ROTATIONAL DYNAMICS OF ANISOTROPIC PARTICLES IN TURBULENCE:
MEASUREMENTS OF LAGRANGIAN VORTICITY AND THE EFFECTS OF ALIGNMENT WITH THE VELOCITY GRADIENT

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DEDICATION

To Katie
For her encouragement, patience, and perspective.
We measure the Lagrangian rotational dynamics of anisotropic particles in turbulent flow with stereoscopic video imaging. The canonical shapes that we choose to study are rods, crosses (two perpendicular rods), and jacks (three mutually perpendicular rods), which we design and fabricate using 3D printing technology. The three-dimensional position of particles is measured using standard stereomatching techniques. To study these particles, we have developed a general methodology for measuring the time-resolved Lagrangian orientation and solid body rotation rate of anisotropic particles in a turbulent flow. We apply these techniques to measurements in a $R_\lambda \approx 110$ flow, where turbulence is generated by two grids oscillating in phase. Since the advected particles have a largest dimension less than 7 times the Kolmogorov length, they are good approximations of tracer particles. Using resistive force theory, we demonstrate that ideal tracer jacks and crosses have the same rotational dynamics as spheres and disks, respectively. Thus, our measurements allow us to probe the dependence of the rotation rates of ellipsoidal particles on aspect ratio ($\alpha$) at qualitatively distinct points spanning the entire range: $\alpha = 0$ (measured by disks), $\alpha = 1$ (jacks), and $\alpha \to \infty$ (rods).

We’ve developed experimental techniques to measure several quantities previously only accessible by direct numerical simulations (DNS). Using these methods, we have measured the mean square rotation rate of neutrally buoyant particles, $\dot{\theta}_i$, spanning the full range of aspect ratios and obtained results that agree with the DNS. By measuring the full solid-body rotation of jacks, we provide a new, extensible way to directly probe the Lagrangian vorticity of the fluid and vortex structure in the flow. Our measurements of the alignment of crosses with the direction of their solid body rotation rate vector are the first direct observation of the alignment of anisotropic particles by the velocity gradients in the flow, which we subsequently verify using data direct numerical simulations of turbulence. The success of our methods demonstrates the possibility of measuring Lagrangian solid-body rotation of anisotropic particles, of any aspect ratio, to a high degree of accuracy. This opens a wide range of research possibilities in turbulence and other problems in fundamental fluid dynamics.
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### ACRONYMS
INTRODUCTION

“... the most important unsolved problem of classical physics.”
— R. P. Feynman on turbulence

A required characteristic of turbulence is the presence of vorticity; an irrotational flow cannot be turbulent. This thesis introduces a new way to experimentally probe statistics of the rotational character of turbulent flows. In physics we use a variety of methods to develop an understanding of the physical world. While the theoretical and experimental models we use to represent, construct, and discover physical phenomena might seem independent of each other at times, rich problems like understanding the small scale properties of turbulence make abundantly clear the importance of a multi-methological approach. Since the Kolmogorov theory of 1941 [1, 2], physicists have used experimental measurements and direct numerical simulations of the Navier-Stokes equations, our best theoretical description of a fluid, to test the predictions about turbulence and understand deviations from the theory.

The smallest scales of turbulence are known as the Kolmogorov scales. This is the scale at which the energy in the fluid is dissipated to heat by microscopic mechanisms. The rate at which this energy is turned into heat is determined by the velocity gradients in the flow and the kinematic viscosity of the fluid. For a turbulent fluid with a
given viscosity, \( \nu \), and energy dissipation rate, \( \epsilon \), one can show that the energy dissipation rate is fully determined by the strain [3]:

\[
\epsilon = 2 \nu \left\langle S_{ij} S_{ij} \right\rangle ,
\]

where

\[
S_{ij} \equiv \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) ,
\]

is the strain. The strain is the part of the velocity gradient tensor \( \frac{\partial u_i}{\partial x_j} \) that produces deformations in the fluid. The energy dissipation rate together with the kinematic viscosity of the fluid sets the Kolmogorov time scale, \( \tau_\eta \), in the flow: \( \tau_\eta = (\nu/\epsilon)^{1/2} \). It also sets the smallest length scale of the flow via \( \eta = (\nu^3/\epsilon)^{1/4} \) [3]. Measuring properties at these scales is the central experimental goal.

One of the dominant techniques in experimental turbulence studies has been Lagrangian particle tracking. Small, neutrally buoyant particles are seeded into a turbulent flow and imaged using high-speed video. Particle trajectories are tracked within a stationary viewing volume. Using a techniques known as stereo-matching, their positions and velocities can be accurately measured.

An important frontier in particle tracking studies in turbulence is the dynamics of anisotropic particles. Understanding their properties is important both for applications, e.g. fiber flows in the paper industry [4] or ice crystal dynamics in clouds [5, 6, 7], and the fundamental properties of turbulence since anisotropic particles rotate in response to the velocity gradient tensor, which has been the focus of much of the work on the dynamics of the small scales of turbulence [8]. Extended particles in fluid respond to the velocity gradient in a known way [9]. Early numerical studies have found that the motion of rods in turbulence is strongly affected by their alignment with the flow [10].
More recently, direct numerical simulations of the rotation rate of particles of various aspect ratios have been shown to be affected by their alignment [11]. One of the focuses in our group over the past couple of years has been the verification of these numerical results for particles in the large aspect ratio limit, where the effects of alignment are the strongest [12, 11]. The smoking gun measurement that shows the effects of alignment is the variance of the rotation rate, \( \langle \dot{\Omega}_i \dot{p}_i \rangle \), where \( p \) is a unit vector pointing along the length of a rod (see Figure 2). The average rotation rate of rods is greatly decreased when they are correlated with the flow as compared to the random orientation case. These experimental measurements were an important step into the field of anisotropic particles in turbulence. The simulations are computed using direct numerical simulations (DNS) of the Navier-Stokes equations to produce a velocity gradient. The relation between the velocity gradient and the rotation rate is given by Jeffery’s Equations:

\[
\dot{p}_i = \Omega_{ij} p_j + \frac{\alpha^2 - 1}{\alpha^2 + 1} [S_{ij} p_j - p_i p_j S_{jk} p_k],
\]

(Jeffery’s Equation)
where $S_{ij}$ is again the strain and $\Omega_{ij} \equiv \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$ is the vorticity. The DNS data is able to access the entire range of aspect ratios. However, there have not yet been experimental measurements of particles spanning the range.

In this thesis, we extend the measurements of rods by developing new techniques for fabricating and measuring the rotational dynamics of particles of any aspect ratio, $\alpha \equiv L/d$ (see Figure 2). To the best of our knowledge, these measurements are the first of their kind and, as a result, we had to develop many new techniques for fabricating particles, extracting their orientation, and measuring their Lagrangian rotation rates. With these new methods, we can access quantities that were previously difficult to access experimentally. One topic that has received much interest in the literature is Lagrangian vorticity \cite{13,14,15,16,17}. There are two approaches that groups have taken to measure this quantity. The “classical” approach—adhering to the standard techniques of particle tracking—is to seed the flow with a relatively high density of particles and measure the velocity gradient directly from the distribution of their velocities and positions in space. These measurements require at least four particles in view (more for reduced noise) within a volume $\propto \eta^3$, which limits their range of applicability to small flows and low Reynolds numbers. Zeff et al. \cite{13} and Luthi et al. \cite{16} implement creative variations to this approach and obtain important results. More recently, Klein et al. \cite{17} and Zimmermann et al. \cite{14,15} measure the rotational dynamics of
spheres significantly larger than the Kolmogorov scale. These measurements can be performed in larger viewing volumes, but are limited to the large scales of the flow. In order to measure the rotation rate of spheres, one has to mark certain positions on the sphere—otherwise it is isotropic and its orientation cannot be determined by imaging. There is no known way to do this for spheres on the scale of $\eta$, so small scale properties of the turbulence are inaccessible with these methods.

The attraction of measuring the rotation of spheres is that their rotation is determined only by the vorticity in the flow, as can be seen by setting $\alpha = 1$ in Jeffery’s Equation. Thus, by measuring their rotation rate one has a direct means of measuring the fluid vorticity. Inspired, we posed the question: are there any particles who posses enough of the same symmetries as a sphere to couple only to the vorticity, that we can fabricate on the scale of $\eta$, and whose orientations we can accurately determine? It is well known that the rotational dynamics of any rigid body is equivalent to that of some ellipsoid as long as that rigid body is unaltered by a $\pi/2$ rotation about any axis followed by a reflection over a plane containing that axis \[18\]. This might be familiar as the symmetry of the $S_4$ crystallographic point group in the Schoenflies notation \[19\]. A jack—a name taken from the child’s game—is an object that possesses this symmetry (see Figure 3). Moreover, it is intuitively apparent that it should rotate just like a sphere since it possesses the same symmetries only in a discrete form. In Chapter 2 we will provide a model calculation that argues that this intuition is correct. An important corollary to this correspondence between jacks and a sphere in terms of their symmetries is that any effects of the arms interacting with the fluid can be neglected on fundamental grounds to the order of accuracy that Jeffery’s Equations are true.
The impact of the realization that jacks rotate like spheres is evident in the context of current 3D printing technology, which allows us to fabricate these particles with an arm diameter of 300 µm. In our flow, this corresponds to largest dimension as small as 5 η. Although our particles are still slightly larger than η, we have reason not to despair. Studies of the dependence of the statistics of rods on the rotation rate provide solid reason to expect that the statistics of second moments should not differ by more than about 10-15% for particle sizes up to 10 η. [10, 20] Moreover, the 3D printing industry is rapidly progressing and may soon enable fabrication of particles with a largest length on the order of 10^2 µm[21].

A natural extension to measurements of jacks and rods is to also measure crosses. In Chapter 2, we will show that crosses rotate just like disks, using the same framework for why jacks behave like spheres. These three shapes span the space of possible ellipsoid-like particles. Understanding their rotational dynamics would provide a first order understanding of how all particles rotate in turbulence. Jacks and crosses in particular offer new information that is not contained in measurements of rods: the full solid body rotation vector, ω. Thus, measurements of jacks and crosses give direct access to their alignment with the vorticity in the flow. Crosses have a wide variety of possible motions and are decidedly anisotropic, so the question of how they align with the flow is very interesting. Although simulations have predicted the effects of alignment with the flow on the
mean square rotation rate \[11\], it is not known what types of motion they undergo on average. Simultaneous measurements of the orientation and the solid body rotation allow us to define a quantity to quantify the degree of alignment: \(\langle |p_i \hat{\omega}_i| \rangle\). This measures the cosine of the angle between the orientation and the axis of rotation.

We have introduced a new framework to experimentally investigate the role that stretching and alignment plays on the smallest scales of turbulence. A full understanding of these properties and how they depend on the larger scales is a central goal of research in turbulence; we have made an important step in that direction. Measurements of jacks provide an efficient and accurate method to measure Lagrangian vorticity. Multi-particle statistics of rotational dynamics are possible within this framework, though we do not explore them in this thesis, and may open the door to more in-depth studies of vortex lines in turbulence. Measurements of rods and, in particular, crosses provide information about the alignment effects of turbulent flows. Moreover, understanding the rotational dynamics of the particles themselves has wide ranging applications. We have only just begun to uncover the physics of anisotropic particles through experimental particle tracking in turbulence—and there is already much that we have learned.
THEORETICAL BACKGROUND

Here we will introduce in more detail the theoretical tools and background knowledge required to understand the results of this thesis.

2.1 VELOCITY GRADIENTS

The velocity gradient tensor exists for any velocity field and describes its spatial variation. In turbulence, it refers to the fluid velocity field and is one the most important quantities to understand because it encodes the deformation of the fluid. In general, the spatial dependence is complex. However, it can be better understood by separating the symmetric and anti-symmetric components. The standard definitions:

\[
\Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right), \quad \text{and} \quad S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]  

(2.1.1)

are the vorticity and strain tensors, respectively. See Figure 4 for a qualitative picture of the type of flow each tensor describes. An important composite flow is uniform shear, \( \Omega_{ij} + S_{ij} \), which is equal parts vorticity and strain. Shear flow serves as a crude, time-static depiction of real flow fields common in turbulence.

The small scale properties of turbulence discussed above are intimately related to the velocity gradients. As such, they are a target of experimental investigation. There are emerging techniques that attempt to measure the them directly with high spatial resolution,
2.2 RIGID BODY ROTATIONS

When solid bodies are placed in velocity gradient fields, they respond to deformations in the fluid. We need a mathematical formalism to describe how they reorient themselves in time \([22]\). Euler’s theorem tells us that the motion of a rigid body can always be decomposed into a rotation and a translation. Rotations can be represented by an orthogonal matrix, with a minimal representation in terms of the Euler angles (see Figure 5). A rotation of the coordinate system can be performed by rotating by \(\phi\) about the \(z\)-axis, then by \(\theta\) about the \(x’\)-axis, and then finally by \(\psi\) about the \(z''\)-axis. The matrix that repre-
2.3 Rotation of_axisymmetric ellipsoids: Jeffery’s equations

Axisymmetric ellipsoids are shapes like those in Figure 2 that are a first order generalization of a sphere. The aspect ratio describes the shape of the particle (see Figure 2), and is given by \( \alpha \equiv d/L \), where \( d \) is the width of the equal axes and \( L \) is the length of asymmetric axis. A large aspect ratio corresponds to a prolate ellipsoid, which becomes a rod in the infinite limit, while a small aspect ratio corresponds to an

\[
A^{-1} = \begin{pmatrix}
\cos \psi \cos \phi - \cos \theta \sin \phi \sin \psi & -\sin \psi \cos \phi - \cos \theta \sin \phi \cos \psi & \sin \theta \sin \phi \\
\cos \psi \sin \phi & -\sin \psi \sin \phi + \cos \theta \cos \phi \cos \psi & -\sin \theta \cos \phi \\
\sin \theta \sin \psi & \sin \theta \cos \psi & \cos \theta
\end{pmatrix}
\]  

(2.2.1)

One can define an orthogonal set of principal axes for any rigid body which completely defines its orientation. We can define the rotation of an arbitrary object by the action of \( A^{-1} \) on each of the directors defining its principal axes.

Experimentally, it is not possible to retain information about all three principle axis vectors for objects with full axial symmetry (i.e., not an \( n \)-fold symmetry like a cross or jack has). For example, a rod only requires \( \phi \) and \( \theta \) to define its orientation. However, this is not to say that the information contained in the particle’s rotation about its symmetry axis is unimportant. On the contrary, it is required in order to know the full solid-body rotation vector, which is necessary to fully measure and understand particle rotational dynamics and alignment of the particle by the velocity gradients in the flow.
oblate ellipsoid, which becomes a planar disk in the limit of zero aspect ratio. These objects are both highly anisotropic in a different way. As \( \alpha \) approaches one, the ellipsoid become increasingly isotropic—a sphere being the ideal case. The rotational dynamics of these different shapes vary widely based on how they couple to the velocity gradient, which is determined by the specifics of the particle shape.

The orientation vector, \( p \), is a unit vector giving the orientation of the ellipsoid. It is defined to lie along the longest axis of a prolate ellipsoid, or the shortest axis of an oblate ellipsoid. One way to keep this straight is to picture a slab of putty. If it is stretched in a long strip, the orientation vector points along it. One can then imagine continuously deforming the putty into other shapes along the range of aspect ratios, all the while keeping the orientation vector pointing in the same direction. Once an aspect ratio near zero is reached, the direction of \( p \) would be perpendicular to the face of the putty-disk. For a sphere, the definition of \( p \) is arbitrary due to its symmetry.

The whole project of studying anisotropic particles in turbulence requires understanding their rotation rates and their relation to the velocity gradients. Jeffery’s Equation gives \( \dot{p} \) for axisymmetric ellipsoidal particles in viscous flow \([9]\). These equations describe how ellipsoids couple to the velocity gradients as a function of orientation and shape. They are found by explicit evaluation for the forces on an ellipsoidal particle due to the velocity gradient. By using DNS to produce a realistic velocity gradient field, one can use Jeffery’s Equation to simulate the rotation rate of any axisymmetric ellipsoid. Understanding how and why these simulations differ from experimental measurements is one of the ongoing goals in this field.
2.3 Rotation of Axisymmetric Ellipsoids: Jeffery’s Equations

2.3.1 Rod Dynamics

Over the past decade, there have been several groups who have studied the rotational dynamics of rods from different vantages. A rod is well approximated as having an aspect ratio of $\alpha \to \infty$. In this case we have,

$$\lim_{\alpha \to \infty} \frac{1 - \alpha^2}{1 + \alpha^2} = -1.$$  \hspace{1cm} (2.3.1)

From Jeffery’s Equation we can then see that the rotation rate of rods depends on both the strain and the vorticity. Referring to Figure 4, intuition tells us that rods should tend to rotate with the vorticity and align with the strain field. The generalization to three dimensions complicates the issue, which part of the motivation for studying such objects experimentally. Still, the important qualitative features remain. Both simulations and experiment find that the alignment with the velocity gradient effectively reduces rotation rate $[10, 11, 20]$. 

Figure 6: Simulation of thin rod rotation rate as a function of length by Shin and Koch in 2005 $[10]$. The black points are simulations of rods incorporating their correlation of the flow.
2.3.2 Isotropic Particle Dynamics

A sphere is the canonical isotropic shape and has an aspect ratio of one since its length and diameter are equivalent. One can quickly verify for the isotropic case that Jeffery’s Equation reduces to:

\[
\dot{p}_i = \Omega_{ij}p_j.
\]  \hspace{1cm} (2.3.2)

That is, the rotation rate of an isotropic particle is completely independent of the strain field. Thus, the solid body rotation of a neutrally buoyant tracer particle with \( \alpha = 1 \) is equivalent to the vorticity measurements of the rotation rates give a direct measurement of the fluid vorticity. However, measuring the rotation rate of a sphere directly is impossible, since the orientation of a sphere is undetermined and more creative techniques are required.

Some groups are able to track the rotation dynamics of inertial size spheres in turbulent flow [17]. They use a super-absorbant polymer that is index matched with water. By placing marker particles inside of the polymer, they can measure the rotation of a finite size sphere. However, we want to be closer to the Kolmogorov scale, and this method, while promising, cannot be easily scaled down. Instead, we propose a jack (Figure 3) as the quintessential isotropic object. In Section 2.4 we show that jacks do, indeed, rotate like spheres. It is a conceptually simple extension to suggest that crosses rotate like disks, which we also show below. Very little is known in the literature about the dynamics of disks experimentally, but we will present our results later in comparison to simulations [11].
Resistive force theory is widely used in soft-condensed matter systems (see, e.g., [23]). We will apply it first to an arbitrarily oriented thin rod of width $a$ in a general velocity field. Then we will generalize the technique to all aspect ratios. Firstly, let us assume that the viscous drag force acting on the rod is of the canonical form, where the constant $C_d$ that scales the force is determined analytically by Gray and Hancock [24]:

$$dF = 2aC_d u_\perp dr$$
$$df = aC_d u_\parallel dr$$

We note here that $df$ is parallel to the rod, while $dF$ is perpendicular to the rod and responsible for producing all of the torque. The vector $u$ is the relative velocity between the rotating rod and the fluid velocity. The subscripts indicate the component parallel or perpendicular to the rod.

We can use these relations in order to calculate the total torque due to fluid flow past the rod. In low Reynolds number flows, the velocity field is linear over distances on the order of the Kolmogorov scale. Additionally, we may take the center of the particle to be at rest. [9]

We can then approximate a general velocity field by a linearization:

$$v_i(x) = v_i(x_\circ) + \left. \frac{\partial v_i}{\partial x_j} \right|_{x=x_\circ} \Delta x_j$$
$$= \left. \frac{\partial v_i}{\partial x_j} \right|_{x=0} r p_j,$$

where $p$ is a normalized vector denoting the orientation, $r$ is the radial distance from the center of the rod, and we have set the origin of our coordinate system equal to $x_\circ$. Now we can compute the force on a
rod with displacement $\Delta x = rp$ from its center. We will denote the orientation by the normalized vector $p$. The velocity responsible for the complete drag force is the relative velocity between the flow field and the tangential velocity due to its rigid body rotation:

$$u = v - \omega^' \times rp$$  \hspace{1cm} (2.4.3)$$

To compute the torque on the rod, we are interested only in the component of velocity perpendicular to the rod. We can write this by subtracting away the component of the relative velocity parallel to the rod from the full relative velocity, $u$. This gives us the $u_\perp$ that we desire and we can enter it into Equation 2.4.1. Thus we have, in tensor notation:

$$dF_i = 2aC_d \left[ \frac{r}{r} \frac{\partial u_i}{\partial x_j} p_j - \left( \frac{r}{r} \frac{\partial u_k}{\partial x_j} p_l p_k \right) p_i - r \epsilon_{imn} \omega^'_m p_n \right] dr$$

$$= \left[ (S_{ij} + \Omega_{ij}) p_j - p_k (S_{kl} + \Omega_{kl}) p_l p_i - \epsilon_{imn} \omega^' m p_n \right] 2aC_d r dr,$$  \hspace{1cm} (2.4.4)$$

where we have written the velocity gradient in terms of its symmetric and antisymmetric parts, $S_{ij}$ and $\Omega_{ij}$, respectively. The strain and vorticity tensors parameterize the problem and are given by:

$$\Omega = \begin{pmatrix}
0 & -\omega_3 & \omega_2 \\
\omega_3 & 0 & -\omega_1 \\
-\omega_2 & \omega_1 & 0
\end{pmatrix}$$  \hspace{1cm} (2.4.5)$$

$$S = \begin{pmatrix}
k_1 & \sigma_1 & \sigma_2 \\
\sigma_1 & k_2 & \sigma_3 \\
\sigma_2 & \sigma_3 & k_3
\end{pmatrix}$$  \hspace{1cm} (2.4.6)$$
The \( \omega_i \)'s make up the components of the vorticity vector in the flow, while the \( k_i \)'s are the eigenvalues of the strain (in diagonalized form) and must sum to zero for an incompressible flow.

We now rearrange (2.4.4), noting that \( p_k \Omega_{kl} p_l = 0 \) since \( p_k p_l \) forms a symmetric tensor while \( \Omega_{kl} \) is antisymmetric, and compute the torque:

\[
d\tau_q = \epsilon_{qti} r p_t dF_i \\
= \epsilon_{qti} p_i [\Omega_{ij} p_j + S_{ij} p_j - p_i p_k S_{kl} p_l - \epsilon_{imn} \omega'_m p_n] 2a C_d r^2 dr \tag{2.4.7}
\]

Integrating along the length of the rod, \( l \), we have:

\[
\tau_q = \epsilon_{qti} p_t [\Omega_{ij} p_j + S_{ij} p_j - p_i p_k S_{kl} p_l - \epsilon_{imn} \omega'_m p_n] 2a C_d \int_{-l/2}^{l/2} r^2 dr \\
= \epsilon_{qti} p_t [\Omega_{ij} p_j + S_{ij} p_j - p_i p_k S_{kl} p_l - \epsilon_{imn} \omega'_m p_n] \frac{a}{6} C_d l^3 \tag{2.4.8}
\]

Now, consider the term \( \epsilon_{imn} \omega'_m p_n = (\omega' \times p)_i \). This is nothing else than the rate at which the orientation of the rod is changing, \( \dot{p} \). Thus we have:

\[
\tau_q = \epsilon_{qti} p_t [\Omega_{ij} p_j + S_{ij} p_j - p_i p_k S_{kl} p_l - \dot{p}_i] \frac{a}{6} C_d l^3 \tag{2.4.9}
\]

For any orientation, \( p_t \) (2.4.9) can be used to compute the \( q \)th component of the torque on a single rod. Moreover, recall the condition for Stokes flow:

\[
\tau_q = 0, \quad \text{for } q=1,2,3 \tag{2.4.10}
\]

When we impose this constraint on (2.4.9), we immediately recover Jeffery’s Equations in the limit of infinite aspect ratio [9]:

\[
\dot{p}_i = \Omega_{ij} p_j + S_{ij} p_j - p_i p_k S_{kl} p_l \tag{2.4.11}
\]
With this understanding in hand, we may now explore the rotation dynamics of objects that are composites of perpendicular rods with arbitrary lengths.

### 2.4.1 Torque on an arbitrary composite of perpendicular rods

Ellipsoids are 3-dimensional objects defined by three perpendicular axes of arbitrary lengths: \( l_1, l_2, l_3 \). Three perpendicular rods define the “scaffold” of such an object. We know from [18] that these objects rotate like ellipsoids of a “some” aspect ratio if they have the proper symmetry. We can determine that aspect ratio within the assumptions of resistive force theory. Referring to (2.4.11), we can read off the \( q \)th component of the torque on such an scaffold due to the \( \alpha \)th arm, \( \tau_q^\alpha \).

The total \( q \) component of the torque is then:

\[
\tau_q = \sum_\alpha \Gamma^\alpha \varepsilon_{q\alpha} p_i^\alpha \left[ \Omega_{ij} p_j^\alpha + S_{ij} p_j^\alpha - p_i^\alpha p_k^\alpha S_{kl} p_l^\alpha - \epsilon_{imn} \omega'_{0m} p_n^\alpha \right] \tag{2.4.12}
\]

Where \( \Gamma^\alpha \) is the pre-factor for the \( \alpha \)th arm:

\[
\Gamma^\alpha \equiv aC_{\alpha} l_3^3 \tag{2.4.13}
\]

Using straightforward tensor analysis, one can show that this gives an analog to Jefferey’s equation for axisymmetric particles, \( \Gamma^\alpha = \Gamma^\beta \neq \Gamma^\gamma \):

\[
\dot{p}_i = \Omega_{ij} p_j + \frac{\alpha^3 - 1}{\alpha^3 + 1} \left[ S_{ij} p_j - p_i p_j S_{jk} p_k \right] \tag{2.4.14}
\]

where \( \alpha \equiv \Gamma^\gamma / \Gamma^\beta \) in this case.

---

1 Greek letters are not summed over unless explicitly indicated.
EXPERIMENTAL METHODS

We use high-speed video stereoscopic video to track the rotational dynamics of anisotropic particles in turbulence. We fabricate and image anisotropic particles on the scale of 5-10η using 3D printing—demonstrating the experimental viability of this technique.

3.1 FABRICATION

Any particles used in an experimental flow must meet several requirements if their statistics are to be connected to properties of turbulence. Firstly, they must be density matched with the fluid. This constraint is to ensure that the dynamics of the particle are dominated by the fluid dynamics so that any inertial effects do not bias the measurements. In past experiments performed in our lab exploring the rotation dynamics of rods in turbulence, the material of choice is Nylon-6. This material was chosen because, of the materials which could be used to fabricate a large number millimeter scale rods in the lab, it has the closest density to water, at about 1.15 g/cm³, of materials known to be compatible with the method of fabrication. In order to improve the density matching, the water is salinated with CaCl₂ to increase its density. This method can be used to match the density of the fluid and the particles to within a small degree of error.

Secondly, the characteristic size of the particles should be close to the Kolmogorov length, η (generally on the order of a few hundred microns). Otherwise, the information contained in the rotation dy-
dynamics of the particle is an average over length scales up to the size of the particle. The dynamics of long particles, length scales of up to $20\eta$ or so, can be measured and understood by including the contribution of all length scales into the rotation rate. This is work that has been done by a graduate student in our lab and will soon appear in the literature \[20\]. In the present case, by matching the particle size to the Kolmogorov length scale as best we can, the dynamics of our particles should be dominated by the smallest scales of the flow. The degree to which this is true can be understood using methods and experience developed in previous work in our lab \[11, 20\].

Lastly, there are two additional constraints enforced by our imaging system. The aspect ratio of particles must be high enough to experimentally distinguish between different orientations. For rods an aspect ratio, $\alpha$, as low as 5 is sufficient. If we consider more complex shapes—e.g., jacks or crosses—as composites of perpendicular rods, it is reasonable to assume the rods that make them up should have an aspect ratio of 10 at the lowest. As such, the sections that extend from the central point of intersection will have an aspect ratio of 5 and can be easily resolved by our cameras. The second imaging constraint is that not only must the material be neutrally buoyant, but it must also hold the fluorescent dye that is excited by the laser. This has been true of all the materials we have come across.

### 3.1.1 3D Printing

Making a large number of complex shapes with sub-millimeter detail, while difficult, is a challenge that can be met using current 3D fabrication technology\(^1\). We use a Computer Aided Design (CAD) program to design the particles we wish to analyze. They are then fabricated

\[^1\] We would like to thank Nadia Cheng and Peko Hosoi (MIT) for useful discussions and guidance during the early parts of the fabrication process.
Figure 7: Shown above is an example of one of the jacks that we fabricated using stereolithography. The nubs on the end of some of the arms are clearly visible.

directly from the file we produce. We found varying degrees of success with different technologies before ultimately settling on a specific 3D printer. The main considerations in this decision were fabrication accuracy, cost, speed of fabrication, capacity of fabrication, available materials, and post-fabrication effort.

Stereolithography is a method of 3D fabrication that builds a UV-curable polymer into the desired object. The object is built up in thin 2D layers—a laser is focused on a small region and rastered to harden the material. We worked with Fine Line Prototyping to produce a few sample jacks in two different sizes: \( d = 300\mu m \) and \( d = 500\mu m \). It is clear from Figure 7 that stereolithography is reasonably effective at this scale. The particles are very sturdy and absorb the Rhodamine dye very well. However, there are still a few concerns. One limitation is the “nubs” that remain on the end of the arms. There are there because the particle has to be supported by small posts during the fabrication process, which are then removed by hand. For making on the order of \( 10^3 \) particles, removal of these nubs by hand would
be prohibitive to the experimental process. Moreover, in Figure 7b the diameter of the post is close to the diameter of an arm of the jack and the arm near the top of the image has become distorted. One might argue that these distortions are small compared to $\eta$ and any asymmetry that they produce will have a negligible effect on the aspect ratio of the particle. While this may be true, other techniques allowed us to meet a higher experimental standard.

*Acrylonitrile Butadiene Styrene* (ABS) is a material that is very commonly used by 3D printing technologies and has a density of 1.05 g/cm$^3$. This is nearly ideal for our fabrication requirements, and would require only minimal water salination. However, at least at the present stage of the technology, the printers that work with this particular material do not have the sufficient accuracy to work at the scales we require. The machine we first used had a posted tolerance of 800$\mu$m. Although this is the minimum part thickness that the printer can handle, complex geometries like the jack do not print cleanly even when we make the arm diameter up to twice that tolerance. The parts we printed at this scale were good enough to take images of a single particle and begin to understand some of the pathologies we might encounter and to begin working on a tomographic algorithm, but much more accurate printers were required for experimentally viable particles.

The machine that we ultimately used to fabricate our particles is the Objet Connex 500, a top of the line 3D printer. It’s quoted tolerances are 20 – 85$\mu$m, but depend on model geometry and orientation. We tried several iterations of fabrication to find the optimal placement orientation. After consulting with engineers at Thogus Products, we determined that printer tolerance limited our smallest reasonable dimension to 300$\mu$m. This is small enough to be within about 10% of
The tracer limit. Besides the ability to print at these scales, another asset of the Objet Connex is that it can print in many different materials with a range of stiffness, transparency, and density. The material that we found best met our needs has a density of approximately 1.2 g/cm³. The density varied among each sample of particles and is also dependent on the particle shape (see Table 1). The variation is only on the order of 1%, so we choose our fluid density to be near the mean. Any resulting inertial forces should be small compared to the drag forces in the flow. We can quantify this effect by looking at the z-component of the velocity when we analyze our data. This is something that we plan to do very soon. The Objet Connex builds along the z-axis in 16 µm layers that lie parallel to the xy-plane. Thus, it seemed reasonable that aligning a jack with the build direction would lead to the most structurally sound object. It turns out that this is not the case; we have not explored any mechanical properties of 3D printed particles, but I understand the reason behind this as follows. We are printing at scales which are very near to the resolution limit. By angling the particles, each 2D print layer of each arm is an ellipse, rather than a circle, and has a larger cross-sectional area. Also, if we consider the possible ways to orient a jack on a flat surface, it is clear that placing at an angle (like a tripod), rather than balancing it on a single arm, is the most stable configuration.

Table 1: Shown above are our experimentally determined spreads in the particle density. This spread may be a result of printing near the resolution limit of the 3D printer.

<table>
<thead>
<tr>
<th>Particle Shape</th>
<th>$\rho_{UL}$: $\frac{3}{4}$ sink (g/cm³)</th>
<th>$\rho_{UL}$: $\frac{3}{4}$ float (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rod</td>
<td>1.205</td>
<td>1.215</td>
</tr>
<tr>
<td>cross</td>
<td>1.205</td>
<td>1.215</td>
</tr>
<tr>
<td>jack</td>
<td>1.215</td>
<td>1.225</td>
</tr>
</tbody>
</table>

2 A tracer particle is one whose length scale is equal to or less than $\eta$. 
The proper way to place crosses and rods is less clear. At first glance, it seems like they should be printed in an orientation that has them lain flat on the print bed. In fact, this leads to particles that are flatter in one radial direction than in the other. The reason behind this is unclear; our best explanation is that it is a combination of geometry and the dominant forces on that scale (e.g. gravity or surface tension). Since the arms we printed we round, the first layer that is laid down is a line. Each successive lay will remain well below the resolution until about halfway up the diameter of the arm. The limited resolution of the printer may have caused more model material to be placed in these regions, flattening out the profile of the arm. While the final particles (rods and crosses) that we used had this asymmetry, it should not significantly affect the rotational dynamics since the difference is not large compared to $\eta$. However, for future, iterations, it will be important to explore other print orientations for rods and crosses.

### 3.1.2 Particle Preparation

Once the particles are printed, it is a separate issue to prepare them for use in the experiment. 3D printers work by using two materials: the model material and the support material (see Figure 9). When the print is complete, the support material must be removed. For objects on a macro scale, Objet has a tested process for doing this that uses a water jet to remove most of the material and a 10% NaOH bath to remove any residual. For our particles, this was not an efficient option because of their smaller size. However, I developed a technique that efficiently removes the particles form the support matrix that draws inspiration from the aforementioned process.
3.1 Fabrication

Figure 9: Shown above is a representation of the Objet 3D printing process. (Obtained from: www.ags-3d.com)

**Step 1:** Dissolving away the support material takes a considerable amount of time and liquid volume. The excess solution from the process for 30,000 particles totals roughly 20L. Because of the large scale effort, it is useful to prepare a stock of 10% NaOH solution before beginning the dissolution process. Another strong base or a stronger solution of NaOH may also be useable, but I did not test that particular parameter.

**Step 2:** Once the NaOH solution is made, the particles are added in order to dissolve away the support material. A large Erlenmeyer flask or beaker is suitable for this purpose. Each group of 1000 particles is combined with 600-800 mL of solution and the container is placed into a sonicator. This scale is determined by the size of the sonicator available and can in principle be scaled up.

**Step 3:** Partway through the sonication process (after about 1 hour), the solution is changed. After the support material begins dissolving, the solution will appear opaque in color and become more viscous since the polymer is dispersed throughout the fluid. A filter and fun-
nel system is used to separate the polymer-base solution from the particles—which have not been completely separated from the support material yet. The new solution and particle combination is then sonicated for another hour or so.

*Step 4:* To tell whether the process is complete, one can simply look into the flask to see if the structure of the particles is visible and no longer obscured by the support material. At this point, the particles should be filtered out of the solution. Even for rods, the yield of fully separated particles should be very good.

*Step 5:* Once the particles are separated from the support material, they are ready to be dyed. The particles are placed in a solution of Rhodamine B and warmed on a hot plate. Their temperature is kept between 60 – 80°C so that the Rhodamine B, which can be toxic, is not released into the air. After several hours, the dye should be absorbed into the particles.

This process is very straightforward and serves as an efficient methodology for processing many thousands of particles of shapes of all aspect ratios. For the rods, I resorted to a double filter system to increase yield. It is easiest to incorporate the filter into the funnel rather than using a flat sieve where it is difficult to retrieve the particles after they have been filtered out of the gooey polymer-base solution. The approach that I took was to build a mesh “bag” and attach it to

![Figure 8: Shown above is an example of a jack and cross both printed with the Objet Connex 500. The width of each arm is 300µm](image)
the end of the funnel. That way, the gooey solution could simply be poured through. After lightly rinsing the complex with water, all that remains is the clean particles.

I will offer some nuggets on a few things of which it is worth being aware. When working with NaOH, it is important to be careful not to leave the particles in solution for too long. This is the main motivation for adding the sonicator to the process. I did not test to see how long is “too long”, but both Objet and Thogus (the engineering company that we ordered from) warn of this.

One thing that I did come up against was the particles sometimes not holding the dye well enough. I was building on methods that were previously developed in our lab for dying Nylon-6 fibers. However, the VeroClear material takes longer to fully absorb the dye. I left the particles in the dye for 6+ hours and often let them soak overnight (with the hot plate off).

If the particles are being stored for later use, they should be kept in sample of the fluid that will ultimately be used in the experiment. If stored in open air, the particles become less dense. We think that this is due to small cavities in the body of the particles.

3.2 Flow and Imaging

Our flow is created in a $1 \times 1 \times 1.5 \text{m}^3$ Plexiglas octagonal prism, shown in Figure 10, using two metal grids that oscillate in phase at a parametrically variable rate on the order of a few Hz. The grids have a mesh size of 8 cm, which sets the largest length scale in our flow, and an oscillation amplitude of 12 cm. Particles are imaged using a stereoscopic camera setup (see Figure 10). A region near the center of the tank was chosen as the imaging volume because that is where the turbulence is most nearly homogenous. With our set camera magnifi-
<table>
<thead>
<tr>
<th>Grid Frequency</th>
<th>$R_\lambda$</th>
<th>$\epsilon \text{ (mm/s}^3\text{)}$</th>
<th>$\eta$ (mm)</th>
<th>$\tau_\eta$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hz</td>
<td>110</td>
<td>128</td>
<td>0.531</td>
<td>130</td>
</tr>
<tr>
<td>3 Hz</td>
<td>260</td>
<td>260</td>
<td>0.245</td>
<td>28</td>
</tr>
</tbody>
</table>

Table 2: Shown above the values that characterize our turbulent flow.

cation and laser illumination, the dimensions of the viewing volume are approximately $3 \times 3 \times 3 \text{ cm}^3$. Each camera is $1280 \times 1024$ square pixels and run at 450 Hz.

The Nd:YAG laser that we use to illuminate the viewing volume excites the fluorescent particles into a specific frequency. We use filters on our lenses to improve our data quality by filtering out scattered light. In Figure 10, the viewing volume is illuminated in green. In past experiments performed on this same setup, the source of the laser was along the $x$-axis. This introduced a sampling bias. Though this bias can be accounted for without changing the hardware, we have since modified the optical setup so that the viewing volume is illuminated from along both the $x$-axis and $y$-axis. This modification was also motivated by the complex structure of the particles we image in this particular study. With tracer particles, used in most experiments on this system, there is no concern of “self-shadowing”, where one part of the particle prevents another part from being illuminated by the laser. With rods, this is, in fact, the source of the sampling bias; if a rod is aligned along the $x$-axis (or at a small angle to it), it will not fluoresce and, hence, not be detected. This problem increases in importance as we go from rods to crosses and jacks since there are a greater number of possible orientations that would lead to at least one arm being aligned with the laser. Moreover, while rods don’t have this problem, the arms of crosses and jacks can block light from reaching other arms of the particle. By having a the laser illuminate the volume from two perpendicular directions, we maximize the fluorescence. We use the Tsai calibration to calibrate the position
3.3 Data Acquisition

We took data at two Reynolds numbers near continuously over the
course of a week and a half. For each particle shape, we have 6 hours
of data at each value of $R_{\lambda}$. Without our image compression system,
this would have amounted to roughly 240 TB worth of data. However,
our lab uses real-time compression hardware that takes in a high
speed video stream in real time and outputs only the pixels above a
fixed threshold, providing a compression factor of several hundred
[26].

Compared to past experiments performed in this lab on the rota-
tional dynamics of rods, our images contain many more pixels due to

Figure 10: Shown above is our experimental setup. The viewing volume of
the octagonal tank is illuminated by an Nd:YAG laser (green),
which is in the focus of the four stereoscopically arranged video

cameras.

and viewing angles of the cameras for 3D reconstruction from the
stereoscopic images [25].
the fact that we use lenses with a larger zoom and because our particles have more surface area. With less than one particle in view per frame on average, the ability to take continuous data was essential to the success of this experiment and would not have been possible with the image compression system.

We planned to perform measurements of the energy dissipation rate in our flow using the standard methods, but we ran into some issues. The tracer particles should be density matched to the fluid. Our fluid density is non-standard and we cannot use dyed polystyrene sphere. PMMA is a plastic with a density of $1.18 \text{ g/cm}^3$ and is also a common material for making tracer size spheres for applications other than particle tracking. We did not find any which were pre-dyed and we needed to have them custom ordered to have the spheres dyed after they’ve already been fabricated. In the end, we were not able to see the tracers in our flow—they were too dim. We have since learned that the Rhodamine dye has to be inserted into the plastic before the spheres are fabricated. This leads to much brighter fluorescence. To define the energy dissipation rate, we use instead our measurements of the solid body rotation of jacks. We will discuss this calculation in Chapter 5.
DATA ANALYSIS

Our experimental data totals on the order of a TB in compressed video format. This corresponds to about $10^6$ tracks total per particle type at each grid frequency. We time resolve the full solid body rotation rate vector along the trajectory of each particle using custom algorithms that we have developed. We will discuss here our methods.

4.1 OVERVIEW OF THE ANALYSIS

Before discussing each component of our analysis in detail, I would like to provide a brief “bird’s-eye” overview of how the pieces fit together conceptually. I have provided a bare-bones pseudocode in Listing 1.

Determining the orientation of particles requires a stereoscopic view and the information from all of the cameras must be utilized in synchrony. Like all experimental data, the information in our custom-format compressed video (CPV) files have defects that are an artifact of the system. The requirement of using the information from all cameras at the same time introduces an additional hurdle: we must identify and deal with any defects as they arise. This is one of the main functional roles of the code that drives our analysis. The CPV files are written with a frame number corresponding to each video frame. In order to analyze the images, we must keep the CPV files in sync as we seek through them. Besides checking the frame number and catching wrong frame numbers that arise from the rare instances when the
image compression circuit records random bits, we also have to make sure that all cameras have at least one particle in view and that the CCD did not surpass its pixel count limit. We must continue to keep the CPV files in sync throughout the analysis.

In order to distinguish our actual measurements from any bright pixels resulting from noise or background gain, we perform a clustering sequence. For a given particle shape, there is a range of numbers of contiguous pixels that they will occupy on the 2D image plane, and we can determine a valid cluster in this way. Since each camera may have more than one cluster, we must determine which quadruples of clusters (from the four cameras) correspond to the same particle in 3D—this is called stereo-matching and can be implemented in just the same way as typical particle tracking experiments. \(^1\)

\(^1\) We actually don’t implement stereomatching for the analysis that we present in this thesis because we rarely have more than a single particle in view at a time. However, it is a trivial matter to incorporate it into the code.
Once we have found the centers of the particles, we can reconstruct the orientation from the stereoscopic image data. We have tested several different methods to do this. We ultimately determined that the only reliable method known to us was a new algorithm that we have developed in-house. I will discuss what the most appealing factors of each method was and what kinds of considerations went into determining which was the best to use.

Once we have determined the orientation of the particles as a function of time, we must resolve the rotation rate along its trajectory. On the scales that are a fraction of the Kolmogorov time, the rotation rate can be approximated as linear. However, linearizing a rotation is not so straightforward since it is defined by 9 elements—a matrix. However, we have developed an algorithm which does just this and provides accurate results. On this topic, we will again discuss the details of its implementation and our methodology for determining its accuracy.

In the final stage of data analysis, we measure statistics of the rotation rate in our flow. Among the things we are currently looking at is the second moment of the rotation rate, $\langle \dot{p}_i \dot{p}_i \rangle$ for different aspect ratios. We also measure directly the alignment of disks and jacks with the velocity gradient by looking at the angle between their $\hat{p}$ vector and their solid body rotation. We present these results, the first of their kind, in Chapter 5 and describe our methods for computing them in Section 4.6.

4.2 ALIGNMENT OF CPV FILES

Our CPV files use a custom format to encode information into the header of the file. Included is the frame number of each image, the number of pixels, and boolean flags denoting whether there are any er-
rors in the data compression. Each of the four cameras function independently from each other so any errors they record are uncorrelated. Thus, in order to use the information from all four cameras simultaneously, we must ensure that the current frame we are analyzing is “valid” for all four cameras. Validity is determined by three criteria.

Firstly, the camera must have a frame number that is in sequence with its previous frame number and all cameras must agree on the same frame number. Sometimes the frame number is wrong because it was recorded wrong by the image compression circuit. When the data is written from the chip to the computer’s hard disk, sometimes a random bit is stored for the frame number. The reason for these rare events is not well understood, but electronic pickup may be a contributing factor. We identify these errors and skip the data for that frame. The number of pixels on each camera must also be above a certain minimum level, which also depends on camera. We determine this level by looking at the data finding the lower end of the range of the size of particles on the 2D image plane. Lastly, there is also an upper limit to the number of pixels that can be retained in an image. This a a value set by the image compression circuit. If the number of pixels exceed this value, no data is recorded for those pixels and the rest of the data in the image is distorted. We must skip these frames as well.

We implement these conditions in the code that seeks through our CPV files. While they are conceptually simple, coding the logic that does this reliably is rather involved. However, these idiosyncrasies are not pertinent to our experimental results.
4.3 PARTICLE CLUSTERING

In each frame, we have to determine which bright pixels are due to particle illumination or noise. In MATLAB, there are built-in functions that find continuous groups of pixels in an image. We ultimately decided to use these because they have been optimized for speed using the vector nature of the scripting language. However, we have also developed some additional methods to increase speed, which we used in early iterations. I will offer an explanation of these here.

Clustering is just as important for measurements of anisotropic particles as it is for typical particle tracking experiments. One of the differences, though, is that the size in pixel number varies for anisotropic particles while for tracers it is roughly constant. This introduces some complications. For one, selecting for contiguous pixel groups within a range of sizes requires surveying the data to get an understanding of this range of validity. For the most part, this is a straightforward thing to do, but there are certain orientations of rods, for example, that may have a pixel size similar to tracer particles. The few straggling particles of shapes not matching that of what are measuring at any given stage constitute a small fraction of the overall population. We can find the center of each cluster using a weighted average of the bright pixels. We find the three-dimensional center of the particle using standard ray matching techniques.

4.4 ORIENTATION EXTRACTION

Determining the orientation of anisotropic particles is the keystone of this thesis. After a few iterations, we have developed a novel and efficient technique for accurately finding the orientation of particles of a known shape. We will also present another technique that we spent
some time on and which may hold promise for certain applications with further development.

4.4.1 Reconstruction Algorithm: a first attempt

Tomography comes from the greek word *tomos*, which translates to *section*. Tomographic reconstruction uses images from several different cameras, all of which view the same imaging volume from a different angle. By using computationally intensive but surprisingly straightforward algorithms, it is possible to reconstruct the three-dimensional object from a series of two-dimensional images. The main challenge is minimizing computational intensity.

The *Multiplicative Algebraic Reconstruction Technique* (MART) is a commonly used and robust algorithm for tomographic image reconstruction [27]. While there are equally effective and more computationally efficient algorithms, some dramatically so (cf. [28]), MART is the simplest to implement. Many of the improvements are simply modifications of this algorithm, and as such can be easily applied. These techniques have mainly been used with tracer particles and other objects without a defined structure. In our case, we may be able to capitalize on the geometric constraints of our jacks and crosses to further improve the algorithm’s effectiveness and/or speed.

The first step is to discretize the imaging volume. The space is divided into 3D cells called voxels, each with coordinates \((X,Y,Z)\). Each voxel has an intensity that we would like to determine; we denote this intensity distribution by \(E(X,Y,Z)\). Physically, the intensity from a given voxel is mainly projected onto several pixels of each camera, but contributes partially to every pixel. In practice, we do not consider the contribution of every voxel to every pixel because it dramatically increases computational requirements, and that contri-
bution rapidly becomes negligible. Instead, it is common to introduce a weighting matrix \( w_{ij} \) that measures the contribution of the \( j \)th voxel to the \( i \)th pixel. To measure this factor, the intersecting volume of the line of sight with a given voxel is normalized by the volume of that voxel. Denoting the intensity distribution from all voxel contributions on a given camera by \( I(x,y) \), we have in general:

\[
\sum_{\{j\}} w_{ij} E(X_j, Y_j, Z_j) = I(x_i, y_i),
\]

(4.4.1)

where \( \{j\} \) is the set of all voxels that contribute to the \( i \)th pixel and \( E(X,Y,Z) \) is the unknown we wish to solve for. The MART algorithm can find a solution by starting from an initial guess for the voxel intensity distribution, \( E^0(X,Y,Z) \), that is chosen to be uniform and nonzero. Structure then emerges from iterative reconstruction of the voxel volume using the camera images, \( I(x,y) \), in the following formula. Looping over all voxels and each pixel from each camera:

\[
E_{k+1}(X_j, Y_j, Z_j) = E_k(X_j, Y_j, Z_j) \left( \frac{I(x_i, y_i)}{\sum_{\{j\}} w_{ij} E_k(X_j, Y_j, Z_j)} \right)^{\mu w_{ij}}.
\]

(4.4.2)

The exponent has two functions: 1) \( \mu \leq 1 \) is a relaxation parameter, and 2) \( w_{ij} \) ensures that only contributing voxels are updated.

We implement a simplified version of the algorithm described above and attack the problem from the perspective of the voxel space: for any given voxel, determine what pixels it contributes to on each camera — assuming a specific Point Spread Function (PSF). To start, we assume the simplest function, where all of the intensity in one voxel maps to a single pixel with coordinates \( (x_i, y_i) = r_j \), determined via projection using Tsai’s algorithm [25]. In Listing 2 is the pseudocode for our implementation. Despite its simplicity, this simple implemen-
Listing 2: Pseudocode for tomographic algorithm

```plaintext
E(X, Y, Z) = ones(VoxelSpaceDimensions) % Uniform, non-zero initial guess
for all cameras:
    for all voxels:
        rj = Tsai(X, Y, Z)
        E(X, Y, Z) = Im(rj) * E(X, Y, Z)
    end
end
```

orientation extraction is can accurately reconstruct the 3-dimensional intensity distribution of jack, crosses and rods form our experimental data.

Once the particles have been tomographically reconstructed, we need to determine, for each frame of the compressed video, a unique set of Euler angles that define the orientation of the object. In order to do this, we take advantage of the structure of our particles and calculate a quantity akin to the moment of inertia—let us call it the moment of inertial intensity, $\Phi$.\(^2\)

Let us first recall how the moment of inertia is calculated. It can be derived from the kinetic energy of a rigid body. We begin with the definition:

\[
T = \frac{1}{2} \int x^2 \rho(x) \, dx
= \frac{1}{2} \int (\omega \times x)^2 \rho(x) \, dx,
\]

where $\omega$ is the angular velocity, $x$ is the vector position of a point in space, $\rho(x)$ gives the mass distribution, and we made use of the relation $\dot{x} = \omega \times x$. Shifting to tensor notation, the kinetic energy becomes:

\[^2\text{We would like to thank Professor Francis Starr for suggesting that we explore an algorithm of this sort.}\]
That is, the moment of inertia is given by:

\[ I_{jk} = \int (\delta_{jk}x_i^2 - x_jx_k)\rho(x) \, dx \]  

(4.4.3)

In general, the moment of inertia for an object in 3-dimensions is a symmetric tensor. As is true of all symmetric tensors, \( I \) can be diagonalized. The eigenvalues of the tensor in this form are mutually orthogonal and are known as the principal axes of the moment of inertia. These vectors generally correspond to the symmetry axes of a rigid body.

Tomography provides us with a 3-dimensional discrete-space intensity distribution, \( E \). As an intuitive but potentially quantifiable approximation, the density of the object, \( \rho(x) \), should be proportional to the light intensity, \( E(x) \). Indeed, for jacks or crosses, the only place where light can come from is their arms, \( i.e. \) not the space between them. Thus, in order to extract \( \Phi \), which has a form very similar to the moment of inertia, we posit a function of the following form:

\[ \Gamma(x) = r^nE(x)^p, \]  

(4.4.4)

where \( r \) is the distance to a point \( x \) in body coordinates. The relative values of the exponents \( n \) and \( p \) control the relative importance of intensity values based on their distance from the center of the reconstructed object. The dependence on “\( r \)” in Equation 4.4.4 is inspired
4.4 ORIENTATION EXTRACTION

(a) Example of a bad fit  
(b) Example of a good fit

Figure 11: Shown here are tomographic reconstructions of a jack for two different orientations. For some orientations, the jack cannot be uniquely reconstructed due to shadowing from the arms.

by the systematic “ghost particles” that tend to cluster around the center of the jack or cross since that is where the angular separation between arms is the smallest. Now we define the moment of inertial intensity as follows:

\[ \Phi_{jk} = \int \left( \delta_{jk} x_i^2 - x_i x_k \right) \Gamma(x) \, dr. \] (4.4.5)

Our initial thinking was that the principal axes of the moment of inertial intensity correspond exactly to arms of jacks, crosses, and rods if we choose \( \Gamma(x) \) correctly. With artificially constructed images (“test jacks”) as well as with real images, the eigenvalues of very nearly correspond to the arms of the object, but never exactly. Even when we perform a nonlinear fit in 3D by interpolating the E-matrix, we get the orientation wrong for a significant number of samples. There are two problems with this approach.

In Figure 11, we show a reconstructed jack with the orientation determined using the method described above. It is evident that the orientation is not quite correct. In Figure 11a the jack is in an orientation that leads to a “ghost-arm”. These errors in reconstruction are the clearest source of inconsistency in this model. More fundamentally, an ideal jack has no unique set of principle eigenvectors due to
its rotational symmetry. The moment of inertial intensity is the same for any choice of eigenvectors. What we compute when we get the eigenvectors of the experimental are the principle axes of any deviations from a perfect jack due to the discretation and the intensity distribution on its surface. This method might be viable for rods and crosses, but we have developed another method that works well for all aspect ratios.

4.4.2 Projection Algorithm

While tomography should in principle provide a 3D reconstruction to which we can fit a model, the limitation of a finite number of cameras makes it difficult to avoid the biases associated with specific orientations. In order to circumvent this issue, we devised a new method by noting that there is additional information that we know about our particle that is not being incorporated into the reconstruction algorithm: we have a known parameterization for its shape. In this section, we will outline the projection algorithm that accurately and efficiently determines its orientation. Conceptually, it works by creating a model particle in 3D with a first guess of its configuration. Then, the same techniques used for camera calibration can be applied to create a 2D projection of the model onto the image plane for each camera—this creates a set of model images. We use a nonlinear fit to search for the orientation that creates the set of model images that are the closest match to the data.

Any solid body rotation can be parameterized by the three Euler angles; we use this scheme to parameterize the orientations of a particle. Take the easiest case to visualize: a jack. We can define the default orientation of the jack to be when its three arms are aligned with the, \( \hat{x}, \hat{y}, \) and \( \hat{z} \) directions. In this case it is easy to see that the rotation ma-
trix that produces this orientation is the identity matrix. It is straightforward to show in general that the columns of a rotation matrix \( R \) taking the identity to a specific orientation correspond to the arms of a jack in that specific orientation and we call it the orientation matrix, \( O \). Thus any orientation \( O \) is also parametrized by the Euler angles. This is easily extended to crosses and rods.

Upon this foundation, we developed a nonlinear fitting algorithm to determine the orientation of a particle from one set of stereoscopic images. We begin by making a first guess at the orientation using the eigenvectors of the moment of intensity tensor, which form an orientation matrix, \( O_\text{g} \). Although this is not in general a reliable final measurement of the orientation, it provides a guess that converges quickly to the correct orientation. This orientation is parameterized by a set of Euler angles, \( \{e\}_\text{g} \). We perform a nonlinear search in Euler angle space to determine the best fit orientation to our data.

Typical nonlinear fit algorithms work by finding the minimum of a user supplied function. In our case, we seek to minimize the difference between \( O_\text{g} \) and the actual orientation of the particle in the data, \( O_d \). From our first guess of orientation and the measurement of the particle’s center, we construct a model of the particle as it would be projected onto the 2D image plane of each camera (see Figure 12a). The total difference in intensity between the model images and our experimental data then provides the residual function values that we seek to minimize. When the difference between the two sets of images is smallest, we have found a good match to the orientation.

The accuracy of this method is partially dependent on the construction of a good model and its speed depends on the implementation of that construction. Consideration of both of these factors has played an important role in our analysis. The 2D projection of our guess of

---

3 We will discuss the tools that we use to verify this claim later in this section.
4.4 ORIENTATION EXTRACTION

The orientation is determined by projecting the 3D endpoints onto the image plane of each camera using the Tsai calibration [25]. Since we treat each arm of the object independently, our algorithm in this form can treat objects with an arbitrary number of arms as long as they form a solid body object and share a common origin so that its rotation is described by a single rotation matrix. We apply a Gaussian intensity distribution along the width of each arm (in 2D) and a Fermi intensity distribution across the length of the arm:

\[ I(x, y) = I_o e^{\frac{x^2}{2d^2}} \times \left( 1 + e^{x/ad} \right)^{-1}, \quad (4.4.6) \]

where \( x \) is measured along the arm, \( y \) is measured perpendicular to the arm, \( d \) is again the width of an arm, and we used a value of \( a = 0.7 \). \( I_o \) is the average intensity on each individual camera for the particular frame in question. We determined that these functions would provide the best known model to our data, which tends to have mostly constant intensity across the arm but drops off rapidly along the edges (see Figure 12b).

Our model is conceptually simple, but it does need to be implemented with care. One of the main concerns is pixelation. All non-linear searches are susceptible to local minima, which can often be non-physical. If the model one uses to compare with the data is
non continuous, the residual function has corresponding discontinu-
ities. These can look like minima to the non-linear search and give
erroneous results. To achieve a continuous model, we evaluate Equation 4.4.6 at a continuous point-set rather than only at pixel points.
We found that this removes any hops in the residual function that are
due to discretizing the space.

Another important consideration is which pixels to include in the
model. The number of pixels on the 2D image plane is dependent
on the orientation of the particle. If the model contains all points in
the 2D image plane for every orientation in the non-linear search,
not only will the intensity at each point change, but the particular
points as well as the total number of points in the model will change.
Since the model is what defines the behavior of the residual function,
this variability in its definition leads again to discontinuities and non-
physical minima. To resolve this issue, we evaluate the model only at
the points in the data. This makes sense because it is how the model
compares with the data that we are ultimately concerned with.

With both of these fixes to our zeroth order attempts implemented,
our algorithm finds the orientation of rods, crosses, and jacks very
consistently. In fact, these methods can, in principle, be adapted to
determining the orientation of arbitrarily shaped particles, which is
something that our group may explore in the near future. The orienta-
tions returned in this way can be compared with the data by project-
ing the endpoints of the arms in 3D onto the 2D plane and plotting
that on top of the data. (see Figure 12b). There are still rare times
when the orientation we find does not match the orientation that we
can tell by eye from the data. When measuring the orientation of jacks,
this corresponds to images that look like an “X” on each camera; the
third arm points perpendicular to the image plane. In plots of the Eu-
ler angles as a function of time, these rare anomalous fits correspond
Listing 3: Pseudeocode for Projection Algorithm

```plaintext
... 
eul_guess = euler(orientation_guess)
eul_final = non_linear_algorithm(@eul_leastsq(eul_guess, ...

function residual = leastsq(eul, ...)
  3D_endpoints = foo(eul, center, ...)
  2D_endpoints = Tsai_Projection(3D_endpoints)
  model_image = foo(2D_endpoints)
  residual = (model_image - data_image)^2
end
```

to an instantaneous small jump in an otherwise smooth trajectory. Figure 12 is actually an example of an orientation that is close to this situation. In this case, our algorithm did a good job of finding the right orientation. As we’ll discuss in more detail later, the jumps corresponding to being slightly off on the measurement of orientations like this produce erroneously high measurements of the rotation rate, but our method of measuring the instantaneous vorticity vector allows us to exclude these contributions from measurements of second moments and other descriptive properties of the rotational dynamics.

To apply this algorithm of orientation finding, we need a first guess. At the beginning of a track, it suffices to use tomography to generate an orientation. For the subsequent frames, we use the previous frame as the first guess at the orientation since the rotation is between frames is very small. We have experimented with other methods of finding a first guess at the orientation, but we have not yet developed a more reliable replacement.

### 4.5 Orientations and Their Symmetries

Symmetries in physics tend to simplify the problem we are trying to solve. In the present case of measuring orientations, the symmetry
of the object determines how careful one has to be in determining its Euler angles. Consider a jack, for example, which is equally well defined for 24 different rotations. If you choose a particular arm and define it by the vector \( \mathbf{p} = \hat{z} \), rotating by \( \pi/2 \) about that arm gives four equivalent definitions of the orientation of that jack. There are four more if you define \( \mathbf{p} = -\hat{z} \). If you consider the other cartesian unit vectors as well, it is easy to see that there are 24 different definitions of a single orientation for a jack - all of which a non-linear search is equally likely to find. Crosses and rods have similar issues.

While all of these definitions are conceptually equivalent, mathematically they set a different definition of \( \mathbf{p} \). This problem is of particular importance for crosses and rods, where there is only one definition of \( \mathbf{p} \) that is consistent with the description of rotation rate given by Jeffery’s Equation.

### 4.6 Vorticity Smoothing

Once we have measurements of orientation at each time frame, the next task is to determine the rotation rates along the trajectories. This goal is similar to measuring accelerations or velocities, except it is a higher order problem: we need a matrix at each time step. We set up the problem as follows. A rotation is linear over a certain time interval, \( \Delta t \), if the rotation between frames is constant during that interval. Making this assumption, for the orientation in every frame we find the best fit rotation matrix that reproduces the orientation in each of the frames \( \Delta t/2 \) to the left and \( \Delta t/2 \) to the right. The pseudocode for the core of this algorithm is shown in Listing 4.

Starting from our measurements of the orientation in each frame, we can obtain a first guess at the rotation rate by finding the rotation matrix, \( \Delta \), that takes \( \mathbf{O}_1 \) to \( \mathbf{O}_{\Delta t} \) and raising it to the power
Listing 4: Pseudeocode for Vorticity Smoothing Algorithm

% There are several measures that we use to identify errors in measuring the orientation. We exclude these subsets of the data from our measurements of the rotation rate.
...
eul_guess = euler(center_ori)
rot_guess = real((ori_end * ori_begin')^(1/delta_t))
vars = [eul_guess, rot_guess]
[eul_final, rot_final] = non_linear_algorithm(@vars leastsq(
vars, ...), ...)

function residual = leastsq(vars, ...)
R0 = ori(vars(1:3)) % orientation at center of track
Delta = vars(4:6)
fitseg = zeros(nseg,3,3)
datseg = zeros(nseg,3,3)
for i=1:nseg
    switch
        case tseg(i)>0
            % tseg holds time from center of each orientation
            fitseg(i,:,:,:) = Delta^tseg(i)*R0
        case tseg(i)<0
            fitseg(i,:,:,:) = (Delta')^tseg(i)*R0
        case tseg(i)==0
            fitseg(i,:,:,:) = R0
    end
    % oritation measured from projection algorithm
    datseg(i,:,:)=gg_ori(eulseg(i,:))
end
residual = sum(sum(sum((fitseg-datseg).^2)));

% Once we have obtained the rotation linearized over a fit range, it is a simple matter to compute the quantities of interest.
of $1/\Delta t$. This gives the average rotation rate across this interval. We then perform a nonlinear search in the components of $\Delta$ and $O_i$ to find the rotation matrix and the set of Euler angles that minimizes the residual, which is defined as the least-squares difference between the respective elements of the rotation matrices that correspond to the physical orientations within the times step $\Delta t$ found using our projection algorithm and the orientations predicted by linearizing the rotation rate.

The value obtained for the solid-body rotation rate, $\omega$, obviously depends on the fit length, $\Delta t$, used. When we measure any quantities that depend on the rotation rate, we must perform the vorticity smoothing at a number of different fit lengths. We discuss the procedure for doing this when we discuss our results in Chapter 5.
RESULTS

The rotational dynamics of anisotropic particles are governed by the velocity gradients in the fluid flow. We study these effects from a few perspectives. Experimental measurements of the mean square rotation rate, $\langle p_i p_i \rangle$, of rods and crosses differ from the randomly aligned case, which gives implicit evidence of their alignment with the velocity gradients in the flow. The measurement of second moments, while difficult, is important because it can be compared directly with analytical predictions about the statistical properties of turbulence. However, this is not the only way to obtain information about the correlation of particle orientation with the flow. Our measurements of the full solid-body rotation of jacks and crosses allow for a direct computation of the degree of alignment, by looking at the angle between $p$ and $\omega$, the solid-body rotation vector. These experimental measurements and data analysis techniques are the first of their kind. Our results are the first to verify predictions about the rotational dynamics of anisotropic particles of all aspect ratios in a turbulent flow and present an experimental methodology for the direct measurement of the degree to which a particle is aligned with the velocity gradient.

5.1 MEASUREMENTS OF THE SOLID BODY ROTATION OF ANISOTROPIC PARTICLES IN TURBULENCE

Lagrangian particle tracking is one of the most common experimental tools in turbulence studies. Standard techniques measure the positions and velocity of small tracer-sized spheres seeded into the tur-
Figure 13: Shown above is the time resolved, reconstructed trajectory and orientation of a jack in three dimensional turbulence. The length of the trajectory is $15\tau_\eta$. A solid black line shows the center trajectory of the particle as well as projections onto perpendicular planes for visualization.

bulent flow. By looking at anisotropic tracer particles, we can obtain direct information about the velocity gradients on the smallest scales of turbulence.

In Figure 13 we show an example jack trajectory. We have reconstructed the orientation of a Kolmogorov scale particle along a trajectory that is $15\tau_\eta$ long. An orientation sample is plotted every $\tau_\eta/2$, but we have resolved orientations down to $\tau_\eta/60$. This track is characteristic of the kind of motion we observe for jacks in turbulence. The jack makes a large sweep through the volume before making a tight curl in the upper right hand corner of the space. The wide range of orientations along this trajectory demonstrates the robustness of our algorithms for determining the orientation.

We can apply our vorticity smoothing algorithm to the measured Lagrangian orientations to measure the solid-body rotation rate, $\omega$, rate along the trajectory. This quantity, along with the instantaneous orientation, fully specifies the particles rotational dynamics. This is
very powerful because, with sufficient experimental accuracy, any statistic in turbulence that can be framed in terms of the solid-body rotation of a particle can be fully determined by applying the methods we present here. In Figure 14, we show the probability density function (PDF) of $\dot{p}_i\dot{p}_i$ for both jacks and crosses computer from our experimental data as well as from DNS data [11]. For values of the mean square rotation rate less than about $5$, our measured distributions agree quite well with the simulations. We expect the PDF for different aspect ratios to be almost identical for these values of $\dot{p}_i\dot{p}_i$. This is a characteristic feature of turbulence, where particle motion is typically very near to the mean values. Values of many times the mean are present in the PDF but are exponentially rarer with increasing $p_i\dot{p}_i/\langle p_i\dot{p}_i \rangle$. This appears in our experimental data as decreasing statistical convergence with increasing $\dot{p}_i\dot{p}_i$. We note that for values of $p_i\dot{p}_i$ further away from the mean, the experiment appears systematically higher than the simulations. This is related to the “jumps” we see in our measurements of the orientation when one of the arms of a jack or cross pass in front of or behind another arm. These jumps cannot be completely removed by our vorticity smoothing algorithm.
and will contribute spuriously high rotation rates. There is also a considerable amount of scatter in the points are larger $p_i$. This is due to poor statistical convergence and we are currently analyzing more of our data to resolve this particular issue.

5.2 MEASUREMENTS OF THE ALIGNMENT OF ANISOTROPIC PARTICLES BY TURBULENCE

Usually turbulence is thought of as a randomizing process and any particles carried along will have their orientations randomly oriented by the flow. However, the velocity gradients in turbulence actually have the net effect of aligning anisotropic particles. Simulations of rod-like particles have shown that they align strongly with the vorticity vector [29]. This can be qualitatively understood as a result of vortex stretching. The rate of change of the orientation, $\dot{p}$, can be rewritten to include a term proportional to the rate of change of the fluid vorticity vector and a second order viscous term. A rigorous quantification of this mechanism would require a better understanding of the viscous term, which has not yet been achieved.

Previous experimental work conducted by a graduate student in our lab has only been able to detect the effects of alignment indirectly by measuring a rotation rate that is smaller than the randomly aligned case. From our measurements of the orientation and full solid body rotation of jacks and crosses, we can directly measure the alignment of the particles with their axis of rotation, $\hat{\omega}$. In so doing, we pioneer an experimental technique to measure the net effect of the alignment of anisotropic particles by the vorticity and strain tensors. The cosine of the angle $\nu$ between $p$ and $\hat{\omega}$, is given by

$$\cos \nu = |p_i \hat{\omega}_i|,$$
5.2 Alignment of Anisotropic Particles by Turbulence

Figure 15: Shown above is a PDF of $\hat{p}_i\hat{\omega}_i$ for both jacks and crosses. Jacks (red) show very weak alignment with the velocity gradient. Crosses show strong anti-alignment in the $p_i$ direction (blue) and moderate alignment in the $p'_i$ direction (green). The fluctuations are a result of not quite reaching statistical convergence.

where $\nu$ ranges from 0 to $\pi/2$. This means that the alignment of a particle with the flow is determined by the component of solid body rotation about $p$, normalized by its total rotation. This information is lost for rods, but can be measured for both jacks and crosses. A particle whose orientation is randomly distributed will have a uniform distribution of $\cos \nu$, since the velocity gradients in the fluid are random. Deviations from a uniform PDF is the result of alignment or anti-alignment. Our measurements of $\cos \nu$ for crosses and jacks reveal a distinct difference in the rotational dynamics and alignment between the two shapes.

The PDF of $\cos \nu$ is shown in Figure 15 for both jacks and crosses. The largest contribution to the PDF of $p_{\text{cross}}$ comes from small values of $\cos \nu$. This tells us about the types of motion crosses typically undergo: they tend to rotate about one of the arms rather than about $p$. That is, $p$ tends to be perpendicular to $\hat{\omega}$ for crosses and they rotate like quarters spun on a table top rather than a frisbee tossed in the air. While this may seem non-intuitive at first, it makes some sense if we think about it in the context of what is already known about...
the rotational dynamics of rods [12, 11]. Rods align with the direction of stretching—they have a single preferred direction. Crosses can be thought of as two rods under the constraint of perpendicularity and a shared center (resistive force theory). The plane shared by the two constrained rods will tend to align with the velocity gradient (we would expect them to, again, align with the direction of vorticity), but their orientation within that plane will be random. The only remaining degree of freedom is rotation about the direction of alignment. If we define the unit vector along one of the arms of a cross as $\mathbf{p}_{\text{cross}}'$, we can also plot the PDF of $\cos \nu$ for this direction (see Figure 15). We see that the strongest contributions to the alignment of $\mathbf{p}_{\text{cross}}'$ come from values of $\cot \nu$ close to 1. This is simply a reformulation of the condition that $\mathbf{p}_{\text{cross}}$ is aligned perpendicularly with the flow.

A post doc in our lab has simulated cross trajectories using resistive force theory and DNS data that we have from our collaborators [11]. In Figure 16, we show our experimental data plotted along with the

Figure 16: Shown above is a PDF of $|\hat{p}_i \hat{\omega}_i|$ for both jacks and crosses. For both ($\mathbf{p}_\text{cross}'$) and ($\mathbf{p}_\text{cross}$), the experimental data and simulations show excellent agreement.
values expected from the simulations. For \( \cos \nu \) near one, the simulation and experiment agree very well. At small values of \( \cos \nu \), theory predicts a higher probability for \( p_{\text{cross}} \) than what appears in the experiment, while at values of \( \cos \nu \) near one theory predicts a slightly lower value. This is expected because of the experimental limitations of measuring orientations and the differences between them at those boundaries.

We can also look at the average value of \( \cos \nu \). This provides a single number that quantifies the degree of alignment. To the best of our knowledge, though there are many numerical studies of the alignment in turbulence, the quantity to which we are now turning our attention has not been studied in detail. Similar to measurements of second moments, like \( \langle p_i \dot{p}_i \rangle \), the value of \( \langle \cos \nu \rangle \) depends on the fit-length used to compute the derivative—in this case, \( \omega \). As one can see in Figure 17 the dependence here, though, is rather weak. This measurement perhaps makes the difference in alignment between jacks and crosses most evident. For jacks we measure \( \langle \cos \nu \rangle = 0.5 \pm 0.01 \), while for crosses we measure \( \langle \cos \nu \rangle = 0.25 \pm 0.02 \). The factor of two relation should be understood as purely incidental at this point—barring any future calculations. If there is an analytical relation between \( \langle \cos \nu \rangle \) and aspect ratio, it should be derivable from Jeffery’s Equation in combination with some assumptions about the properties of isotropic turbulence.

### 5.3 Measurements of the Second Moment of Rotation Rate

Second moment measurements are notoriously difficult. This is because they are sensitive to being skewed by noise and low statistics. To measure \( \langle \dot{p}_i \dot{p}_i \rangle \), we use a technique developed by Voth et al. [30]
Figure 17: Shown above is a plot of $\langle \cos \nu \rangle$ as a function of fit length. The difference between jacks and crosses is evident. The angle $\nu$ is pulled toward $\pi/2$ for crosses as they are perpendicularly aligned with the flow. $\tau_\eta \approx 30$ Half lengths. Dashed lines are to guide the eye.

and extended to measurements of $\langle \dot{p}_i \dot{p}_i \rangle$ conducted by a graduate student in our lab (details in[31]). To measure a rotation rate from discrete data, one has to perform a fit over a finite range. The value measured depends on the length of the fit. If there were no noise at all and the data resembled a pure function, the fit length could go to zero and rotation rate measurements would provide the physically correct value. However, this is never the case. Short fit lengths are skewed by noise in the data overestimate the rotation rate, while longer fit lengths will smooth over fast rotations and underestimate the rotation rate.

To make reliable measurements, we have to measure the rotation rate as a function of the fit length (see Figure 18). The longer fit lengths, though they predict too small of a rotation rate, will average over the noise. From previous work conducted by a graduate student our lab [11, 20, 31], we expect that the mean square rotation rate will follow an approximately exponential dependence in $\tau$. By fitting to the exponential tail and extrapolating back to a fit length of zero, we can measure $\langle \dot{p}_i \dot{p}_i \rangle$ with a minimum of noise.
Using this method, we measure the mean square rotation rate to be $10.2 \pm 1.0 \, \text{s}^{-2}$ for jacks and $13.9 \pm 1.0 \, \text{s}^{-2}$ for crosses. The flatness of our curves as a function of fit length (see Figure 18) demonstrates the accuracy of our measurements. The exponential tails of observed from measurements of mean square acceleration or rotation rate statistics in previous experimental work have been noticeably steeper $[30, 20]$. These measurements tease out the dependence of $\langle \dot{p}_i \dot{p}_i \rangle$ and are the first of their kind to span the space of ellipsoidal particles. To compare our results to numerical simulations, we first need to compute the energy dissipation rate. We showed in Section 2.4 that jacks are expected to rotate just with the vorticity. Our measurements of $\langle \cos \nu \rangle$ (see Figure 17 or Figure 16) suggest that this is indeed the case. Previous work on the dependence of the rotation rate variance on the length of rods suggest that we are only about 10% off the tracer limit in this case $[20, 10]$. Thus, solid body rotation rate squared of the jacks should be within 10% of the enstrophy of fluid, or the square of the fluid vorticity. We can then use the well known result of isotropic turbulence, $\langle S_{ij} S_{ij} \rangle = \langle \Omega_{ij} \Omega_{ij} \rangle$, to estimate the energy dissipation rate in our flow. Our measurement of mean square solid body rotation rate follows the same procedure as finding the mean square
rotation rate, so we won’t explicate it here. The value that we measure is $15.5 \pm 1.0 \text{s}^{-2}$ We give the Kolmogorov scales according to this value for $R_\lambda = 110$ in Table 2 back in Chapter 3.

Now we can compare our measurements of $\langle \dot{p}_i \dot{p}_i \rangle$ for crosses and jacks to direct numerical simulations and previous experiments on the rotation rate of rods performed by a graduate student in our lab. In Figure 19, we present what is milestone in the field of particle tracking in turbulence: the mean square rotation rate for rigid bodies spanning the range of aspect ratios. Despite the strong alignment of crosses with the velocity gradients in the flow, their mean square rotation rate differs from the random alignment case to a lesser degree than do the rods. This agrees with our understanding of the kinds of motion crosses undergo in turbulence. If the plane of the cross is aligned with the velocity gradient, $p$ is still free to rotate in any direction within a perpendicular plane. This is very different from the way that rods align because rod alignment effectively reduces $\langle \dot{p}_i \dot{p}_i \rangle$ for the duration of the alignment event.

![Figure 19](image_url)

Figure 19: Shown above is a plot of $\langle \dot{p}_i \dot{p}_i \rangle$ as a function of the aspect ratio. Our new measurements agree well with the simulation data. This represents a first-order understanding to the rotational dynamics of all solid body particles in turbulence.
CONCLUSIONS

We have demonstrated that it is possible to accurately measure the Lagrangian rotation rate of anisotropic particles of various shapes near the Kolmogorov scale in turbulent flows. The methods that we present for doing so are general and can be extended to measure the orientation of particles of more general shapes. 3D printing provides a means for fabricating particles whose shapes is only limited by the imagination and whose size is limited by the technology frontier—which is allowing for ever smaller scales. Going beyond the proof of principle, we have measured the rotation rate variance for sphere- and disk-like particles to complete what amounts to a very detailed sketch of the rotational dynamics of solid bodies in turbulence.

6.1 CONTINUED DATA ANALYSIS

We have presented in this thesis only a small fraction of the data that we have available from the experiments we took this past summer. We have yet to analyze all data sets for the experiment at $R_\lambda \approx 110$ and have not begun to analyze data at the higher $R_\lambda$. Completing this analysis will be the main priority over the coming months.

Our methods for measuring the rotation rate were conceived for particles where you can measure the full solid body rotation vector. As such, the problem is over specified when the techniques are applied to rods without modification. We have determined away to avoid these issues without circumventing the functional core of the vorticity smoothing analysis. This is important for maintain its gen-
erality as well to our understanding of any issues with our experimental analysis. The graduate student in our lab who has directed the effort on the measurements of rotational dynamics of rods is also analyzing our rod data using her methods. Comparison between our independent results will help to the degree of accuracy of our new methods.

I have been collaborating with a post doc in our lab who is obtaining information about the \( \cos \nu \) of crosses from DNS data. As I analyze the rest of the experimental data, we will continue to compare notes on our findings. Preliminary results suggest that the qualitative features of the PDF of \( \cos \nu \) are the same between simulation and experiment, but we are still working on this part of the analysis. These results will be very interesting because in the DNS we have access to the eigenvectors of strain and the fluid vorticity in addition to all the solid body rotation vector and orientation of the particle. Thus, we can determine to what degree the alignment of the orientation with the solid body rotation is due to the fluid vorticity and to what degree the strain field is also contributing. This will also help to inform future experiments in our lab that aim to measure the Lagrangian orientation and rotation rate of anisotropic particles simultaneously with the full velocity gradient tensor of the fluid.

To accompany our measurements of \( \langle p, p_i \rangle \) for crosses, we are also in the midst of nailing a down measurement for \( \langle p', p_i' \rangle \). A preliminary estimate puts the mean square rotation rate of \( p' \) in the range \([0.9, 1.5]\), which is on par with \( \langle p, p_i \rangle \) for particles with \( \alpha \gg 1 \). A first glance at a Figure 19 might suggest that rods are much more strongly aligned with the flow because the rotation rate differs most form the randomly aligned case. However, a measurement of \( \langle p', p_i' \rangle \) for crosses roughly equal to \( \langle p, p_i \rangle \) for rods suggests that the difference in rotation rate is due to the definition of \( p \) rather than a fundamental
difference in the kind of alignment crosses experience in turbulence as opposed to rods. Exploring and understanding the connections between rods and crosses will be an interesting question to answer as we continue to analyze our data.

To produce Figure 19, we made an estimate of the Kolmogorov time using the solid body rotation of jacks and the assumption of homogeneous and isotropic turbulence. This is a very good approximation, although it is of course not strictly true. We can check the accuracy of this estimate and perhaps improve on its accuracy by using an alternate method to measure the Kolmogorov time. The auto-correlation function of the jack vorticity should have a zero-crossing at a lag time equal to the eddy turnover time-scale corresponding to the size of the particle (this builds on some concepts introduced in [20]). In the coming weeks, we will investigate this method as a diagnostic for other measurements of $\tau_\eta$.

6.2 IMPROVEMENTS TO EXPERIMENTAL AND ANALYSIS METHODS

During the process of data analysis, we have recognized a number of ways that our methods could be improved. Some of these we have implemented along the way, but the others I will discuss here.

The vorticity smoothing algorithm is effective, but the use of a non-linear search is inefficient. The problem should be straightforward to specify as a set of linear equations which can be solved for the solid body rotation rate. We have begun to work on the formal aspects to reformulate the problem in this way because there are two important benefits. The first is speed. Computers are much better at performing linear operations than a nonlinear algorithm—especially in MATLAB. Secondly, a linear algebra formulation is much more robust and has
no possibility of being caught in a local minima. As such, its patholo-
gies are easier to understand and span a smaller parameter space.

One of the largest hurdles in our analysis is how to deal with times when we fail to accurately measure an orientation like the one in Figure 12, where there may be a small set of orientations that have close to the same residual. One approach to dealing with this problem is to modify the model intensity function. A preliminary idea is to find a way to choose the intensity of the model for each arm individually, which should help to create sharper minima in the non-linear search. One implementation is to have a second non-linear search after the first one has found a (very nearly!) reliable orientation. Then, we can take the average value of the pixels along each of the arms and re-iterate. A second implementation is to make the model intensity a function of the angle, \(\theta\), between the unit vector defining the 2D image plane and the unit vector defining the orientation of each arm. It is not immediately obvious what specific functional form to use, but a qualitative assessment of the data suggests that the intensity is greater for small \(\theta\).

However, there is a more radical approach also, which seems like it should be the ultimate direction that we should head. At this point, the orientation and solid body rotations are determined at different stages. In fact, the orientation is found twice because that is required to compare the results of the vorticity smoothing to the projection algorithm. It seems that the most accurate algorithm would simultaneously measure the solid body rotation and orientation, using the data to compute a least squares residual. The solid body rotation rate is more resilient to the frames with difficult to measure orientations because it inherently smooths over several frames and forces a continuous rotation. This could also have the added benefit of being able
to linearize the entire problem of calculating Lagrangian orientation and solid body rotation rates.

Another barrier in our code is our first guess at the orientation. Our projection algorithm has proved robust, but our first guess at the orientation of a particle is rather crude. A post doc in our lab has experimented with ways to find the tips of particles in 2D. With this method, it may be possible to reduce the problem of finding the orientation is reduced to typical particle tracking experiments with tracer particles. It is not obvious whether a method of this designed would be able to compare the techniques we have developed since the freedom of the points on the arms makes it more susceptible to noise, but it would certainly be sufficient to find a first guess.

6.3 Future Extensions

There a number of interesting projects that one could pursue related to anisotropic particles. I will highlight here the ones that I think are most promising.

One of the recent realizations in turbulence is that certain properties of the small scales of turbulence can be dependent on the intermittency of energy input and anisotropies on the largest scales [32]. Experimental measurements of the role that large scale plays on the dynamics of Kolmogorov scale anisotropic particles would provide a new way to look at physics that is still no completely understood.

Recently, experimental progress has been made in another component of fluid dynamics in general that also has an important role in turbulence: vortex tubes and, more specifically, vortex knots [33]. Kleckner et al. are able to generate and image complex vortex structures on a macroscopic scale using high speed video. It is known that vortex tubes have a central role in turbulence and are related to the
enstrophy production. Using a considerably higher seeding density than we chose for our experiments, it should be possible to image the formation and evolution of vortex tubes in turbulences by looking at the collective dynamics of jacks. These techniques could even be applied to systems like those in [33], where the goal is to understand the complex topology of vortex knots.

The broad applications of anisotropic particles make this an exciting time to be a soft condensed matter physicist. I think the most important extension of the techniques we have developed is their application to more general objects. There has been a lot of recent interest in active particles and biological swimmers. In principle, the techniques we have developed can be generalized in their non-linear form to measure the time resolved orientational dynamics of an arbitrary dynamic shape, which encompasses both of the aforementioned systems. In a different vein, some recent theoretical work has shown that there are some particles which can stop rotating in a simple shear flow. [34] A description of their rotation rate in the more general flow fields found in turbulence is outside of the scope of Jeffery’s Equation, but may still be accessed by experiment. Moreover, particles outside the domain of Jeffery’s Equation are a particularly interesting class to study experimentally. Inspired by the design of particles to perform a specific function [18, 34], I pose the question: is it possible to design a multi-arm composite particle that follows, for example, the eigen-frame of strain as at changes in time and space? The implicit question here is: can we discover any fundamental symmetries of turbulence? We have already found one in the symmetry of a jack (sphere). However, other interesting physics with fundamental implications might be found in the dynamics of chiral particles, where the arms are of different lengths, or in other non-axisymmetric particles, which would be of interests to theorists, experimentalists, and engi-
neers alike. A theoretical description of the rotation rates of these particles would require a generalization of Jeffery’s equations. Our simple arguments based on resistive force theory may provide a starting point for a more general, formal calculation of the rotation rate of particles composed of an arbitrary number of arms in an arbitrary configuration. Recent work has shown that measures of particle chirality can be dependent on the particle orientation \[35\]. Thus, simultaneous measurements of orientation, velocity, and solid-body rotation would be required. Moreover, since some DNS data shows that chiral flow structures (e.g., Lagrangian coherent structures) are expected to have fundamental implications for dispersion in isotropic turbulence \[36\], understanding how chiral particles behave in turbulence could provide an experimental foothold.

6.4 CLOSING REMARKS

In this thesis, we have developed a general methodology for measuring the time-resolved Lagrangian orientation and solid body rotation rate of anisotropic particles in a turbulent flow. Our measurements of the alignment of crosses with the direction of their solid body rotation rate vector are the first direct observation of the alignment of anisotropic particles by the velocity gradients in the flow. Our measurements of jacks provide a way to directly probe the vorticity of the fluid and vortex structure in the flow. Our measurements of the mean square rotation rate spanning the range of aspect ratios agrees with DNS data of the same quantity and represents a cohesive understanding of the rotational dynamics of anisotropic particles in turbulence. This foray into the realm of anisotropic particles opens a wide range of possibilities for new developing new experimental techniques for probing the smallest scales of turbulence—and brings us that much
closer to solving one of the longest standing problems in classical physics.


DECLARATION

This thesis is a presentation of my original research work. Wherever contributions of others are involved, every effort is made to indicate this clearly, with due reference to the literature, and acknowledgement of collaborative research and discussions.

Middletown, CT, December 2011

Guy Geyer