large scale energy fluctuations on small scale as suggested by previous research [22]. The conditional structure function has the capability of identifying the fluctuations of energy injection even when the fluctuations is small. We find that the curvature of the dependence of the second order conditional structure function is closely related to the change in Kolmogorov constant. A more detailed study has to be done to better understand how the curvature can be used to quantify the effect of fluctuations of large scale energy input on small scales.

The maximum of the change of the Kolmogorov constant that we observed is more than 20% below the value for the continuously driven case. Our observations indicate that the small scale statistics are indeed influenced by the large scale energy fluctuations, and the Kolmogorov constant is therefore not universal in the presence of large scale energy fluctuations. We can conclude the fluctuations of the large scale energy injections should be included as a large factor when considering the interactions between large and small scales, and we can quantify the effect of this large scale phenomenon on small scale through our refined model and the conditional structure function.


large scales in conditional structure functions from various turbulent flows”.
